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A NOTE ON PRIMENESS IN NEAR-RINGS AND MATRIX NEAR-RINGS

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Strictly equiprime, jth row strictly equiprime concepts were introduced in Near-rings and obtained that (i) if N is a strictly equiprime near-ring then the matrix near-ring $M_n(N)$ is the jth row strictly equiprime for $1 \le j \le n$; (ii) if I is a prime left ideal of a near-ring N then the corresponding ideal I^* is a prime left ideal in $M_n(N)$.

INTRODUCTION

Many authors[2, 7-14] studied different types of prime ideals and related concepts in Near-rings, Gamma near-rings, rings and gammarings. Booth and Groenewald[1], Groenewald[3,4] and Meldrum and Van der Walt[5] studied prime ideals and related concepts in matrix near-rings.

In section 1, we introduce the concept 'prime left ideal' in Near-rings, and obtain some equivalent conditions for a left ideal of N to be a prime left ideal. In section 2, we introduce the notion jth row equivalence of two matrices in matrix near-rings and obtain few related results. In section 3, we prove the two main theorems.

Throughout this paper N-stands for a zero symmetric right near-ring. For standard definitions and notations which are not mentioned here, we refer to Booth and Groenewald[1], Groenewald[3, 4], Meldrum and Van der Walt[5] and Pilz[6]. For any subset X of a near-ring N, the ideal (left ideal) generated by X is denoted by $\langle X \rangle$ ($\langle X \rangle_l$ respectively). If $X = \{a\}$ then $\langle X \rangle$ ($\langle X \rangle_l$) is denoted by $\langle a \rangle$ ($\langle a \rangle_l$).

Meldrum and Van der Walt[5] introduced the concept of matrix near-ring. The definition is as follows :

Consider N with identity 1. N^n denotes the direct sum of n-copies of $(N, +)$. For any $r \in N$, $1 \le i \le n$ and $1 \le j \le n$ define $f_{ij}^r : N^n \to N^n$ as $f_{ij}^r(a_1, ..., a_n) = (b_1, ..., b_n)$ where $b_k = 0$ if $k \ne i$ and $b_k = ra_j$ if $k = i$. If (i) $f^r : N \to N$ defined by $f^r(x) = rx$ for all $x \in N$; (ii) $I_i : R \to R^n$ is the canonical monomorphism; and (iii)

$\pi_j : R^n \to R$ is the canonical epimorphism; then it is clear that $f_{ij} = I_i f \pi j$, and $f_{ij} \in M(N^n)$ where $M(N^n)$ is the set of all mappings from N^n to N^n. It is a known fact that $M(N^n)$ is a near-ring. The subnear-ring $M_n(N)$ of $M(N^n)$ generated by $\{f_{ij}^r / r \in N,\ 1 \le i, j \le n\}$ is called the near-ring of $n \times n$ matrices over N. $M_n(N)$ is called a matrix near-ring.

If I is an ideal of N then the related ideal I^* in $M_n(N)$ is defined as : $I^* = \{A \in M_n(N) / Ap \in I^n$ for all $p \in N^n\}$. Meldrum and Van der Walt[5] have proved that $(I_1)^* \subsetneq (I_2)^*$ for any two ideals I_1 and I_2 of N with $I_1 \subsetneq I_2$. They have also proved that if N is a prime near-ring then so is $M_n(N)$.

Now we state some definitions from the literature which are used in this paper.

Definitions (Booth and Groenewald[1]) — (i) A near-ring N is said to be equiprime if for all $0 \neq a \in N$ and $x, y \in N$, $a\ r\ x = a\ r\ y$ for all $r \in N$ implies $x = y$.

(ii) An ideal P of N is called an equiprime ideal of N if N/P is an equiprime near-ring.

(iii) A near-ring N is called strongly equiprime if for all $0 \neq a \in N$, there exists a finite subset F of N such that $x, y \in N$, $a f x = a f y$ for all $f \in F$ implies $x = y$.

