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STEADY FLOW OF AN ELASTICO-VISCOUS FLUID IN POROUS COAXIAL CIRCULAR CYLINDERS

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In the present paper, we have discussed the flow of an elastico-viscous fluid in the annulus of two porous coaxial circular cylinders when both the boundaries are rotating with different angular velocities, secondly if one is moving axially and the other is at rest. While dealing with the above cases, we have taken into account the angular velocity together with the radial velocity which is because of porosity or the axial velocity together with the radial velocity which is due to a porosity parameter. Here we have obtained the exact solution i.e. for all values of the parameters involved in a second order fluid.

INTRODUCTION

The flow through porous boundaries is of great importance both in technological as well as bio-physical fields, examples of which are soil mechanics, transpiration cooling, food preservation, cosmetic industry, blood flow and artificial dialysis. In recent years the problem of fluid flow past porous media or in channels with mass transfer, heat transfer have gained more importance because of varied application. For example, I.V.Fluid containers made of PVC are commonly used these days. Water from inside permeates out thus increasing the concentration of drug inside and sometimes becoming hazardous to life. Therefore, the study relating to suction or injection is very important. The early researchers considered the blood to be a Newtonian fluid but being a suspension of cells it behaves as a non-Newtonian fluid at low shear rates in small arteries.

A large number of theoretical investigations dealing with the steady incompressible laminar flow with either injection or suction, have appeared during the last few decades. Berman[1], Sellars[2], Yuan[3] have studied the two dimensional steady state flow in a rectangular channel with porous walls. Terrill[4,5] obtained series solutions for small Reynolds number, large positive Reynolds number corresponding to large suction and large negative Reynolds number corresponding to large blowing. In a recent paper Terrill[6] has also found the exact solution for a flow in a porous pipe.

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Berman[7] considered the steady state laminar flow of incompressible fluid in an annulus with porous walls. Nanda[8] has obtained exact solution of the Navier-Stokes equations and energy equation for the case of a steady state flow of the fluid through an annulus with porous walls, the inner wall of which is moving with a constant velocity parallel to the axis while the other is at rest. Kapur and Malik[9] and Sinha and Chaudhary[10] have discussed the steady state laminar flow of a viscous incompressible fluid betwen two coaxial porous cylinders rotating with constant angular velocities.

Gupta and Gupta[11] considered the unsteady flow of a fluid through the annular space between two porous coaxial cylinders.

Gupta and Singh[12] have considered the steady problem of porous cylinders where both the cylinders are rotating with different uniform angular velocities about the common axis and the cylinders are in relative motion along the axis and the visco-elastic fluid which is a second order fluid is allowed to flow in the annulus under constant axial pressure gradient. They have assumed that the suction and injection parameters are small.

The solution obtained by Gupta and Singh is valid only for small values of k (the suction and injection parameter). The striking feature of the solutions to follow is that they are valid for all values of k (positive or negative, small or large) and a few more new results have also been obtained.

FORMULATION OF THE PROBLEM

Let us suppose that the fluid is flowing in the annulus of coaxial circular cylinders of radii a_1, b_1 $(a_1 < b_1)$. The inner cylinder is moving with uniform velocity W_0 along its axis, also the inner and outer cylinders are rotating about the common axis of rotation with angular velocities ω_1 and ω_2 respectively.

Cylindrical polar coordinate system (r, θ, z) with z axis along the common axis of cylinders is most suitable for the solution of the problem. Due to symmetry about the axis of the pipe one finds that velocity components should depend only on the radial distance r and therefore, we have

$$u = u\,(r),\ v = v(r),\ w = w(r) \qquad \dots (1)$$

where v and w are angular and axial velocities. Whereas the boundary conditions are

$$u = U_{a_1},\ v = a_1\,\omega_1,\ w = W_0 \quad \text{on} \quad r = a_1$$

$$u = U_{b_1},\ v = b_1\,\omega_2,\ w = 0 \quad \text{on} \quad r = b_1 \qquad \dots (2)$$

where U_{a_1} and U_{b_1} are the uniform injection and suction velocities, and ω_1, ω_2 are the angular velocities.

