Indian J. pure appl. Math., 27(6) : 621-631, June 1996

HEAT AND MASS TRANSFER ALONG A VERTICAL PLATE IN THE PRESENCE OF A MAGNETIC FIELD

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(*Received* 14 *February* 1995; *after revision* 24 *October* 1995; *accepted* 4 *January* 1996)

Heat and mass transfer along a vertical plate under the combined buoyancy force effects of thermal and species diffusion in the presence of a transverse magnetic field is investigated. The boundary layer equations are transformed to ordinary differential equations. The solution is dependent on the major parameters, Prandtl number, Schmidt number, magnetic parameter and the relative buoyancy force effect between species and thermal diffusion. Numerical results are presented. The maximum value. of velocity, the local wall shear stress, local Nusselt number and local Sherwood number are found for different values of magnetic parameters.

INTRODUCTION

Many transport processes occur in industrial applications in which the transfer of heat and mass takes place simultaneously as a result of combined buoyancy effects of thermal diffusion and diffusion of chemical species. Many studies have been reported for vertical[1], horizontal[2, 3] and inclined plate[4]. In this paper is considered the temperature and concentration of the fluid at infinity as T_∞ and C_∞ every where and the temperature and concentration at the plate are T_w and C_w.

The same problem was solved without mass transfer by Gupta[5]. In this study the induced magnetic field is neglected in the analysis and the mass transfer due to temperature gradient is neglected also.

EQUATIONS

The coordinates (x', y') are measured along the semi-infinite plate and normal to it respectively with the origin at the leading edge. The x'-axis being parallel to the direction of gravity but directed upward. The magnetic field is in the y'-direction. The differential equations for fluid flow can be written as :

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$$\frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \qquad \text{... (1)}$$

$$u'\frac{\partial u'}{\partial x'} + v'\frac{\partial u'}{\partial y'} = \nu\frac{\partial^2 u'}{\partial y'^2} + \beta\, g(T' - T_\infty) + \beta^*\, g\,(C' - C_\infty) - \frac{\sigma B^2}{\rho}u' \qquad \text{... (2)}$$

$$u'\frac{\partial T'}{\partial x'} + v'\frac{\partial T'}{\partial y'} = \alpha\frac{\partial^2 T'}{\partial y'^2} \qquad \text{... (3)}$$

$$u'\frac{\partial C'}{\partial x'} + v'\frac{\partial C'}{\partial y'} = D\frac{\partial^2 C'}{\partial y'^2} \qquad \text{... (4)}$$

where C' is the concentration, T' the temperature, (β, β^*) are the temperature and concentration coefficients of volumetric expansion, ν is the kinematic viscosity, ρ the density, σ electrical conductivity, g gravitational acceleration further (u', v') are the components of velocity along the (x', y') axes, α is the thermal diffusivity and B the magnetic field strength.

Following Elbashbeshy[6] the following dimensionless variables are introduced

$$x = \frac{x'}{L}, \; y = \frac{y'}{L}G^{1/4}, \; \psi = \frac{\psi' G^{1/4}}{UL}, \; B^2 = B_0 x^{-1/2} \; (B_0 \text{ constant})$$

$$T = \frac{T' - T_\infty}{T_w - T_\infty}, \quad C = \frac{C' - C_\infty}{C_w - C_\infty}$$

where ψ' is the stream function such that the continuity equation is satisfied. i.e.

$$u' = \frac{\partial\psi'}{\partial y'}, \; v' = -\frac{\partial\psi'}{\partial x'}.$$

L is a typical length along the plate, $U = \sqrt{g\beta L(T_w - T_\infty)}$ is quantity with the dimension of speed and $G = g\beta L^3(T_w - T_\infty)/\nu^2$.