Definition (Groenewald[4]) — An ideal P of N is said to be 3-prime ideal if $a, b \in N$, $aNb \subseteq P$ implies $a \in P$ or $b \in P$. If (0) is a 3-prime ideal in N then N is called a 3-prime Near-ring.

Definition (Meldrum and Van der Walt[5]) — An element $A \in M_n(N)$ is called a diagonal matrix if $A = f_{11}^{r_1} + f_{22}^{r_2} + ... + f_{nn}^{r_n}$ where $r_i \in N$ for $1 \le i \le n$.

1. Prime Left Ideals in Near-Rings

Now we introduce the notion "prime left ideal" in near-rings.

Definition 1.1 — A left ideal P of N is said to be a prime left ideal if it satisfies the following condition :

I, J are two left ideals of N such that $IJ \subseteq P$ implies $I \subseteq P$ or $J \subseteq P$.

Proposition 1.2 — The following conditions are equivalent :

(i) P is prime left ideal;

(ii) $\langle a \rangle_1 \langle b \rangle_1 \subseteq P \Rightarrow a \in P$ or $b \in P$;

(iii) $a \langle b \rangle_1 \subseteq P \Rightarrow a \in P$ or $b \in P$;

(iv) $\langle a \rangle_1 (P + \langle b \rangle_1) \subseteq P \Rightarrow a \in P$ or $b \in P$; and

(v) $A(P + B) \subseteq P \Rightarrow A \subseteq P$ or $B \subseteq P$, where $a, b \in N$ and A, B are left ideals of N.

PROOF : (i) $\Rightarrow$ (ii) : Follows from the definition.

(ii) $\Rightarrow$ (iii) : Suppose $a \langle b \rangle_1 \subseteq P$. By Proposition 1.42 of Pilz[6], we have that $\langle a \rangle_1 \langle b \rangle_1 \subseteq P$. Now by (ii), we have that either $a \in P$ or $b \in P$.

(iii) $\Rightarrow$ (iv) : Suppose $\langle a \rangle_1 (P + \langle b \rangle_1) \subseteq P$.

Now $\langle a \rangle_1 \langle b \rangle_1 \subseteq \langle a \rangle_1 (P + \langle b \rangle_1) \subseteq P \Rightarrow a \in P$ or $b \in P$.

(iv) $\Rightarrow$ (v) : Suppose $A(P + B) \subseteq P$. In a contrary way assume that $A \nsubseteq P$ and $B \nsubseteq P$. Then there exist $a \in A \setminus P$ and $b \in B \setminus P$. Now $\langle a \rangle_1 (P + \langle b \rangle_1) \subseteq A(P + B) \subseteq P$. By (iv), we have that either $a \in P$ or $b \in P$, a contradiction. Therefore $A \subseteq P$ or $B \subseteq P$.

(v) $\Rightarrow$ (i) : Suppose A and B are two left ideals such that $AB \subseteq P$. Let $a \in A$, $p \in P$ and $b \in B$. Then $a(p + b) - ab \in P$ (since P is a left ideal), $ab \in P$. So $a(p + b) - ab + ab \in P$ and so $A(P + B) \subseteq P$. Now by (v), we have that either $A \subseteq P$ or $B \subseteq P$.

Now the proof of the following Remark is clear.

Remark 1.3 : Suppose P is a left ideal which is not right ideal. For any $b \in N$ such that $Pb \nsubseteq P$, the following are equivalent :

(i) $\langle a \rangle_1 \langle b \rangle_1 \subseteq P \Rightarrow a \in P$;

(ii) $\langle a \rangle_1 (P + \langle b \rangle_1) \subseteq P \Rightarrow a \in P$; and

(iii) A, B are left ideals, $A(P + B) \subseteq P$, $b \in B \Rightarrow A \subseteq P$.