Let us now non-dimensionalize the parameters in the following manner :

$$u' = \frac{u}{U_0},\ v' = \frac{v}{U_0},\ w' = \frac{w}{U_0},\ p' = \frac{p}{\rho U_0^2},$$

$$R = \frac{\rho U_0 L}{\mu_1},\ r' = \frac{r}{L},\ z' = \frac{z}{L},\ a' = \frac{a_1}{L};$$

$$b' = \frac{b_1}{L},\ \Omega_1' = \frac{\omega_1 L}{U_0},\ \Omega_2' = \frac{\omega_2 L}{U_0},\ W_0' = \frac{W_0}{U_0}, \qquad \dots (3)$$

$$\alpha = \frac{\mu_2 U_0}{\mu_1 L},\ \beta = \frac{\mu_3 U_0}{\mu_1 L},\ U_a' = \frac{U_{a_1}}{U_0},\ U_b' = \frac{U_{b_1}}{U_0}$$

where L is the typical length and α, β are the non-dimensional parameters governing elastico-viscosity and cross-viscosity of the fluid. U_0 is a characteristic velocity.

For the problem under consideration the basic equations as given in Coleman and Noll[13] assume, when non-dimensionalized, the following form

$$\frac{du}{dr} + \frac{u}{r} = 0 \qquad \dots (4)$$

$$-R\left[\frac{k^2}{r^2} + \frac{v^2}{r^2}\right] = -R\frac{\partial p}{\partial r} - 2\alpha\left[2\frac{d^2v}{dr^2}\left(\frac{dv}{dr} - \frac{v}{r}\right) - \frac{1}{r}\frac{dv}{dr}\left(\frac{dv}{dr} - \frac{2v}{r}\right)\right.$$
$$\left. - \frac{v^2}{r^3} + \frac{dw}{dr}\left(2\frac{d^2w}{dr^2} + \frac{1}{r}\frac{dw}{dr}\right) - \frac{12k^2}{r^5}\right]$$
$$+ 4\beta\left[\frac{1}{2}\frac{d^2v}{dr^2}\left(\frac{dv}{dr} - \frac{v}{r}\right) - \frac{1}{2r}\frac{dv}{dr}\left(\frac{dv}{dr} - \frac{2v}{r}\right)\right.$$
$$\left. - \frac{v^2}{2r^3} + \frac{1}{2}\frac{dw}{dr}\left(\frac{d^2w}{dr^2} + \frac{1}{2r}\frac{dw}{dr}\right) - \frac{4k^2}{r^5}\right] \qquad \dots (5)$$

$$\frac{kR}{r}\left[\frac{dv}{dr} + \frac{v}{r}\right] = \left[\frac{d^2v}{dr^2} + \frac{1}{r}\frac{dv}{dr} - \frac{v}{r^2}\right] - \frac{\alpha k}{r}\left[\frac{d^3v}{dr^3} + \frac{2}{r}\frac{d^2v}{dr^2} - \frac{1}{r^2}\frac{dv}{dr} + \frac{v}{r^3}\right] \qquad \dots (6)$$

and

$$\frac{kR}{r}\frac{dw}{dr} = -R\frac{\partial p}{\partial z} + \frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right) - \frac{\alpha k}{r}\left(\frac{d^3w}{dr^3} - \frac{1}{r}\frac{d^2w}{dr^2} + \frac{1}{r^2}\frac{dw}{dr}\right)$$
$$- \frac{2k\beta}{r^2}\left(\frac{d^2w}{dr^2} - \frac{1}{r}\frac{dw}{dr}\right). \qquad \dots (7)$$

Equation (4) on integration yields

$$u = k/r \qquad \dots (8)$$

where k is the non-dimensional constant related to injection and suction velocities such as

$$k = U_a \cdot a = U_b \cdot b \qquad \text{... (9)}$$

where k is positive for injection on the inner cylinder and suction on the outer cylinder and negative for the reverse order.

