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COMMENTS AND REALISTIC APPROXIMATIONS TO PDES OF UNSTEADY THERMAL BOUNDARY LAYER FLOWS OF VERTICAL PLATE

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Several defects of an analytical model concerned to porous plate under free and forced convection by (Lohurikar and Jahagirdar[5] (*Indian J. pure appl. Math.* **21** (4), April 1990), are brought out after a critical study. Consequently, the problem of thermal boundary layer of vertical plate under different fluid medium have been addressed. Structure of mathematical formulation has been examined, when plate is made up of porous material. To understand flow and heat transfer mechanism of two-phase flows under free/forced convection, numerical algorithms, for generation of parametric databases, are described.

1. NOMENCLATURE

a = Radius of particle
Br = Brinkman number
c = Ratio of specific heats of particle to fluid
c_p = Specific heat of fluid at constant pressure
c_s = Specific heat of particle at constant pressure
f = Non-dimensional concentration of particles
f_0 = Initial mass concentration of particles
g = Gravitational acceleration
Gr = Grashof number
i = Grid point along x axis
j = Grid point for time step
k = Thermal conductivity of fluid
k' = Permeability constant
l = Length of plate
m = Mass of the particle
m' = Maximum grid point along y axis
N = Number density of particles per unit volume
Nu = Nusselt Number
Pr = Prandtl Number
Q = Heat exchange from fluid to particle per unit volume
q = Thermal constant associated with particles

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Re = Reynold's number
t = Non-dimensional time
t_{max} = Maximum time step
Δ_t = Grid difference along t Direction
u = Velocity of fluid-particle mixture
v = Normal velocity of fluid-particle mixture
x = Flow along x direction
y = Flow along y direction
Δ_y = Grid difference in y direction.
θ = Non-dimensional Temperature of fluid particle mixture
θ = Temperature at $y = 0$
θ_1^0 = Tempeature at $y = \infty$
$\Delta\theta$ = $\theta_0 - \theta_t$
ρ = Density
μ = Viscosity
ν = Kinametic Viscosity
τ_x = Skin friction
λ = Ratio of viscosity to permeability constant
β = Volumetric expansion
δ = Boundary layer thickness.

2. Introduction

The phenomena of thermally driven fluid past a isothermal vertical plate finds several applications in industrial equipment. The analysis of such component, which is subjected to different material and multiphase fluid mixture, call for extensive mathematical models with advanced numerical techniques. Though, the literature dealing with fluid past a isothermal vertical plate is adequate[1-3], its solution lacks in 2D/3D environment due to non-linearity and coupling between velocity and temperature terms. When the vertical plate is of porous medium, subjected to flowing fluid, its momentum balance modifies with material permeability factor.

There have been intense studies in the area of flows subjected to body forces. In particular, flows generated and sustained by buoyancy, i.e. natural convection, have been the focus of attention because of their wide range applications. For most fluids, such as air and particles, the problem can be formulated in terms of the well-known Navier Stokes' and energy equations. While the steady-state problems are governed by an elliptic system of equations, the character changes to parabolic, when the unsteady terms are retained. The steady-state equations only approximate the original governing system, under the limiting case of long evolution time for the flow field. In this manner, all transition phenomena and the possible presence of turbulence are ignored. Even when the flow is laminar, the existence of a steady state is not guaranted a priori for free-convection problems. Hence, it is clear that the unsteady version of the Navier-Stokes and energy equations more correctly define the flow and heat transfer phenomena. However, if the steady state is realizable, then these equations can be (1) analytically solved, (2) simplified, as with boundary-layer equations, or (3) solved by well developed "elliptic solvers".

The solutions to such class of problems through exhaustive review[4] generates interesting applications in process equipment. But the findings made by Lohurikar and Jahagirdar[5] are not meaningful, because of several discrepancies in analysis. If the fluid composed of gas-dust particles flows past isothermal vertical plate, structure of mathematics is more complicated and its effects are reported by Lee[6] and Bhasker[7]. In order to understand the problems associated to thermal convection especially with the class of two-phase flows, the paper highlights as to how the physical process of such a phenomena can be understood through numerical tecniques.