Note 1.4 : If P is a left ideal then the following conditions are equivalent :

(i) $PL \subseteq P$, L is a left ideal $\Rightarrow L \subseteq P$;

(ii) $P \langle b \rangle_1 \subseteq P$, $b \in N \Rightarrow b \in P$; and

(iii) P is the largest left ideal contained in the set X where

$$X = \{x \in N / P \langle x \rangle_1 \subseteq P\}.$$

Note 1.5 : If P is a prime left ideal which is not a two sided ideal, then P is the largest left ideal contained in $X = \{x \in N / P \langle x \rangle_1 \subseteq P\}$.

PROOF : Suppose the left ideal P is not two sided ideal. Then there exist an element $b \in N$ such that $Pb \nsubseteq P$. Suppose that L is a left ideal of N such that $PL \subseteq P \Rightarrow (Pb)L \subseteq P(bL) \subseteq PL \subseteq P \Rightarrow \langle Pb \rangle_1 L \subseteq P$ (by Proposition 1.42 of Pilz[6]) $\Rightarrow \langle Pb \rangle_1 \subseteq P$ or $L \subseteq P \Rightarrow Pb \subseteq P$ or $L \subseteq P \Rightarrow L \subseteq P$ (since $Pb \nsubseteq P$). Therefore, for any left ideal L of P, we have that $PL \subseteq P \Rightarrow L \subseteq P$. Now by above result we have that P is the largest left ideal that is contained in X.

Definition 1.6 — (i) Let $M \subseteq N$. M is said to be a left m-system-1 if either $M = \phi$ or $a, b \in M \Rightarrow a^1 b^1 \in M$ for some $a^1 \in \langle a \rangle_1$ and $b^1 \in \langle b \rangle_1$.

(ii) M is said to be a left m-system-2 if either $M = \phi$ or a, $b \in M \Rightarrow ab^1 \in M$ for some $b^1 \in \langle b \rangle_1$.

Note 1.7 : It is clear that if M is a left m-system-2 then it is a left m-system-1.

Proposition 1.8 — If P is a left ideal of N, then the following conditions are equivalent :

(i) P is prime left ideal;

(ii) $N \setminus P$ is a left m-system-2; and

(iii) $N \setminus P$ is a left m-system-1.

PROOF : (i) $\Rightarrow$ (ii) : Suppose P is prime. Consider $N \setminus P$. If $P = N$ then $N \setminus P = \phi$ is a left m-system-2. Suppose $N \setminus P \neq \phi$. To show that $N \setminus P$ is a left m-system-2, take $a, b \in N \setminus P \Rightarrow a, b \notin P$. Since P is prime left ideal, by the Proposition 1.2, we have that $a \langle b \rangle_1 \nsubseteq P$. Therefore there exists $b^1 \in \langle b \rangle_1$ such that $ab^1 \notin P$, which implies $ab^1 \in N \setminus P$.

(ii) $\Rightarrow$ (iii) : follows from Note 1.7.

(iii) $\Rightarrow$ (i) : Suppose $N \setminus P$ is a left m-system-1. If $N \setminus P = \phi$ then $P = N$ and hence P is a prime left ideal. Suppose $\langle a \rangle_1 \langle b \rangle_1 \subseteq P$. Now we have to show that $a \in P$ or $b \in P$. In a contrary way, suppose that $a \notin P$ and $b \notin P$. Then $a \in N \setminus P$ and $b \in N \setminus P \Rightarrow a^1 b^1 \in N \setminus P$ for some $a^1 \in \langle a \rangle_1$ and $b^1 \in \langle b \rangle_1$ $\Rightarrow$ $a^1 b^1 \notin P \Rightarrow \langle a \rangle_1 \langle b \rangle_1 \nsubseteq P$, a contradiction to the supposition. Hence P is a prime left ideal.