From the above equations as well as from the equation of continuity (4), we infer that $(\partial p/\partial z)$ should be a function of r and z and let it be given by

$$-p = -\lambda z + g(r) \qquad \text{... (10)}$$

where λ is an absolute constant and $g(r)$ is any arbitrary function of r. Also we find from eqn. (6) that v does not depend on β.

Solution of the Problem

To determine (λ being a constant) v and w, we have to solve the eqns. (6) and (7) subject to the following boundary conditions :

$$u = U_{a_1}, v = a\Omega_1, w = W_0 \quad \text{on} \quad r = a_1$$

$$u = U_{b_1}, v = b\,\Omega_2, w = 0 \quad \text{on} \quad r = b_1. \qquad \text{... (11)}$$

Angular Velocity

First we shall solve eqn. (6) for toroidal (angular or rotational) velocity for both the cases where k is positive or negative.

Case I : When $k \geq 0$

Equation (6) can be put in the following form

$$\frac{kR}{r^2}\frac{d}{dr}(vr) = \frac{1}{r^2}\frac{d}{dr}\left(r^2\frac{dv}{dr} - vr\right) - \frac{k\alpha}{r^2}\frac{d}{dr}\left(r\frac{d^2v}{dr^2} + \frac{dv}{dr} - \frac{v}{r}\right) \qquad \text{... (12)}$$

which on integration yields

$$\frac{d^2v}{dr^2} + \left(\frac{1}{r} - \frac{r}{\alpha k}\right)\frac{dv}{dr} - \frac{v}{r^2} + \left(\frac{1+kR}{\alpha k}\right)v = \frac{A}{\alpha k r} \qquad \text{... (13)}$$

where A is a constant of integration.

Put $v = \dfrac{A'}{r} + v_1$ in (13),

where $A' = \dfrac{A}{2+kR}$ and v_1 satisfies the equation

$$\frac{d^2v_1}{dr^2} + \left(\frac{1}{r} - \frac{r}{\alpha k}\right)\frac{dv_1}{dr} - \frac{v_1}{r^2} + \left(\frac{1+kR}{\alpha k}\right)v_1 = 0. \qquad \text{... (14)}$$

Substituting $v_1 = r\phi(r)$ in (14), we get

$$\phi''(r) + \left(\frac{3}{r} - \frac{r}{\alpha k}\right)\phi'(r) + \frac{R}{\alpha}\phi(r) = 0. \qquad \text{... (15)}$$

Hence the complete and appropriate solution of eqn. (13) is

$$v = \frac{A'}{r} + rB'\, U\left(-\frac{kR}{2}, 2, \frac{r^2}{2\alpha k}\right) \qquad \text{... (16)}$$

where

$$A' = \frac{A}{2+kR} = \frac{a^2b^2\left[\Omega_2\, U\left(-\frac{kR}{2}, 2, \frac{a^2}{2\alpha k}\right) - \Omega_1\, U\left(-\frac{kR}{2}, 2, \frac{b^2}{2\alpha k}\right)\right]}{\left[a^2\, U\left(-\frac{kR}{2}, 2, \frac{a^2}{2\alpha k}\right) - b^2\, U\left(-\frac{kR}{2}, 2, \frac{b^2}{2\alpha k}\right)\right]} \qquad \text{... (17)}$$

$$B' = \frac{(b^2\,\Omega_2 - a^2\,\Omega_1)}{\left[b^2\, U\left(-\frac{kR}{2}, 2, \frac{b^2}{2\alpha k}\right) - a^2\, U\left(-\frac{kR}{2}, 2, \frac{a^2}{2\alpha k}\right)\right]} \qquad \text{... (18)}$$

when $\alpha \to 0$ (keeping k finite) i.e. $\frac{r^2}{2\alpha k} / \frac{a^2}{2\alpha k} \to \infty$, the limiting solution/asymptotic solution for the Newtonian fluid is obtained as follows by using the asymptotic values of $U(a', b', x)$ i.e. Re $x \to \infty$, then following Slater[14], we have

$$\lim_{x \to \infty} U(a', b', x) \sim x^{-a'}$$

then we get

$$v = \frac{A'}{r} + B'\, r^{(1+kR)} \qquad \text{... (19)}$$

where

$$A' = \frac{a^2b^2\,[\Omega_2\, a^{kR} - \Omega_1\, b^{kR}]}{[a^{2+kR} - b^{2+kR}]} \qquad \text{... (20)}$$

and

$$B' = \frac{[a^2\,\Omega_1 - b^2\,\Omega_2]}{[a^{2+kR} - b^{2+kR}]}. \qquad \text{... (21)}$$