The equations (1-4) must be solved subject to the boundary conditions :

At $\quad y = 0, \;\; u = v = 0, \; T' = T_w, \;\; C' = C_w \qquad$ ... (5)

$y \rightarrow \infty, \;\; u = 0, \; T' = T_\infty, \;\; C' = C_\infty.$

We introduce the variables :

$$\psi' = x^{3/4} f(\eta), \quad \eta = x^{-1/4} y, \quad T = \theta(\eta), \quad C = \phi(\eta).$$

Substituting these into eqns. (1)-(4) give rise to the following ordinary differential equations

$$f''' = \frac{3}{4}ff'' - \frac{1}{2}f'^2 + \theta + e\phi - Mf' = 0 \qquad \text{... (6)}$$

$$\theta'' + \frac{3}{4} Pr f \theta' = 0 \quad \text{... (7)}$$

$$\phi'' + \frac{3}{4} Sc f \phi' = 0 \quad \text{... (8)}$$

where $e = \beta^* (C_w - C_\infty)/\beta(T_w - T_\infty)$ represents the relative effect of chemical diffusion on thermal diffusion. When $e = 0$ there is no mass diffusion and the buoyancy force arises solely from the temperature difference. Pr, Sc and M are Prandtl number, Schmidt number and magnetic field parameter. Primes denote differentiation with· respect to η.

The boundary conditions in the new variables are :

$$\eta = 0, \quad f = f' = 0, \quad \theta = \phi = 1$$

$$\eta \to \infty, \quad f' = 0, \quad \theta = \phi = 0 \quad \text{... (9)}$$

The governing equations (6)-(8) with the boundary conditions given by eqn. (9) are solved numerically by using the fourth order Runge-Kutta integration scheme with the Newton-Raphson shooting technique.

The major physical quantities of interest are the local shear stress τ_w, local Nusselt number Nu and the local Sherwood number Sh. They are defined, respectively, by

$$\tau_w = \mu \left. \frac{\partial u'}{\partial y'} \right|_{y=0}$$

$$Nu = -\alpha \frac{\partial T'}{\partial y'} / (T_w - T_\infty)$$

$$Sh = -D \frac{\partial C'}{\partial y'} / (C_w - C_\infty).$$

Numerical results are presented for $Pr = 0.73$, 1, 10, $Sc = 0.6$, 1.10, $e = 0.0$, 0.01, 0.1 and $M = 0.0$, 0.1, 1 (Figs. 1-9).

RESULTS

In Figs. 1-5, we note that the maximum value of velocity decreases with increasing the magnetic parameter M and increases with increasing the parameter e and Prandtl number. The temperature function $\theta(\eta)$ and the concentration function $\theta(\eta)$ increases with increasing magnetic parameter M. The effect of the parameter e on the temperature and concentration is very small.

In Tables I and II we note that the local wall shear stress decreases with increasing the magnetic parameter M and increases with increasing parameter e and Prandtl number.

The local Nusselt number and local Sherwood number decrease with increasing magnetic parameter M and parameter e.

In case $M = 0$, the maximum value of velocity, local shear stress, local Nusselt number and local sherwood number have the same behaviours.

TABLE I

Results for f'', θ', ϕ' *at* $e = 0.1$, $Sc = 1$, $Pr = 0.73$ *and* $M = 0.0, 0.1, 1$

	$M = 0.0$			$M = 0.1$			$M = 1$		
ETA	F**	THITA	PHAY*	F**	THITA	PHAY*	F**	THITA	PHAY*
0.0000	0.9421	− 0.3904	− 0.4275	0.9388	− 0.3899	− 0.4269	0.9100	− 0.3857	− 0.4213
0.1000	0.8343	− 0.3903	− 0.4275	0.8310	− 0.3899	− 0.4268	0.8026	− 0.3857	− 0.4213
0.2000	0.7311	− 0.3901	− 0.4272	0.7279	− 0.3896	− 0.4265	0.7005	− 0.3854	− 0.4210
0.3000	0.6326	− 0.3895	− 0.4263	0.6296	− 0.3891	− 0.4257	0.6037	− 0.3849	− 0.4202
0.4000	0.5391	− 0.3885	− 0.4247	0.5363	− 0.3880	− 0.4241	0.5125	− 0.3839	− 0.4186
0.5000	0.4507	− 0.3968	− 0.4222	0.4482	− 0.3863	− 0.4215	0.4268	− 0.3823	− 0.4162
0.6000	0.3676	− 0.3844	− 0.4186	0.3654	− 0.3839	− 0.4180	0.3467	− 0.3800	− 0.4128
0.7000	0.2899	− 0.3812	− 0.4138	0.2881	− 0.3808	− 0.4132	0.2721	− 0.3770	− 0.4083
0.8000	0.2178	− 0.3772	− 0.4078	0.2163	− 0.3768	− 0.4073	0.2032	− 0.3732	− 0.4027
0.9000	0.1511	− 0.3723	− 0.4006	0.1500	− 0.3719	− 0.4001	0.1399	− 0.3685	− 0.3879
1.0000	0.0900	− 0.3665	− 0.3922	0.0892	− 0.3662	− 0.3917	0.0821	− 0.3631	− 0.3859
1.1000	0.0344	− 0.3599	− 0.3826	0.0339	− 0.3596	− 0.3822	0.0296	− 0.3568	− 0.3788
1.2000	− 0.0157	− 0.3526	− 0.3718	− 0.0159	− 0.3523	− 0.3715	− 0.0175	− 0.3498	− 0.3686
1.3000	− 0.0606	− 0.3444	− 0.3601	− 0.0605	− 0.3442	− 0.3598	− 0.0596	− 0.3421	− 0.3575
1.4000	− 0.1003	− 0.3356	− 0.3475	− 0.0999	− 0.3354	− 0.3473	− 0.0967	− 0.3337	− 0.3456
1.5000	− 0.1350	− 0.3261	− 0.3342	− 0.1344	− 0.3260	− 0.3341	− 0.1291	− 0.3248	− 0.3329