2. jth ROW EQUIVALENCE IN MATRIX NEAR-RINGS

In this section we introduce the Notions (i) "A is jth row equivalent to B" for two elements $A, B \in M_n(N)$; (ii) jth row strictly equiprime near-ring; and (iii) jth column scalar-r matrix. We prove that (i) For any $A, B \in M_n(N)$, $A = B$ if and only if A is jth row equivalent to B for all $1 \leq j \leq n$; and (ii) Every equiprime ideal is a 3-prime ideal.

We start with the following definition.

Definitions 2.1 — (i) Let $A, B \in M_n(N)$. Then we say that A and B are jth row equivalent matrices (or A is jth row equivalent to B) if $(f_{1j}^{1} + \ldots + f_{nj}^{1})A = (f_{1j}^{1} + \ldots + f_{nj}^{1})B$.

(ii) Take j such that $1 \leq j \leq n$. Then $M_n(N)$ is said to be jth row strictly equiprime if for any $0 \neq A \in M_n(N)$ there exists $\phi \in M_n(N)$ such that $A \phi U = A \phi V$, imply U and V are jth row equivalent.

(iii) N is said to be strictly equiprime if for any $0 \neq a \in N$ there corresponds an element $x_a \in N$ such that $u, v \in N$ and $ax_a u = ax_a v \Rightarrow u = v$.

Note 2.2 : (i) jth row equivalence is an equivalence relation; and

(ii) Every strictly equiprime near-ring is strongly equiprime.

Definition 2.3 — Let $1 \leq j \leq n$ and $A \in M_n(N)$. Then (i) A is said to be a jth Column matrix if there exists $r_i \in N$, $1 \leq i \leq n$ such that $A = f_{1j}^{r_1} + f_{2j}^{r_2} + \ldots + f_{nj}^{r_n}$.

(ii) A is said to be jth Column Scalar matrix if there exists $r \in N$ such that $A = f_{1j}^{r} + f_{2j}^{r} + \ldots + f_{nj}^{r}$. This matrix is also called the jth column scalar r-matrix.

Note 2.4 — Let $U, V \in M_n(N)$. Then

$$(f_{1j}^{1} + \ldots + f_{nj}^{1})\, U = (f_{1j}^{1} + \ldots + f_{nj}^{1})\, V$$

$$\Leftrightarrow [(f_{1j}^{1} + \ldots + f_{nj}^{1})\, U]\,(s_1, \ldots, s_n) = [(f_{1j}^{1} + \ldots + f_{nj}^{1})\, V]\,(s_1, \ldots, s_n)$$

$$\forall\ (s_1, \ldots, s_n) \in N^n$$

$$\Leftrightarrow (f^1_{1j} + \dots + f^1_{nj})\, U(s_1, \dots, s_n) = (f^1_{1j} + \dots + f^1_{nj})\, V(s_1, \dots, s_n) \quad \forall\ (s_1, \dots, s_n) \in N^n$$

$$\Leftrightarrow ([U(s_1, \dots, s_n)]_j, \dots, [U(s_1, \dots, s_n)]_j) = ([V(s_1, \dots, s_n)]_j, \dots, [V(s_1, \dots, s_n)]_j) \ \forall\ (s_1, \dots, s_n) \in N^n$$

$$\Leftrightarrow [U(s_1, \dots, s_n)]_j = [V(s_1, \dots, s_n)]_j \quad \forall\ (s_1, \dots, s_n) \in N^n.$$

A straight forward verification shows the following Remark 2.5 and Proposition 2.6. Corollary 2.7 is immediate consequence of Proposition 2.6.

Remark 2.5 : $A = B$ iff A and B are jth row equivalent for all j with $1 \le j \le n$.

Proposition 2.6 — In a zero symmetric near-ring N, every equiprime ideal is a 3-prime ideal.

Corollary 2.7 — Let N be a zero symmetric near-ring. If N is equiprime, then N is 3-prime.