Particular case : If $k \to 0$, then we have

$$v = \frac{A'}{r} + B'\, r \qquad \text{... (22)}$$

where

$$A' = \frac{a^2b^2\,(\Omega_2 - \Omega_1)}{(a^2 - b^2)} \qquad \text{... (23)}$$

$$B' = \frac{(a^2\,\Omega_1 - b^2\,\Omega_2)}{(a^2 - b^2)}. \qquad \text{... (24)}$$

Case II : When $k < 0$

In this case, the solution is of the same form except that $U(a', b', x)$ is replaced by ${}_1F_1(a', b', x)$ where x is negative because of $k < 0$ i.e. $k = -k'$ and $k' > 0$.

Then the solution is given by

$$v = \frac{A^1}{r} + B^1 r\, {}_1F_1\left(\frac{k'R}{2}, 2, \frac{-r^2}{2\alpha k'}\right) \quad \text{... (25)}$$

where

$$A^1 = \frac{a^2 b^2\left[\Omega_2\, {}_1F_1\left(\frac{k'R}{2}, 2, \frac{-a^2}{2\alpha k'}\right) - \Omega_1\, {}_1F_1\left(\frac{k'R}{2}, 2, \frac{-b^2}{2\alpha k'}\right)\right]}{\left[a^2\, {}_1F_1\left(\frac{k'R}{2}, 2, \frac{-a^2}{2\alpha k'}\right) - b^2\, {}_1F_1\left(\frac{k'R}{2}, 2, \frac{-b^2}{2\alpha k'}\right)\right]} \quad \text{... (26)}$$

$$B^1 = \frac{(b^2\Omega_2 - a^2\Omega_1)}{\left[b^2\, {}_1F_1\left(\frac{k'R}{2}, 2, \frac{-b^2}{2\alpha k'}\right) - a^2\, {}_1F_1\left(\frac{k'R}{2}, 2, \frac{-a^2}{2\alpha k'}\right)\right]} \quad \text{... (27)}$$

If $\alpha \to 0$ and k remains finite, the results quoted above and represented by eqns. (19), (20) and (21) are again available where we have used the asymptotic values of ${}_1F_1(a', b', x)$ i.e. Re $x \to \infty$ then

$$\lim_{\text{Re}\, x \to \infty} {}_1F_1(c, d, -x) = (-x)^{-c}.$$

Particular case : If $k' \to 0$, then we get the same result represented by eqn. (22).

Axial Velocity

In this section, we shall solve eqn. (7) representing axial velocity for both the cases. Now eqn. (7) can be put in the following form :

$$\frac{kR}{r}\frac{dw}{dr} = -R\frac{\partial p}{\partial z} + \frac{1}{r}\frac{d}{dr}\left(r\frac{dw}{dr}\right) - \frac{k\alpha}{r}\frac{d}{dr}\left(\frac{d^2w}{dr^2} - \frac{1}{r}\frac{dw}{dr}\right) - \frac{2k\beta}{r}\frac{d}{dr}\left(\frac{1}{r}\frac{dw}{dr}\right) \quad \text{... (28)}$$

which can be solved by putting

$$w = w_1 + \lambda_1 r^2 \quad \text{... (29)}$$

where

$$\lambda_1 = \frac{R(\partial p/\partial z)}{(4 - 2kR)} \quad \text{... (30)}$$

and w_1 satisfies the following equation

$$\frac{kR}{r}\frac{dw_1}{dr} = \frac{1}{r}\frac{d}{dr}\left(r\frac{dw_1}{dr}\right) - \frac{\alpha k}{r}\frac{d}{dr}\left(\frac{d^2w_1}{dr^2} - \frac{1}{r}\frac{dw_1}{dr}\right) - \frac{2k\beta}{r}\frac{d}{dr}\left(\frac{1}{r}\frac{dw_1}{dr}\right). \quad \text{... (31)}$$