3. Comments on Published Paper

The title of paper under reference[5] "unsteady forced and free convection flows past infinite vertical plate through porous medium[1]" wherein the statement of problem is not matching to the mathematical analysis described. The account of non-linear terms in momentum and energy balances has not taken care nor concerned to give justification in formulating the problem. The absence of Grashof number with temperature term in momentum balance or viscous dissipation factor in energy equations to study free and forced convection effects of flat plate clearly shows, that the reported paper has no relevance to thermal analysis under porous medium.

On mathematics side, when the governing equations are transformed into non-dimensional quantities authors have not defined the terms u.w.g.η. Missing of boundary conditions to transformed equations certainly affects the problem description to understand the phenomena physically. As a result, the solution sought to their eqn. (6) as $\theta = \text{erfc}\,(g\eta\,\sqrt{Pr}\,)$ is erroneous, because g is gravitational constant and it is not clear how the above expression was obtained using integration or Laplace transforms. In any case, the quantity g will not exist when flow is forced convection and under free convection g will be accommodated in non-dimensional parameter Grashof number. In view of this, solution obtained for w in eqn. (8) develops serious doubts for it's accuracy. On looking at their results in all figures, the levels indicated for both the axis are μ and η, which has no relevance to it's graphical representations and in particular their Fig. 3, on which, missing of vertical axis indication will not speak anything in the graph. The variation λ, defined as ratio of fluid viscosity to permeability constant was chosen initially as 0.2 and there on jumped to higher magnitudes for which, no basis has been explained.

The computations of profiles for $Pr = 1,7,100$, has not indicated, what type of fluid they have considered, though some where, it was mentioned as air at page 399 of Lahurikar and Jahagirdar[5]. If it so, they have missed the very fundamental concept of Prandtl numbers and its significance to type of fluids as shown in the Fig. 1 of this paper. The authors findings for variations P/λ with τ_x is shown in Table 1 are absurd, as no mathematical relation exist between them according to their eqn. (10) and once again P is no where defined. Due to interchange of page 401 by 400, the paper supplements further defects, thereby falls under below standard for publication. Finally, the model which is supposed to establish criteria for free and forced convection as per it's title, have been thoroughly missed. To this effect the problem would have been purposeful, it following aspects was thought over in the reported analysis.

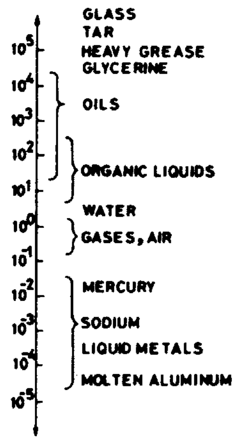

FIG. 1. Scope of Prandtl number values for different substances.

In any convective heat transfer process, density differences arise, as a result of differences in temperature, and under the influence of gravitational force field natural convection effects result in the flow system. In a forced convection case, associated with large Reynold's numbers connected with large flow velocities, where the forces and momentum transport rates are very large, the effects of natural convection are negligible. If on the other hand, buoyancy forces arising from density differences are relatively large, as exemplified by large Grashof numbers, the forced-convection effects may be ignored. However, in many cases of practical interest, both the effects of forced convection and natural convection may be of comparable order. An indication of the relative magnitude of the two effects can be obtained from the differential equations describing the flow and this is best accomplished by the use of non-dimensional parameters such as $Gr/(Re{*}Re)$. With the comparatively small velocities associated with laminar motion, the heat transfer is substantially affected by buoyancy forces and resulting velocity fields.

4. Mechanism of Heat Flow by Convection

The elaborate above physics, consider fluid past a isothermal vertical plate shown in Fig. 2. Due to viscous nature of fluid, a boundary layer is formed near the heated wall. On analysing the flow pattern for classical fluid, the velocity increases from zero at wall to a maximum and then decreases, as ambient conditions are reached. The effect of buoyancy force decreases after reaching the velocity maximum because of less heating of fluid. The temperature and velocity gradient approaches zero at the same location and this buoyancy forces causing the upward flow zero at the edge of boundary and cannot sustain any shear force. If temperature gradient does not approach zero, heat would flow out of boundary providing the density gradient and additional upward flow. This implies that thermal and fluid dynamic boundary layers are of equal thickness, which is not the case in forced convection.