3. Main Theorems

Throughout this section N stands for a near-ring with 1.

Proposition 3.1 — Let N be a strictly equiprime near-ring with 1. Then $M_n(N)$ is jth row strictly equiprime for $1 \le j \le n$. More precisely, if $A \in M_n(N)$ such that (i) $A \ne 0$ with $a = [A(r_1, \dots, r_n)]_i \ne 0$ then there exists a diagonal matrix B, and (ii) for each $j\,(1 \le j \le n)$ there is a jth column scalar-r_a matrix C_j such that $U, V \in M_n(N)$ and $A(BC_j)U = A(BC_j)V \Rightarrow U$ is jth row equivalent to V, for $1 \le j \le n.$'

Proof : Given that $A \in M_n(N)$ such that $A \ne 0$ with $a = [A(r_1, \dots, r_n)]_i \ne 0$. Since N is strictly equiprime there exists $x_a \in N$ such that $a \cdot x_a \cdot u = a \cdot x_a \cdot v \Rightarrow u = v$.
Consider the diagonal matrix $B = f^{r_1}_{11} + \dots + f^{r_n}_{nn}$ and jth column scalar r-matrices $C_j = f^{r}_{1j} + \dots + f^{r}_{nj}$ for $1 \le j \le n$, where $r = x_a$. Write $D = BC_j \in M_n(N)$.
Now suppose $ADU = ADV$ (That is $ABC_j\, U = ABC_j\, V$).
Now we have to show that U is jth row equivalent to V.
In a contrary way suppose that U is not jth row equivalent to V.
This means $(f^1_{1j} + \dots + f^1_{nj})\, U \ne (f^1_{1j} + \dots + f^1_{nj})\, V$

$$\Rightarrow [(f^1_{1j} + \dots + f^1_{nj})\, U]\,(s_1, \dots, s_n) \ne [(f^1_{1j} + \dots + f^1_{nj})\, V]\,(s_1, \dots, s_n)$$

for some $(s_1, \dots, s_n) \in N^n$

$$\Rightarrow [U(s_1, \dots, s_n)]_j \ne [V(s_1, \dots, s_n)]_j \text{ (by above note)}$$

$$\Rightarrow a\, x_a\, [U(s_1, \dots, s_n)]_j \ne a\, x_a\, [V(s_1, \dots, s_n)]_j$$

$$\Rightarrow [A(r_1, ..., r_n)]_i x_a [U(s_1, ..., s_n)]_j \neq [A(r_1, ..., r_n)]_i x_a [V(s_1, ..., s_n)]_j$$

$$\Rightarrow [A(r_1, ..., r_n)] [x_a (U(s_1, ..., s_n))_j] \neq [A(r_1, ..., r_n)] [x_a (V(s_1, ..., s_n))_j]$$

$$\Rightarrow A \ [(f_{11}^{r_1} + \dots + f_{nn}^{r_n})(f_{1j}^{x_a} + \dots + f_{nj}^{x_a}) U(s_1, ..., s_n)]$$

$$\neq A \ [(f_{11}^{r_1} + \dots + f_{nn}^{r_n})(f_{1j}^{x_a} + \dots + f_{nj}^{x_a}) V(s_1, ..., s_n)]$$

$$\Rightarrow A \ [(f_{11}^{r_1} + \dots + f_{nn}^{r_n})(f_{1j}^{r} + \dots + f_{nj}^{r}) U(s_1, ..., s_n)]$$

$$\neq A \ [(f_{11}^{r_1} + \dots + f_{nn}^{r_n})(f_{1j}^{r} + \dots + f_{nj}^{r}) V(s_1, ..., s_n)]$$

(Since $r = x_a$)

$$\Rightarrow ABC_j U \neq ABC_j V, \text{ a contradiction to our supposition.}$$

Hence U and V are jth row equivalent for $1 \leq j \leq n$.