After integration of (31), we have

$$\frac{d^2w_1}{dr^2} + \left\{\left(\frac{2\beta}{\alpha} - 1\right)\frac{1}{r} - \frac{r}{\alpha k}\right\}\frac{dw_1}{dr} + \frac{R}{\alpha}w_1 = \frac{D}{\alpha k} = D' \quad \text{... (32)}$$

where D is the constant of integration.

Putting $z = r^2/2\alpha k$, we have

$$z\frac{d^2w_1}{dz^2} + (\beta/\alpha - z)\frac{dw_1}{dz} + \frac{kR}{2}w_1 = \frac{D'\alpha k}{2}. \quad \text{... (33)}$$

The appropriate solution of the above equation (33) for the two cases when k is positive or k is negative are given as follows:

Case I : When k > 0

Equation (33) is of Kummer's type equation whose solution is a confluent hypergeometric function. Hence the complete solution is

$$w = \lambda_1 r^2 + A_2 U\left(-\frac{kR}{2}, \beta/\alpha, \frac{r^2}{2\alpha k}\right) + D_2, \quad \text{... (34)}$$

where

$$A_2 = \frac{[W_0 - \lambda_1(a^2 - b^2)]}{\left[U\left(-\frac{kR}{2}, \beta/\alpha, \frac{a^2}{2\alpha k}\right) - U\left(-\frac{kR}{2}, \beta/\alpha, \frac{b^2}{2\alpha k}\right)\right]} \quad \text{... (35)}$$

$$D_2 = -\lambda_1 b^2 - \frac{[W_0 - \lambda_1(a^2 - b^2)]\left[U\left(-\frac{kR}{2}, \beta/\alpha, \frac{a^2}{2\alpha k}\right)\right]}{\left[U\left(-\frac{kR}{2}, \beta/\alpha, \frac{a^2}{2\alpha k}\right) - U\left(-\frac{kR}{2}, \beta/\alpha, \frac{b^2}{2\alpha k}\right)\right]} \quad \text{... (36)}$$

Making use of $U(a, b, x) = x^{1-b}\, U(1 + a - b, 2 - b, x)$, eqn. (34) can be written in the following form

$$w = \lambda_1 r^2 + A_2'(r^2)^{1-(\beta/\alpha)}\, U\left(1 - \frac{kR}{2} - \frac{\beta}{\alpha}, 2 - \frac{\beta}{\alpha}, \frac{r^2}{2\alpha k}\right) + D_2' \quad \text{... (37)}$$

where

$$A_2' = \frac{[W_0 - \lambda_1 (a^2 - b^2)]}{\left[(a^2)^{1-\beta/\alpha} U\left(1 - \frac{kR}{2} - \frac{\beta}{\alpha}, 2 - \frac{\beta}{\alpha}, \frac{a^2}{2\alpha k}\right) - (b^2)^{1-(\beta/\alpha)} U\left(1 - \frac{kR}{2} - \frac{\beta}{\alpha}, 2 - \frac{\beta}{\alpha}, \frac{b^2}{2\alpha k}\right)\right]} \quad \text{... (38)}$$

and

$$D_2' = -\lambda_1 b^2 - \frac{[W_0 - \lambda_1 (a^2 - b^2)]\left[(b^2)^{1-(\beta/\alpha)} U\left(1 - \frac{kR}{2} - \frac{\beta}{\alpha}, 2 - \frac{\beta}{\alpha}, \frac{b^2}{2\alpha k}\right)\right]}{\left[(a^2)^{1-\beta/\alpha} U\left(1 - \frac{kR}{2} - \frac{\beta}{\alpha}, 2 - \frac{\beta}{\alpha}, \frac{a^2}{2\alpha k}\right) - (b^2)^{1-\beta/\alpha} U\left(1 - \frac{kR}{2} - \frac{\beta}{\alpha}, 2 - \frac{\beta}{\alpha}, \frac{b^2}{2\alpha k}\right)\right]}. \quad \text{... (39)}$$