1.6000	– 0.1650	– 0.3162	– 0.3203	– 0.1641	– 0.3161	– 0.3202	– 0.1570	– 0.3153	– 0.3197
1.7000	– 0.1904	– 0.3058	– 0.3059	– 0.1894	– 0.3057	– 0.3060	– 0.1807	– 0.3054	– 0.3061
1.8000	– 0.2116	– 0.2950	– 0.2913	– 0.2104	– 0.2950	– 0.2914	– 0.2004	– 0.2952	– 0.2921
1.9000	– 0.2288	– 0.2840	– 0.2765	– 0.2275	– 0.2841	– 0.2767	– 0.2163	– 0.2847	– 0.2780
2.0000	– 0.2423	– 0.2729	– 0.2618	– 0.2409	– 0.2730	– 0.2620	– 0.2289	– 0.2740	– 0.2638
2.1000	– 0.2523	– 0.2617	– 0.2471	– 0.2508	– 0.2618	– 0.2474	– 0.2383	– 0.2633	– 0.2497
2.2000	– 0.2592	– 0.2504	– 0.2327	– 0.2577	– 0.2507	– 0.2331	– 0.2448	– 0.2525	– 0.2359
2.3000	– 0.2632	– 0.2393	– 0.2187	– 0.2617	– 0.2396	– 0.2190	– 0.2486	– 0.2418	– 0.2223
2.4000	– 0.2646	– 0.2283	– 0.2050	– 0.2631	– 0.2286	– 0.2054	– 0.2501	– 0.2312	– 0.2090
2.5000	– 0.2637	– 0.2175	– 0.1919	– 0.2622	– 0.2178	– 0.1923	– 0.2494	– 0.2208	– 0.1962
2.6000	– 0.2607	– 0.2070	– 0.1792	– 0.2593	– 0.2073	– 0.1797	– 0.2468	– 0.2106	– 0.1839
2.7000	– 0.2559	– 0.1967	– 0.1672	– 0.2545	– 0.1971	– 0.1677	– 0.2425	– 0.2006	– 0.1721
2.8000	– 0.2494	– 0.1868	– 0.1557	– 0.2481	– 0.1872	– 0.1563	– 0.2386	– 0.1910	– 0.1609
2.9000	– 0.2415	– 0.1772	– 0.1449	– 0.2403	– 0.1777	– 0.1455	– 0.2297	– 0.1817	– 0.1502
3.0000	– 0.2324	– 0.1680	– 0.1347	– 0.2313	– 0.1685	– 0.1353	– 0.2214	– 0.1727	– 0.1401
3.1000	– 0.2223	– 0.1592	– 0.1251	– 0.2212	– 0.1597	– 0.1257	– 0.2122	– 0.1640	– 0.1306
3.2000	– 0.2112	– 0.1508	– 0.1162	– 0.2103	– 0.1513	– 0.1167	– 0.2022	– 0.1558	– 0.1217
3.3000	– 0.1994	– 0.1428	– 0.1078	– 0.1986	– 0.1433	– 0.1084	– 0.1914	– 0.1479	– 0.1133
3.5000	– 0.1741	– 0.1280	– 0.0928	– 0.1736	– 0.1285	– 0.0933	– 0.1682	– 0.1332	– 0.0982

