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HALL EFFECTS ON HYDROMAGNETIC CONVECTIVE FLOW IN A ROTATING CHANNEL

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Hall effects on combined free and forced convective flow of a viscous incompressible electrically conducting fluid between the horizontal perfectly conducting plates under the action of a uniform transverse magnetic field applied parallel to the axis of rotation, is studied. Exact solution of the governing equation is obtained in closed form. It is observed that Hall current exerts stabilizing influence on the primary flow at the upper plate due to shear stress while at the lower plate Grashof number causes separation on the secondary flow. The rate of heat transfer at both the plates are derived. It is found that Hall current and rotation exert reverse flow of heat at the upper plate when the numerical value is equal to two as the Grashof number is being referred.

1. INTRODUCTION

The theory of rotating fluid is highly important in various technological situations determine the behaviour of conducting fluid with low Prandtl number to which the interaction between electromagnetic force to coriolis force is subjected to the action to modify the mechanical behaviour of the system. Mazumder *et al.*[1], Datta and Jana[2] and Seth and Ghosh[3] investigated the combined effects of free and forced convection flow with Hall effects in a non-rotating system neglecting induced magnetic field under different conditions. However, the influence of such fluid flow problem which lie in their application of geophysical and astrophysical interest, is the study of a steady free and forced convection flow with Hall effects in a rotating system, has not received attention in literature where induced magnetic field is taken into account.

In the present paper, we consider the effect of Hall current on the combined free and forced convection flow of an electrically conducting viscous incompressible fluid between two horizontal perfectly conducting plates rotating with an uniform angular velocity about an axis normal to their planes under the action of a uniform transverse magnetic field applied parallel to the axis of rotation. Exact solution of the governing equations for the fully-developed flow is obtained in closed form. The solution in dimensionless form contains four flow parameters viz. M^2 (the square of the Hartmann number), K^2 (the rotation parameter), G (Grashof number) and m (the

Hall parameter). Asymptotic behaviour of the solution is analysed for large values of K^2 and M^2. The shear stress at both the plates due to primary and secondary flows are derived. The rate of heat transfer at both the plates are presented numerically. It is found that there arise flow reversals in the primary as well as secondary flow directions for $G \neq 0$ while Hall current and rotation exert a destabilizing influence on the primary flow whereas the rotation has a stabilizing influence on the secondary flow. Also it is noticed that there is a reverse flow of heat at the upper plate on increasing m, K^2 for $G = 2$.

2. Formulation of the Problem and its Solutions

Consider the steady fully-developed combined free and forced convection flow of an electrically conducting viscous incompressible fluid between two infinite horizontal perfectly conducting parallel plates $y = \pm L$ under the influence of a constant pressure gradient acting along x-axis and a uniform transverse magnetic field H_0 applied parallel to y-axis about which both the fluid and plates are in a state of rigid body rotation with uniform angular velocity Ω. The plates are cooled or heated by a constant temperature gradient along the x-direction so that the temperature varies linearly along the plate.

Since the plates are infinite along x and z directions all physical quantities except pressure will be the function of y only. It may be easily shown that the following assumptions are compatible with the fundamental equations of magnetohydrodynamics.

$$q = (u', 0, w'), H = (H_x', H_0, H_z'). \qquad \text{... (2.1)}$$

Under the assumptions (2.1) which correspond to the fundamental equations of magnetohydrodynamics in a rotating frame of reference, the equation of momentum and the Ohm's law for a moving conductor taking Hall current into account

$$(q \cdot \nabla)q + 2\Omega \times q = \frac{1}{\rho} \nabla p + \nu \nabla^2 q + \frac{\mu_e}{\rho} J \times H + g \; \{1 - \beta(T - T_0)\} \; \hat{K} \qquad \text{... (2.2)}$$

$$J + \frac{\omega_e \tau_e}{H_0} (J \times H) = \sigma[E + \mu_e \, q \times H] \qquad \text{... (2.3)}$$

where q, E, J and H are respectively, the velocity vector, electric field vector, current density vector and magnetic field vector. $\rho, \nu, \mu_e, p, \sigma, \omega_e, \tau_e, g, \beta, T$ and T_0 are, respectively, the fluid density, kinematic coefficient of viscosity, magnetic permeability, modified pressure including centrifugal force, electrical conductivity, cyclotron frequency, electron collision time, gravity, the coefficient of thermal expansion, the fluid temperature and the temperature in the reference state. $\hat{K}$ is the unit vector along y-axis.