Corollary 3.2 — Suppose N is a near-ring with 1. If N is strictly equiprime near-ring then $M_n(N)$ is strongly equiprime.

PROOF : Suppose N is strictly equiprime near-ring. Let $0 \neq A \in M_n(N)$. Since $M_n(N)$ is the strictly equiprime near-ring for each $1 \leq j \leq n$, there exists $D_j \in M_n(N)$ such that $AD_j U = AD_j V \Rightarrow U$ and V are jth row equivalent. Now write $F = \{D_j / 1 \leq j \leq n\}$. Suppose $AfU = AfV$ for all $f \in F \Rightarrow AD_j U = AD_j V$ for all j $(1 \leq j \leq n)$ $\Rightarrow U$ and V are jth row equivalent for all j $(1 \leq j \leq n) \Rightarrow U = V$ (by Remark 2.5).

Theorem 3.3 — If I is prime left ideal of N then I^* is prime left ideal in $M_n(N)$.

PROOF : Suppose I is a prime left ideal in N. Since I is a left ideal in N, by Corollary 4.2 of Meldrum and Van der Walt[5], we have I^* is an ideal in $M_n(N)$.

Now we have to show that I^* is prime left ideal. In a contrary way, suppose that I^* is not a prime left ideal. Then there exist two left ideals L_1, L_2 in $M_n(N)$ such that $L_1 L_2 \subseteq I^*$, $L_1 \nsubseteq I^*$ and $L_2 \nsubseteq I^*$. Then there exists $A \in L_1 \setminus I^*$, $B \in L_2 \setminus I^*$ which implies that there exist $p, q \in N^n$ such that $Ap \notin I^n, Bq \notin I^n$. Write $Ap = (a_1, ..., a_n)$ and $Bq = (c_1, ..., c_n)$. Since $Ap \notin I^n$ and $Bq \notin I^n$, there exist k, s such that $a_k \notin I$ and $c_s \notin I$. $AL_2 \subseteq L_1 L_2 \subseteq I^*$ and I^* is an ideal

$$\Rightarrow \langle A \rangle L_2 \subseteq I^* \text{ (by Proposition 1.42 of Pilz[6])}$$

$$\Rightarrow \langle A \rangle \langle B \rangle_1 \subseteq \langle A \rangle L_2 \subseteq I^* \text{ (since } B \in L_2)$$

$$\Rightarrow \langle A \rangle \langle B \rangle_1 \subseteq I^* \Rightarrow \langle A \rangle \langle B \rangle_1 M_n(N) \subseteq I^*$$

(because I^* is an ideal)-Fact (1), say. Since $\langle A \rangle$ is right invariant by Corollary 3.10 of Meldrum and Van der Walt[5], $\langle A \rangle N^n = \langle A \rangle e_1$ where $e_1 = (1, 0, ..., 0)$. Now $Ap \in \langle A \rangle N^n = \langle A \rangle e_1 \Rightarrow$ There exist $C \in \langle A \rangle$ such that $Ap = Ce_1$. Since $Ap = (a_1, ..., a_n)$ we have $(a_1, ..., a_n) = Ap = Ce_1$

$= C\,[f_{1\,1}^{1} + f_{2\,2}^{0} + \dots + f_{n\,n}^{0}]\,e_1 = [f_{1\,1}^{20}b_1 + f_{2\,2}^{b_2} + \dots + f_{n\,n}^{b_n}]\,e_1$

(by Lemma 3.7 of Meldrum and Van der Walt[5])

$= (b_1, b_2, \dots, b_n)$

$\Rightarrow \quad a_i = b_i \qquad \forall \quad 1 \le i \le n.$

$C\,[f_{1\,1}^{1} + f_{2\,1}^{0} + \dots + f_{n\,1}^{0}] = [f_{1\,1}^{b_1} + f_{2\,1}^{b_2} + \dots + f_{n\,1}^{b_n}] = [f_{1\,1}^{a_1} + f_{2\,1}^{a_2} + \dots + f_{n\,1}^{a_n}].$