TABLE II

Results for f'', θ', ϕ' *at different values of* e, Sc *and* $M = 0.1$

	$Sc = 1$, $Pr = 0.73$, $e = 0.0$			$Sc = 1$, $Pr = 0.73$, $e = 0.1$			$Sc = 1$, $Pr = 1$, $e = 0.0$			$Sc = 1$, $Pr = 1$, $e = 0.1$		
ETA	F**	THITA*	PHAY*	F**	THITA*	PHAY*	F**	THITA*	PHAY*	F**	THITA*	PHAY*
0.0000	0.9421	– 0.3904	– 0.4275	0.9173	– 0.4219	– 0.4219	0.8730	– 0.3842	– 0.4193	0.8486	– 0.4137	– 0.4137
0.1000	0.8343	– 0.3903	– 0.4275	0.8097	– 0.4218	– 0.4218	0.7750	– 0.3842	– 0.4193	0.7507	– 0.4137	– 0.4137
0.2000	0.7311	– 0.3901	– 0.4272	0.7069	– 0.4215	– 0.4215	0.6809	– 0.3840	– 0.4190	0.6571	– 0.4134	– 0.4134
0.3000	0.6326	– 0.3895	– 0.4263	0.6091	– 0.4207	– 0.4207	0.5911	– 0.3835	– 0.4182	0.5680	– 0.4126	– 0.4126
0.4000	0.5391	– 0.3885	– 0.4247	0.5166	– 0.4192	– 0.4192	0.5057	– 0.3825	– 0.4168	0.4836	– 0.4112	– 0.4112
0.5000	0.4507	– 0.3868	– 0.4222	0.4295	– 0.4167	– 0.4176	0.4248	– 0.3809	– 0.4144	0.4039	– 0.4090	– 0.4090
0.6000	0.3676	– 0.3844	– 0.4186	0.3479	– 0.4133	– 0.4133	0.3486	– 0.3787	– 0.4112	0.3291	– 0.4059	– 0.4059
0.7000	0.2899	– 0.3812	– 0.4138	0.2719	– 0.4088	– 0.4088	0.2772	– 0.3758	– 0.4068	0.2594	– 0.4018	– 0.4018
0.8000	0.2178	– 0.3772	– 0.4078	0.2016	– 0.4031	– 0.4031	0.2107	– 0.3721	– 0.4013	0.1946	– 0.3965	– 0.3956
0.9000	0.1511	– 0.3723	– 0.4006	0.1370	– 0.3962	– 0.3962	0.1490	– 0.3676	– 0.3947	0.1349	– 0.3902	– 0.3902
1.0000	0.0900	– 0.3665	– 0.3922	0.0781	– 0.3881	– 0.3881	0.0923	– 0.3623	– 0.3869	0.0803	– 0.3828	– 0.3828
1.1000	0.0344	– 0.3599	– 0.3826	0.0248	– 0.3790	– 0.3790	0.0404	– 0.3562	– 0.3780	0.0306	– 0.3744	– 0.3744
1.2000	– 0.0157	– 0.3526	– 0.3718	– 0.0230	– 0.3687	– 0.3687	– 0.0066	– 0.3494	– 0.3681	– 0.0141	– 0.3649	– 0.3649
1.3000	– 0.0606	– 0.3444	– 0.3601	– 0.0655	– 0.3576	– 0.3576	– 0.0489	– 0.3418	– 0.3572	– 0.0541	– 0.3546	– 0.3546
1.4000	– 0.1003	– 0.3356	– 0.3475	– 0.1029	– 0.3455	– 0.3455	– 0.0865	– 0.3336	– 0.3455	– 0.0894	– 0.3434	– 0.3434
1.5000	– 0.1350	– 0.3261	– 0.3342	– 0.1353	– 0.3328	– 0.3328	– 0.1197	– 0.3248	– 0.3331	– 0.1203	– 0.3316	– 0.3316