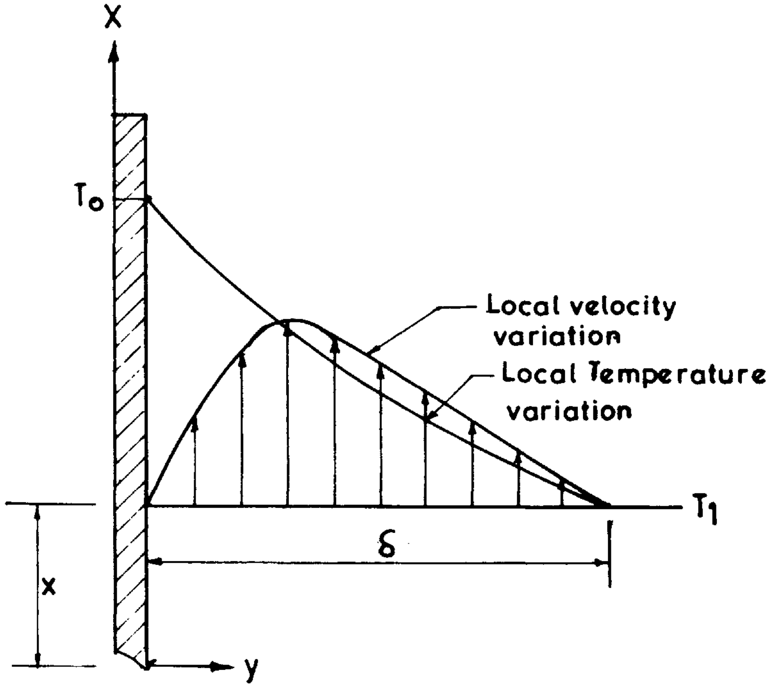

FIG. 2. Profiles of velocity and temperature in thermal convection.

5. MATHEMATICAL FORMULATIONS

For constant fluid properties, the conservation equations of vertical plate in non-dimensional quantities under free convection are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial g} = 0 \qquad ...(1)$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = Gr\theta + \nu\frac{\partial^2 u}{\partial y^2} \qquad ...(2)$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y^2} \qquad ...(3)$$

if the plate is of porous material, then the eqn. (2) modifies into

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial x} = Gr\theta + \nu\frac{\partial^2 u}{\partial y^2} + \frac{\lambda}{\rho}u \qquad ...(4)$$

in case, the plate is subjected to two-phase fluid and dust particles then for small particle flow and thermal relaxation times, eqns. apart from (1) will become:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{Gr\theta}{1+f} + \frac{\nu}{1+f}\frac{\partial^2 u}{\partial y^2} \quad ...(5)$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{Pr(1+q)}\frac{\partial^2 \theta}{\partial y^2} \quad ...(6)$$

$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} + f\frac{\partial u}{\partial y} + v\frac{\partial f}{\partial x} + f\frac{\partial f}{\partial y} = 0. \quad ...(7)$$

The corresponding equations to study the forced convection flow and heat transfer are: –

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \quad ...(8)$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{Pr}\frac{\partial^2 \theta}{\partial y} + \frac{Br}{Pr}\left|\frac{\partial u}{\partial y}\right|^2 \quad ...(9)$$

if the plate is of porous material, then the eqn. (8)

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} + \frac{\lambda}{\rho}u \quad ...(10)$$

in case, the plate is subjected to two-phase fluid, then for small particle flow and thermal relaxation, apart from (1) will becomes:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\nu}{(1+f)}\frac{\partial^2 u}{\partial y^2} \quad ...(11)$$

$$\frac{\partial \theta}{\partial t} + u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{Pr\ (1+q)}\frac{\partial^2 v}{\partial y^2} + \frac{Br}{Pr\ (1+q)}\left|\frac{\partial u}{\partial y}\right|^2 \quad ...(12)$$

$$\frac{\partial f}{\partial t} + u\frac{\partial f}{\partial x} + f\frac{\partial u}{\partial y} + v\frac{\partial f}{\partial x} + f\frac{\partial u}{\partial y} = 0 \quad ...(13)$$

if the vertical plate is subjected to Mixed Convection, then, the eqn. (2) modifies to : –