Assuming uniform axial temperature variation along the channel walls, the fluid temperature may be considered as

$$T - T_0 = Nx + \phi(y). \qquad \text{... (2.4)}$$

Under the assumption (2.1), taking y-component on integrating the momentum equation (2.2), reduces to

$$P = -\rho g + \beta g \int (T - T_0)\, dy - \frac{1}{2}(H_x'^2 + H_z'^2) + F(x) \quad \text{... (2.5)}$$

Combining eqns. (2.2) and (2.3) with the help of eqn. (2.5) in dimensionless form, we obtain

$$\frac{d^2F}{d\eta^2} + M^2 \frac{dh}{d\eta} - G\eta = -1 - 2i\,K^2F, \quad \text{... (2.6)}$$

$$\frac{d^2h}{d\eta^2} + \frac{1}{(1+im)}\frac{dF}{d\eta} = 0, \quad \text{... (2.7)}$$

where $\eta = y/L,\ u_1 = u'\,L/\nu\, P_x,\ W_1 = W'\,L/\nu\, P_x$,

$$H_x = H_x'/\sigma\,\mu_e\,\nu\, H_0\, P_x,\ H_z = H_z'/\sigma\,\mu_e\,\nu\, H_0\, P_x, \quad \text{... (2.8)}$$

$P_x = L^3\left(-\frac{dF}{dx}\right)/\rho\,\nu^2$, $G = g\beta\, NL^4/\nu^2 P_x$ is the Grashof number,

$M = \mu_e H_0 L(\sigma/\rho\nu)^{\frac{1}{2}}$ is the Hartmann number, $K^2 = \Omega L^2/\nu$ is the rotation parameter which is reciprocal of Ekman number, $m = \omega_e \tau_e$ is the Hall current parameter, $F = u_1 + iw_1$ and $h = H_x + iH_z$.

Equation (2.4) shows that positive or negative values of N correspond to heating or cooling along the channel walls. Considering $P_x > 0$, it follows from the definition of G that G is less than or greater than 0 according as the channel walls are heated or cooled in the axial direction.

The boundary conditions for the velocity field are

$$F = 0 \quad \text{at} \quad \eta = \pm\, 1. \quad \text{... (2.9)}$$

Since the plates are perfectly conducting, the boundary conditions for the magnetic field are

$$\frac{dh}{d\eta} = 0 \text{ at } \eta = \pm\, 1. \quad \text{... (2.10)}$$

Since the channel is symmetric at $\eta = 0$, the boundary condition for the magnetic field at $\eta = 0$ may be assumed as (see Nanda and Mohanty[4])

$$h = 0 \text{ at } \eta = 0. \quad \text{... (2.11)}$$

Equations (2.6) and (2.7) together with the boundary conditions (2.9) to (2.11) can be solved and the solution for the velocity and induced magnetic field are

$$F(\eta) = \frac{1}{m_1^2}\left[\left\{1 - \frac{\cos h\, m_1\eta}{\cos hm_1}\right\} + G\left\{\frac{\sin hm_1\,\eta}{\sin hm_1} - \eta\right\}\right] \quad \text{... (2.12)}$$

$$h(\eta) = \frac{(m_1^2 - 2iK^2)}{m_1^2 M^2}\left[\frac{1}{m_1}\left\{\frac{\sin hm_1\eta}{\cos hm_1} + \frac{G(1-\cos hm_1\eta)}{\sin hm_1}\right\} - \eta\left(1 - \frac{G}{2}\eta\right)\right], \quad \dots (2.13)$$

where $m_1 = \alpha - i\beta$,

$$\alpha = \frac{1}{\sqrt{2}}\left[\left\{\frac{M^4}{1+m^2} + \frac{4m}{1+m^2}M^2K^2 + 4K^4\right\}^{\frac{1}{2}} + \frac{M^2}{1+m^2}\right]^{\frac{1}{2}}, \quad \dots (2.14a)$$

$$\beta = \frac{1}{\sqrt{2}}\left[\left\{\frac{M^4}{1+m^2} + \frac{4m}{1+m^2}M^2K^2 + 4K^4\right\}^{\frac{1}{2}} - \frac{M^2}{1+m^2}\right]^{\frac{1}{2}}. \quad \dots (2.14b)$$