Applying the following two techniques, we have obtained the particular cases in which some are again quite new results, whereas if $\beta \to 0$ and later $\alpha \to 0$ or vice-versa give the well-known results for a Newtonian fluid, we get

$$w = \lambda_1 (r^2 - b^2) + \frac{[W_0 - \lambda_1 (a^2 - b^2)]\,(r^{kR} - b^{kR})}{(a^{kR} - b^{kR})} \quad \text{... (40)}$$

the detailed calculation have been avoided to save space.

Further if $k \to 0$, the well known result is

$$w = \lambda_1 (r^2 - b^2) + \frac{[W_0 - \lambda_1 (a^2 - b^2)]\,(\log r - \log b)}{(\log a - \log b)}. \quad \text{... (41)}$$

If $W_0 \to 0$, then the following known results are

$$w = \lambda_1 (r^2 - b^2) - \frac{\lambda_1 (a^2 - b^2)\,(\log r - \log b)}{(\log a - \log b)}. \quad \text{... (42)}$$

If $a = 1$

$$w = \lambda_1 (r^2 - b^2) + \frac{\lambda_1 (1 - b^2)\,(\log r - \log b)}{\log b}. \quad \text{... (43)}$$

Case II : When $k < 0$, i.e. $k = -k'$, where $k' > 0$

Following the same procedure as before and replacing U by ${}_1F_1$, the solution can be written in the following form

$$w = \lambda_1 r^2 + A_2 \frac{1}{\Gamma(\beta/\alpha)}\, {}_1F_1\left(\frac{k'R}{2}, \beta/\alpha, \frac{-r^2}{2\alpha k'}\right) + D_2 \quad \text{... (44)}$$

which implies that

$$w = \lambda_1 r^2 + A_2\, \psi\left(\frac{k'R}{2}, \beta/\alpha, -\frac{r^2}{2\alpha k'}\right) + D_2 \quad \text{... (45)}$$

where

$$\psi\left(\frac{k'R}{2}, \beta/\alpha, -\frac{r^2}{2\alpha k'}\right) = \frac{1}{\Gamma(\beta/\alpha)}\, {}_1F_1\left(\frac{k'R}{2}, \beta/\alpha, -\frac{r^2}{2\alpha k'}\right).$$

Also

$$A_2 = \frac{[W_0 - \lambda_1 (a^2 - b^2)]}{\left[\psi\left(\frac{k'R}{2}, \beta/\alpha, -\frac{a^2}{2\alpha k'}\right) - \psi\left(\frac{k'R}{2}, \beta/\alpha, -\frac{b^2}{2\alpha k'}\right)\right]} \quad \ldots (46)$$

$$D_2 = -\lambda_1 b^2 - \frac{[W_0 - \lambda_1 (a^2 - b^2)]\, \psi\left(\frac{k'R}{2}, \beta/\alpha, -\frac{b^2}{2\alpha k'}\right)}{\left[\psi\left(\frac{k'R}{2}, \beta/\alpha, -\frac{a^2}{2\alpha k'}\right) - \psi\left(\frac{k'R}{2}, \beta/\alpha, -\frac{b^2}{2\alpha k'}\right)\right]} \quad \ldots (47)$$

If $\alpha \to 0$ and $\beta \to 0$ this leads to a Newtonian case, so we have

$$w = \lambda_1 (r^2 - b^2) + \frac{[W_0 - \lambda_1 (a^2 - b^2)]\,(r^{-k'R} - b^{-k'R})}{(a^{-k'R} - b^{-k'R})}. \quad \ldots (48)$$

(This is a part of the thesis submitted to University of Rajasthan by Gupta[15].)