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{\partial p}{\partial x} + \frac{1}{Re}\frac{\partial^2 u}{\partial y^2} + \frac{Gr\theta}{Re^2} \quad ...(14)$$

the above eqns. are subjected to boundary conditions:

$$t = 0\,;\ u = v = 0 \quad \theta = 0 \quad f = f_0 \text{ for all } y$$

$$t > 0\,;\ u = v = 0 \quad \theta = 0 \quad f = f_0 \text{ at } y = 0 \quad ...(15)$$

$$u = v = u \quad \theta = \theta_0 \text{ at } y = \infty,\ f = 0$$

where $f = \dfrac{mN}{\rho}$; $\lambda = \dfrac{\mu}{k'}$; $\mathrm{Re} = \dfrac{ul}{\nu}$; $\mathrm{Pr} = \dfrac{\mu c}{k}p$; $Q = 4\pi ka$;

$$Br = \frac{\mu}{k\Delta T}\ \theta = \frac{T - T_0}{\Delta T}\ ;\ q = \frac{QN}{\rho}\ c\ ;\ c = \frac{c_s}{c_p}\ ;$$

$$Gr = \frac{g\beta l^3 (T_0 - T_1)}{\nu^2 T_1}$$

having solved the above eqns. for u, v, θ, f, the heat transfer coefficient, defined as

$$Nu = \frac{-\dfrac{d\theta}{dy}}{\Delta\theta}\Bigg|_{y=0}\ \frac{x}{\delta} \qquad \ldots(16)$$

6. Approximations to PDES – Vertical Plate in Free Convection

The equations shown in (1) – (15) are mostly parabolic and can be elliptic PDEs in time doman, if the analysis is extended to wake portion of plate. The structure of equations are so complicated by non-linear and coupling terms in such a way, that its closed form solutions are impossible and hence it needs numerical solution. The process of obtaining the computational solution consists of two stages. The first stage converts the continuous PDEs and auxiliary initial and boundary conditions into system of algebraic equations are known as discretization. The process of discretization is easily identified[8] if finite difference method is used, but is slightly less, with finite element, finite volume and spectral method. The second stage of solution process requires an equation solver to provide the solution to the system of algebraic equations. This stage can introduce an error but it is usually negligible compared with error encountered at discretization stage unless, the method is unstable. To ensure the numerical scheme (FDM) for accuracy, there is always necessary, that one has to take care the algorithms for convergence, consistency and stability.

The equations described in (1) – (15) are approximated through difference formulas, whose end algorithms are detailed in the flow chart (Fig. 3) to study flow pattern, when gas-particle flow is past an isothermal vertical plate. Detailed parametric results was studied by Bhasker and Bangad[9] through computer program based on explicit finite difference method in time domain. The same has been extended to study gas-particle flow between parallel vertical plates[10] to predict parametric study for which the above computer program was used with minor modifications.

Several mathematical models have been developed using the eqns. (1) – (16) to study particle dynamics and thermodynamics in incomressible fluid flows. The results of parametric study obtained are documented in Bhasker[7], which is expected to be useful to approach types of multiphase flow problems. A separate paper dealing with compressible flow and heat transfer process of Rayleigh's problem under two-phase flows, extending the above formulations, to gain insights into turbine blade losses, is being sent, for publication else where.