Shear Stress at the plates $\eta = \pm 1$

The non-dimensional shear stress components τ_x and τ_z at the plates $\eta = \pm 1$ due to primary and secondary flows, respectively, are

$$\tau_x = \frac{1}{(\alpha^2+\beta^2)}\left[\mp\left(\frac{\alpha\sin h\,2\alpha + \beta\sin 2\beta}{\cos h\,2\alpha + \cos 2\beta}\right) + G\left(\frac{\alpha\sin h\,2\alpha - \beta\sin 2\beta}{\cos h\,2\alpha - \cos 2\beta} - \frac{\alpha^2-\beta^2}{\alpha^2+\beta^2}\right)\right], \quad \dots (2.15)$$

and

$$\tau_z = \frac{1}{(\alpha^2+\beta^2)}\left[\mp\left(\frac{\beta\sin h\,2\alpha - \alpha\sin 2\beta}{\cos h\,2\alpha + \cos 2\beta}\right) + G\left(\frac{\beta\sin h\,2\alpha + \alpha\sin 2\beta}{\cos h\,2\alpha - \cos 2\beta} - \frac{2\alpha\beta}{\alpha^2+\beta^2}\right)\right]. \quad \dots (2.16)$$

The upper and lower signs in the first term of eqns. (2.15) and (2.16) correspond to the values at the upper plate $\eta = 1$ and that at the lower plate $\eta = -1$ respectively.

It may be noted from (2.15) and (2.16) that the shear stress components τ_x and τ_z due to primary and secondary flows respectively, vanish neither at the upper plate nor at the lower plate and depend on the Hartmann number M, rotation parameter K^2 and Hall parameter m when $G = 0$. Thus it concludes that for perfectly conducting plates there is no flow reversal when $G = 0$.

Asymptotic Solutions

Case I : $K^2 >> 1$ and $M^2 \sim O(1)$ — Since K^2 is very large and M^2 and m are small orders of magnitude, it can expect boundary layer type flow. For the boundary layer at the upper plate $\eta = 1$, writing $(1 - \eta) = \xi$, we obtain from (2.12) and (2.13).

$$u_1 = \frac{e^{-\alpha\xi}}{2K^2}(1-G)\sin\beta\xi, \quad \dots (2.17)$$

$$w_1 = \frac{1}{2K^2}[(1-G\eta)+e^{-\alpha\xi}(G-1)\cos\beta\xi], \qquad \text{... (2.18)}$$

$$H_x = \frac{1}{M^2}\left[1/K\ \{e^{-\alpha\xi}(1+G)(\cos\beta\xi-\sin\beta\xi)\} - \eta\left(1-\frac{G\eta}{2}\right)\left(2+\frac{1-m}{2(1+m^2)K^2}\right)\right], \qquad \text{... (2.19)}$$

$$H_z = \frac{e^{-\alpha\xi}}{M^2K}(1+G)(\sin\beta\xi+\cos\beta\xi), \qquad \text{... (2.20)}$$

where

$$\alpha = K\left\{1+\frac{(1+m)M^2}{4(1+m^2)K^2}\right\},\quad \beta = K\left\{1+\frac{(1-m)M^2}{4(1+m^2)K^2}\right\}. \qquad \text{... (2.21)}$$

It is evident from the expressions (2.17) to (2.20) that there arise boundary layer of thickness $O(\alpha)^{-1}$ which decreases with the increase in either K^2 or M^2. This boundary layer may be identified as modified hydromagnetic Ekman layer.

The exponential terms in eqns. (2.17) to (2.20) damp out quickly on ξ increases. When $\xi \geq 1/\alpha$, we have

$$u_1 \approx 0,\ w_1 \approx (1-G\eta)/2K^2, \qquad \text{... (2.22)}$$

$$H_x \approx -\frac{\eta}{M^2}(1-G\eta/2)\left(2+\frac{1-m}{2(1+m^2)K^2}\right),\ H_z \approx 0. \qquad \text{... (2.23)}$$

The expressions (2.22) and (2.23) show that in a certain core, given by $\xi \geq 1/\alpha$ about the axis of the channel, the velocity in the direction of pressure gradient given by u_1 and the induced magnetic field H_z vanish away while the velocity in the secondary flow direction given by w_1 and the induced magnetic field H_x persist. Also the velocity w_1 is unaffected by the Hall current and magnetic field. The velocity w_1 and primary induced magnetic field H_x vary linearly with η and the effect of Grashof number on the velocity and induced magnetic field become insignificant in the central region.