Discussion of Result

We have taken into consideration the boundary layer near the inner cylinder as well as the outer cylinder. We will discuss the effect of the various parameters α and β the coefficient of viscosity and cross-viscosity and k the porosity parameter. The following conclusions are arrived at as a result of the present investigations :

Figure 1 represents the angular velocity against r, the radial distance for fixed values of R and α, the Reynolds number and visco-elastic parameter respectively but for different values of k ($k \geq 0$, $k < 0$). From this figure, it is evident that when $k \geq 0$, the velocity field decreases continuously with the increase of k upto $k = 3$ and its behaviour changes at $k = 4$. On the other hand when $k < 0$, the velocity field increases continuously with the increase of k' ($k = -k'$, $k' > 0$) which continues upto $k' = 3$ and its behaviour changes when $k' = 4$.

From Fig. 2, fixing α and k, the visco-elastic parameter and porosity parameter respectively, it is observed that when $k > 0$, the contours become dramatically distorted on increasing R i.e. in certain cases the velocity increases and in other decreases. When $k < 0$, the angular velocity increases.

In certain cases, the fluid rotates faster than the boundary which is also in motion. Although, at first, it may seem to be incorrect but this behaviour can be shown to be possible by recalling conservation of angular momentum.

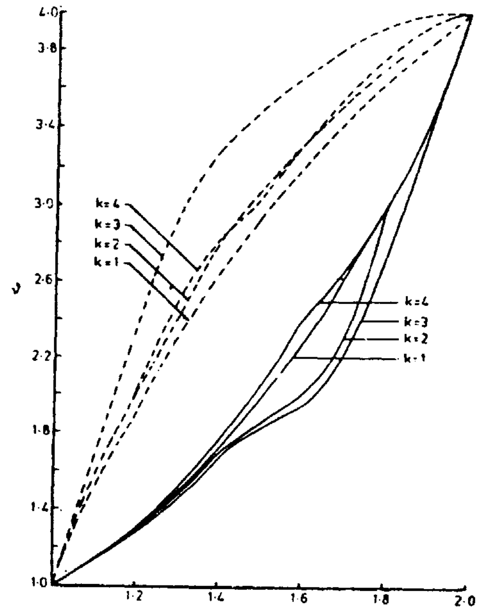

FIG. 1. Variation of angular velocity with porosity parameters k (R = 1, α = 1), — k, ... ($-$ k).

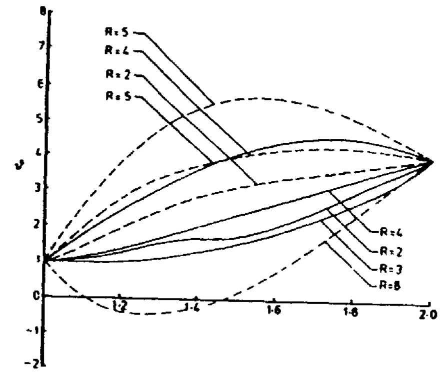

FIG. 2. Variation of angular velocity with Reynold number R. — k, ----- ($-k$) (α = 2, k = ± 1).

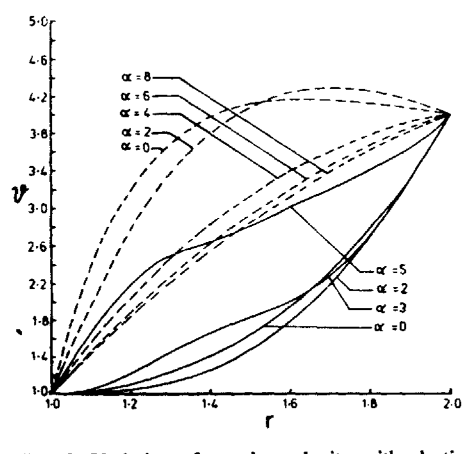

FIG. 3. Variation of angular velocity with elastic parameter α. — k (k = 1, R = 3), ---- ($-k$), k = $-$ 1, R = 4.