7. Conclusions

After studying serious deficiencies in the analysis[5] concerned to free and forced convection, a meaningful problem for multi-phase flows has been addressed, from the first principles. While describing the mathematical formulations, numerical algorithms in time domain have been highlighted to study free convection effects

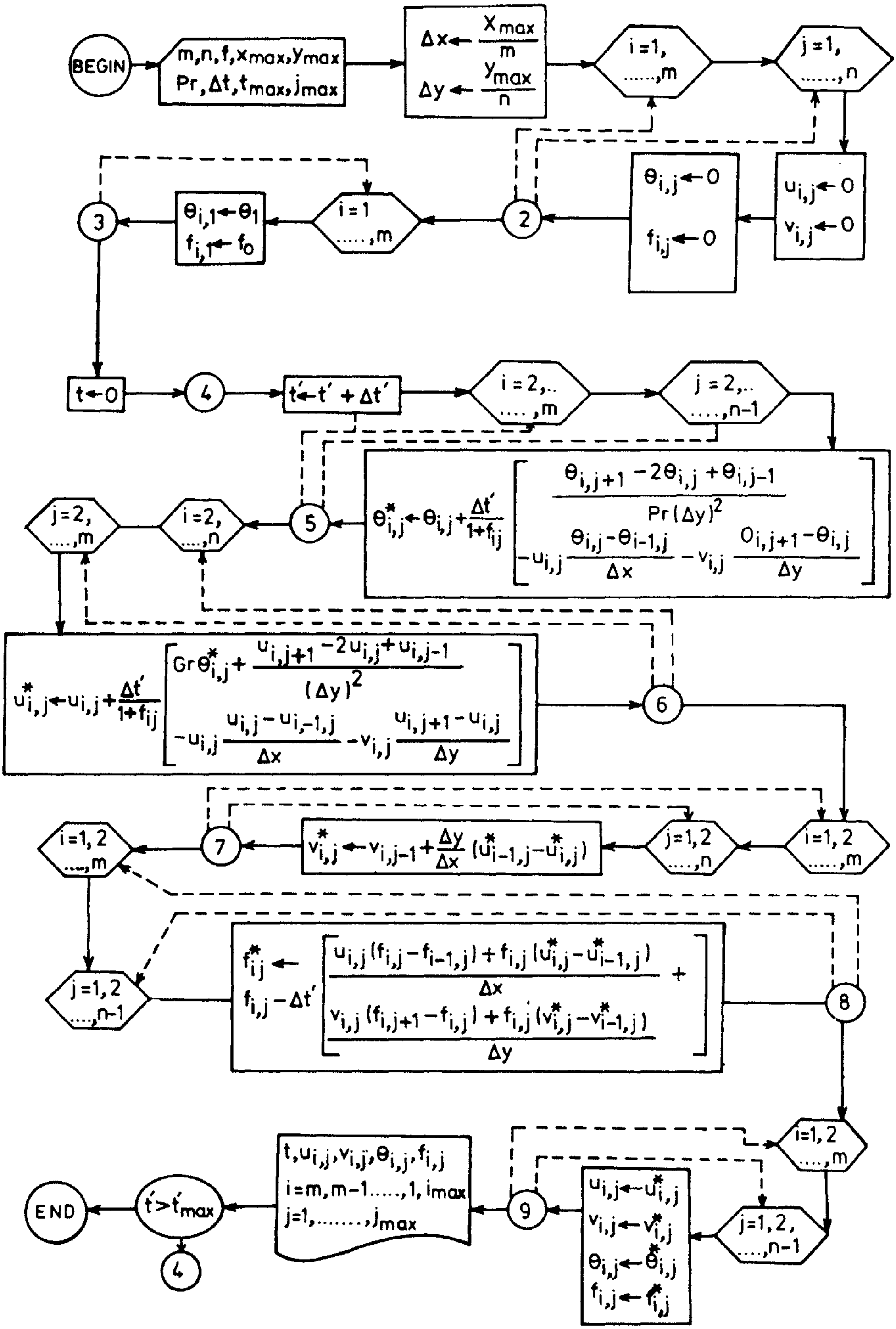

FIG. 3. Flow diagram.

of two-phase gas-particle flows past a vertical plate. Automation procedure explored for prediction of thermo-fluid mechanics of two-phase (gas-particle/porous medium) over heated plates are expected to be useful very much in estimating the performance losses in turbomachinery equipment.

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