Now consider

$$E_{1k}\,C\,[f_{1\,1}^{1} + f_{2\,1}^{0} + \dots + f_{n\,1}^{0}]\,(\alpha_1, \dots, \alpha_n)$$

$$= E_{1k}\,[f_{1\,1}^{a_1} + f_{2\,1}^{a_2} + \dots + f_{n\,1}^{a_n}]\;(\alpha_1, \dots, \alpha_n) \;=\; f_{11}^{a_k}\;(\alpha_1, \dots, \alpha_n).$$

Therefore, $\quad E_{1k}\,C\,[f_{1\,1}^{1} + f_{2\,1}^{0} + \dots + f_{n\,1}^{0}] = f_{11}^{a_k} \Rightarrow f_{11}^{a_k} \in \langle A \rangle$

(since $C \in \langle A \rangle$ and E_{1k} is zero symmetric)

Therefore $f_{11}^{a_k} \in \langle A \rangle \setminus I^*$ (Since $a_k \notin I$, we have $f_{11}^{a_k} \notin I^*$).

Suppose $q = (x_1, \dots, x_n)$. By Lemma 3.7 of Meldrum and Van der Walt[5] there exists y_i, $1 \le i \le n$ in N such that $B\,[f_{1\,1}^{x_1} + f_{2\,1}^{x_2} + \dots + f_{n\,1}^{x_n}] = [f_{1\,1}^{y_1} + f_{2\,1}^{y_2} + \dots + f_{n\,1}^{y_n}]$.

Consider $\quad (c_1, \dots, c_n) = Bq = B\;[f_{1\,1}^{x_1} + f_{2\,1}^{x_2} + \dots + f_{n\,1}^{x_n}]\,(1, 0, \dots, 0)$

$$= [f_{1\,1}^{y_1} + f_{2\,1}^{y_2} + \dots + f_{n\,1}^{y_n}]\,(1, 0, \dots, 0) = (y_1, \dots, y_n)$$

$\Rightarrow \quad c_i = y_i$ for $1 \le i \le n$.

Now $\quad B\,[f_{1\,1}^{x_1} + f_{2\,1}^{x_2} + \dots + f_{n\,1}^{x_n}] = [f_{1\,1}^{y_1} + f_{2\,1}^{y_2} + \dots + f_{n\,1}^{y_n}] = [f_{1\,1}^{c_1} + f_{2\,1}^{c_2} + \dots + f_{n\,1}^{c_n}].$

Consider $\quad E_{1s}\,B\,[f_{1\,1}^{x_1} + f_{2\,1}^{x_2} + \dots + f_{n\,1}^{x_n}]\,(\alpha_1, \dots, \alpha_n)$

$$= E_{1s}\,[f_{1\,1}^{c_1} + f_{2\,1}^{c_2} + \dots + f_{n\,1}^{c_n}]\,(\alpha_1, \dots, \alpha_n) = f_{11}^{c_s}\;(\alpha_1, \dots, \alpha_n).$$

Therefore $\quad f_{11}^{c_s} = E_{1s}\,B\,[f_{1\,1}^{x_1} + f_{2\,1}^{x_2} + \dots + f_{n\,1}^{x_n}] \in \langle B \rangle_1\, M_n(N).$

Hence $\quad f_{11}^{a_k} \in \langle A \rangle$ and $f_{11}^{c_s} \in \langle B \rangle_1\, M_n(N)$

$\Rightarrow f_{11}^{a_k c_s} = f_{11}^{a_k} f_{11}^{c_s} \in \langle A \rangle \langle B \rangle_1\, M_n(N) \subseteq I^*$ (by Fact-(1)). $\Rightarrow a_k c_s \in I$, a contradiction. Hence I^* is a prime left ideal of $M_n(N)$.

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