Case II : $M^2 >> 1$ and $K^2 \sim O(1)$ — In this case also boundary layer type flow is expected. For the boundary layer at the upper plate we obtain from (2.12) and (2.13)

$$u_1 = \frac{1}{M^2}[(1-G\eta)+(G-1)e^{-\alpha\xi}(\cos\beta\xi+m\sin\beta\xi)], \qquad \text{... (2.24)}$$

$$w_1 = \frac{1}{M^2}[-m(1-G\eta)+(G-1)e^{-\alpha\xi}(\sin\beta\xi-m\cos\beta\xi)], \qquad \text{... (2.25)}$$

$$H_x = \frac{1}{M^2}\left[\frac{(1+G)}{M^2}e^{-\alpha\xi}(\alpha\cos\beta\xi-\beta\sin\beta\xi)-\eta\left(1-\frac{G\eta}{2}\right)\right], \qquad \text{... (2.26)}$$

$$H_z = \frac{(1+G)}{M^4} e^{-\alpha\xi} (\alpha \sin \beta\xi + \beta \cos \beta\xi), \quad \text{... (2.27)}$$

where

$$\alpha = \frac{M}{\sqrt{1+m^2}}, \quad \beta = \frac{mM}{2\sqrt{1+m^2}} \quad \text{... (2.28)}$$

The expressions (2.24) to (2.27) demonstrate the existence of a boundary layer of thickness $O(\alpha)^{-1}$ which depends on both the Hall current and magnetic field. The thickness of this layer decreases with the increase in M^2 while it increases with the increase in Hall parameter m. This boundary layer may be identified as modified Hartmann boundary layer. In the central core given by $\xi \geq 1/\alpha$ about the axis of the channel, the velocity field and magnetic field become

$$u_1 = (1 - G\eta)/M^2, \; w_1 = -m(1 - G\eta)/M^2, \quad \text{... (2.29)}$$

$$H_x = -\eta\left(1 - \frac{G\eta}{2}\right)/M^2, \; H_z = 0. \quad \text{... (2.30)}$$

It is evident from the expressions (2.29) and (2.30) that in the central region the secondary velocity is weak in comparison to the primary velocity. In the absence of Hall current the secondary velocity w_1 vanishes away and the fluid will be moving in the direction of pressure gradient only. Also both the velocity and the induced magnetic field H_x vary linearly with η and the effect of Grashof number on the velocity and induced magnetic field become insignificant.

Heat Transfer Characteristics

The energy equation for the fully-developed flow including viscous and Joule dissipations, reduces to

$$u \frac{\partial(T - T_0)}{\partial x} = \frac{K}{\rho c_p} \frac{d^2(T - T_0)}{dy^2} + \frac{\mu}{\rho c_p}\left\{\left(\frac{du'}{dy}\right)^2 + \left(\frac{dw'}{dy}\right)^2\right\}$$
$$+ \frac{1}{\rho c_p \delta}\left\{\left(\frac{dH'_x}{dy}\right)^2 + \left(\frac{dH'_z}{dy}\right)^2\right\}, \quad \text{... (2.31)}$$

where the fluid temperature T is a function of y only and other symbols have their usual meanings.

Using non-dimensional variables (2.8) and introducing dimensionless quantities

$$\theta(\eta) = \frac{\phi(y)}{NL\,P_x}, \; K_1 = \frac{\nu^3 P_x}{KNL^3} \text{ and } P_r = \mu\, c_p/K, \quad \text{... (2.32)}$$

in eqn. (2.31) we obtain

$$\frac{d^2\theta}{d\eta^2} = -K_1\left[\left\{\left(\frac{du_1}{d\eta}\right)^2 + \left(\frac{dw_1}{d\eta}\right)^2\right\} + M^2\left\{\left(\frac{dH_x}{d\eta}\right)^2 + \left(\frac{dH_z}{d\eta}\right)^2\right\}\right] + P_r u_1. \quad \text{... (2.33)}$$

The boundary conditions become

$$\theta(-1) = 0 \text{ and } \theta(1) = \frac{\phi(1)}{NLP_x} = N_1, \text{ (say)}, \quad \text{... (2.34)}$$

where N_1 is the temperature at the upper plate. Substituting the values of u_1, w_1, H_x and H_z from (2.12) and (2.13) in eqn. (2.33) and solving the resulting differential equation subject to the boundary conditions (2.34), the solution for $\theta(\eta)$ may be represented as

$$\theta(\eta) = P_r\{\phi_1(\eta) + \phi_2(\eta) - A_1\eta - A_4\} - K_1\ \{\phi_3(\eta) + \phi_4(\eta) + \phi_5(\eta) + \phi_6(\eta) + A_2\eta - A_3\eta - A_5 - A_6 - A_7\} + \frac{N_1}{2}(1+\eta) + \phi_7(\eta), \quad \text{... (2.35)}$$

where $\phi_i(\eta)$ (i = 1, 2, ..., 7) are functions of M, K^2, m, P_r and η. A_i (i = 1, 2, ..., 7) are functions of M, K^2 and m.