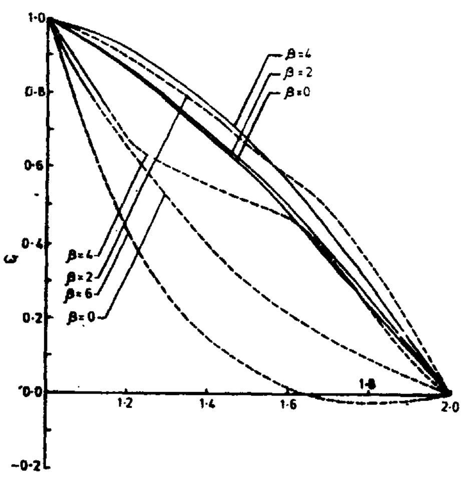

FIG. 4. Response of axial component with an increase in cross viscous force. — k, ---- ($-k$), α = 2, k = ± 1, R = 1.

Figure 3, depicts the effect of α, for fixed values of k and R. The velocity increases as α increases for $k > 0$ in contrast to that of the velocity obtained by Gupta and Singh for $k < 0$.

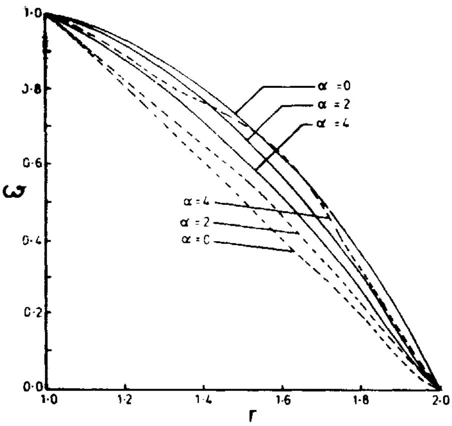

FIG. 5. Variation of axial velocity with elastic parameter α. — k, ------ ($-k$), $\beta = 4$, $k = \pm 1$, $R = 1$.

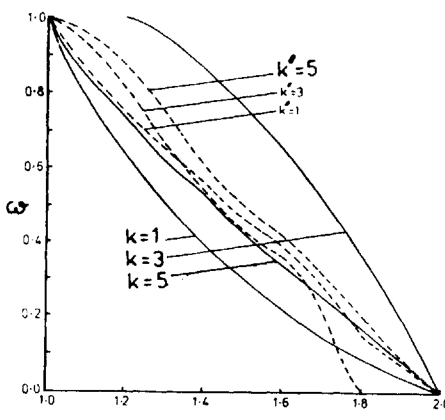

FIG. 6. Response of axial velocity with an increase in porosity parameter $k = -k'$ ($k > 0$), $\alpha = 1$, $R = 1$, $\beta = 2$.

It is further evident that if $\alpha = 2$, the angular velocity decreases in comparison to Newtonian fluid. Now in case of $k < 0$, generally the angular velocity decreases on increasing α (fixing $k = -1$ and $R = -4$) whereas in case of $\alpha = 2$, the velocity increases at a few points in comparison to the Newtonian fluid i.e. $\alpha = 0$ and the maxima shifts towards the outer boundary.

Figure 4 represents the axial velocity w by changing cross-viscosity β (fixing $\alpha = 2$, $R = 1$ and $k = \pm 1$). For $k > 0$ and $\beta > 0$, the velocity increases with the increase of β and when $k < 0$ and $\beta > 0$, the velocity decreases with the increase of β and as $k < 0$, $\beta = 0$, the velocity is increasing in some cases while decreasing in others.

Figure 5 depicts the effect of α, for fixed values of β, k and R. In this case, the axial velocity decreases when $k > 0$ with the increase of α and increases when $k < 0$.

From Fig. 6, assuming $R = 1$, $\alpha = 1$ and $\beta = 2$, it is concluded that for $k' = 1$ and $k' = 5$, the velocity increases continuously as compared to $k = 1$ and $k = 5$ respectively but in case when $k' = 3$, the velocity decreases continuously in contrast to $k = 3$. The curve has a distorted behaviour i.e. increasing for some values and decreasing for others.

The present study could be of much interest to the Bio-Engineers involved in design and construction of artificial organ, e.g. artificial dialysis.

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