The expressions of rate of heat transfer at both the plates i.e. $(d\theta/d\eta)\,\eta = \pm 1$ are also derived. We omit these expressions because of quite lengthy.

3. Discussion of Results

The numerical solutions of the velocity and the induced magnetic field are presented graphically versus η for various values of m^2 taking G, K^2 and M fixed in Figs. 1 and 2. It is evident from Fig. 1 that for $G > 0$, the primary and secondary velocities change its direction as it move away from the upper half to the lower half of the channel and the velocity attains its maximum near the lower plate of the channel. Thus the free convection causes flow reversal in both the direction. It is observed from Fig. 1 that the primary velocity u_1 is of oscillatory nature in the region $-1 \le \eta \le 0.25$ as m increases whereas it decreases with the increase in m for $0.35 \le \eta \le 1$ and the secondary velocity w_1 increases in the region $-1 \le \eta \le 0.4$ with the increase in m and, as changes its direction near $\eta = 0.6$, again increases with the increase in m. In this case there arise a flow reversal in the primary flow direction when $\eta \ge 0.35$ while for secondary flow it appears $\eta \ge 0.55$. Fig. 2 reveals that the induced magnetic field H_x increases numerically with the increase in m while the induced magnetic filed H_z increases numerically near the lower plate and decreases in magnitude in the region $-0.5 < \eta < 0.7$ and again it increases in magnitude near the upper plate.

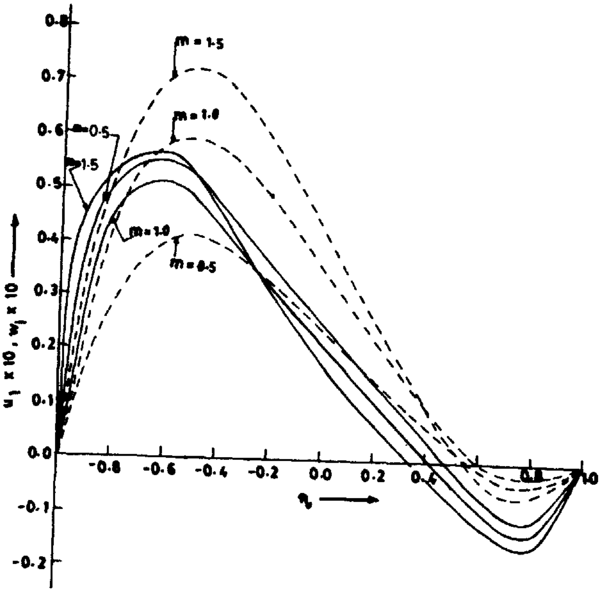

FIG. 1. Velocity component (— $10u_1$, - - - $10w_1$) against η for $G = 2$, $M = 5$, $K^2 = 5$.

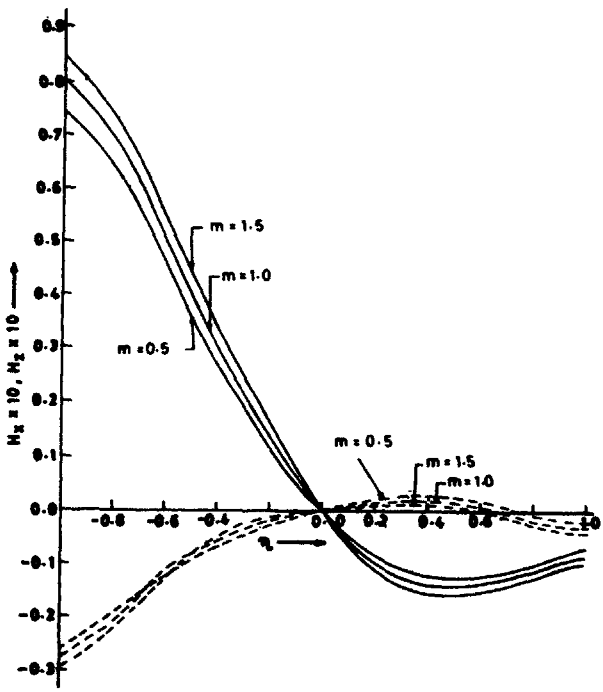

FIG. 2. Magnetic field components (— $10H_x$, - - - $10H_z$) against η for $G = 2$, $M = 5$, $K^2 = 5$.

TABLE I

Shear stress τ_x and τ_z at $\eta = 1$ for $M = 5$ and $K^2 = 3$

G	τ_x			τ_z		
m^2	0.5	1.0	1.5	0.5	1.0	1.5
0.0	−0.18648	−0.18688	−0.19111	− 0.06542	− 0.09938	− 0.12522
2.0	0.12541	0.13681	0.14989	− 0.11420	− 0.17360	− 0.22086
4.0	0.43729	0.46050	0.49091	− 0.16298	− 0.24782	− 0.31649
6.0	0.74918	0.78419	0.83192	− 0.21176	− 0.32205	− 0.41214

TABLE II

Shear stress τ_x and τ_z at $\eta = -1$ for $M = 5$ and $K^2 = 3$

G	τ_x			τ_z		
m^2	0.5	1.0	1.5	0.5	1.0	1.5
0.0	0.18648	0.18688	0.19111	0.06542	0.09938	0.12522
2.0	0.49836	0.51057	0.53212	0.01663	0.02515	0.02958
4.0	0.81025	0.83426	0.87314	− 0.03217	− 0.04906	− 0.06605
6.0	1.12214	1.15796	1.21415	− 0.08092	− 0.12529	− 0.16169

The numerical results of shear stress components τ_x and τ_z at both the plates due to primary and secondary flows, respectively are presented in Tables I and II for various values of G, m^2 taking $M = 5$ and $K^2 = 3$. Table I shows that the shear stress component τ_x at $\eta = 1$ due to primary flow increases with the increase in either m^2 or G while the shear stress component τ_z at $\eta = 1$ increases in magnitude with the increase in either m^2 or G. Table II shows that the shear stress component τ_x at $\eta = -1$ due to primary flow increases with the increase in either m^2 or G while the shear stress component τ_z at $\eta = -1$ increases with the increase in G for fixed m^2 whereas it decreases with the increase in m^2 for fixed value of G. Also it is noticed that there arise separation of flow on increasing G for fixed value of m^2.

The rate of heat transfer at both the plates i.e. $(d\theta/d\eta)\eta = \pm 1$ are presented in Table III, for various values of m and G taking $M = 5$, $K^2 = 5$, $N_1 = 0.5$, $K_1 = 1$ and $P_r = 0.25$. It is observed from Table III that the rate of heat transfer $(d\theta/d\eta)_{\eta=1}$ decreases with the increase in either m of G whereas the rate of heat transfer $(d\theta/d\eta)_{\eta=-1}$ increases with the increase in G. Also it increases with the increase in m for $G = 0$ (forced convection) and is of oscillatory nature on increasing m for $G \neq 0$. It is noticed that there arise reverse flow of heat at the upper plate on increasing m for $G = 2$. Thus we conclude that the rotation, Hall current and heat transfer by free convection induce reverse flow of heat at the upper plate.

TABLE III

The rate of heat transfer $\left(\frac{d\theta}{d\eta}\right)_{\eta=\pm 1}$ *for* $M = 5$ *and* $K^2 = 5$

G	$(d\theta/d\eta)_{\eta=1}$				$(d\theta/d\eta)_{\eta=-1}$			
m^2	0	0.5	1.0	1.5	0	0.5	1.0	1.5
0	0.16203	0.14530	.12817	.11252	.33797	.35469	.37183	.38747
2	.06223	.00335	–.01608	–.04228	.056297	.58753	.52640	.54701
4	–.22873	–.39892	–.45460	–.53436	.97919	1.08069	.97523	1.04382
10	–1.02249	–3.16775	–3.53574	–4.03421	3.37513	4.12218	4.08732	4.55787

REFERENCES

1. B. S. Mazumder, A. S. Gupta and N. Datta, *Int. J. Engng. Sci.* **14** (1976), 285.
2. N. Datta and R. N. Jana, *Int. J. Engng. Sci.* **15** (1977), 561.
3. G. S. Seth and S. K. Ghosh, *Proc. Math. Soc.* **3** (1987).
4. R. S. Nanda and H. K. Mohanty, *Appl. Sci. Res.* **24** (1971), 65.