1.6000	– 0.1650	– 0.3162	– 0.3203	– 0.1630	– 0.3159	– 0.3159	– 0.1485	– 0.3154	– 0.3200	– 0.1469	– 0.3192	– 0.3192
1.7000	– 0.1904	– 0.3058	– 0.3059	– 0.1863	– 0.3058	– 0.3058	– 0.1733	– 0.3057	– 0.3066	– 0.1696	– 0.3063	– 0.3063
1.8000	– 0.2116	– 0.2950	– 0.2913	– 0.2055	– 0.2918	– 0.2918	– 0.1942	– 0.2956	– 0.2927	– 0.1884	– 0.2931	– 0.2931
1.9000	– 0.2288	– 0.2840	– 0.2765	– 0.2208	– 0.2777	– 0.2777	– 0.2113	– 0.2852	– 0.2788	– 0.2037	– 0.2797	– 0.2797
2.0000	– 0.2423	– 0.2729	– 0.2618	– 0.2325	– 0.2635	– 0.2635	– 0.2251	– 0.2746	– 0.2647	– 0.2157	– 0.2663	– 0.2663
2.1000	– 0.2523	– 0.2617	– 0.2471	– 0.2411	– 0.2694	– 0.2694	– 0.2357	– 0.2640	– 0.2507	– 0.2248	– 0.2529	– 0.2529
2.2000	– 0.2592	– 0.2504	– 0.2327	– 0.2466	– 0.2355	– 0.2355	– 0.2433	– 0.2533	– 0.2369	– 0.2310	– 0.2396	– 0.2396
2.3000	– 0.2632	– 0.2393	– 0.2187	– 0.2496	– 0.2219	– 0.2219	– 0.2481	– 0.2426	– 0.2234	– 0.2348	– 0.2265	– 0.2266
2.4000	– 0.2646	– 0.2283	– 0.2050	– 0.2501	– 0.2087	– 0.2087	– 0.2506	– 0.2321	– 0.2102	– 0.2362	– 0.2138	– 0.2138
2.5000	– 0.2637	– 0.2175	– 0.1919	– 0.2485	– 0.1959	– 0.1959	– 0.2507	– 0.2217	– 0.1974	– 0.2357	– 0.2014	– 0.2014
2.6000	– 0.2607	– 0.2070	– 0.1792	– 0.2451	– 0.1836	– 0.1836	– 0.2489	– 0.2115	– 0.1851	– 0.2333	– 0.1895	– 0.1895
2.7000	– 0.2559	– 0.1967	– 0.1672	– 0.2400	– 0.1718	– 0.1718	– 0.2452	– 0.2016	– 0.1733	– 0.2294	– 0.1780	– 0.1780
2.8000	– 0.2494	– 0.1868	– 0.1557	– 0.2336	– 0.1606	– 0.1606	– 0.2399	– 0.1919	– 0.1620	– 0.2240	– 0.1670	– 0.1670
2.9000	– 0.2415	– 0.1772	– 0.1449	– 0.2259	– 0.1500	– 0.1500	– 0.2332	– 0.1826	– 0.1513	– 0.2175	– 0.1565	– 0.1565
3.0000	– 0.2324	– 0.1680	– 0.1347	– 0.2172	– 0.1399	– 0.1399	– 0.2253	– 0.1736	– 0.1412	– 0.2099	– 0.1466	– 0.1466
3.1000	– 0.2223	– 0.1592	– 0.1251	– 0.2077	– 0.1304	– 0.1304	– 0.2162	– 0.1650	– 0.1317	– 0.2014	– 0.1372	– 0.1372
3.2000	– 0.2112	– 0.1508	– 0.1162	– 0.1974	– 0.1215	– 0.1215	– 0.2063	– 0.1567	– 0.1227	– 0.1922	– 0.1283	– 0.1283
3.3000	– 0.1994	– 0.1428	– 0.1078	– 0.1866	– 0.1132	– 0.1132	– 0.1955	– 0.1488	– 0.1143	– 0.1823	– 0.1199	– 0.1199
3.4000	– 0.1870	– 0.1352	– 0.1000	– 0.1754	– 0.1054	– 0.1054	– 0.1840	– 0.1412	– 0.1065	– 0.1720	– 0.1121	– 0.1121

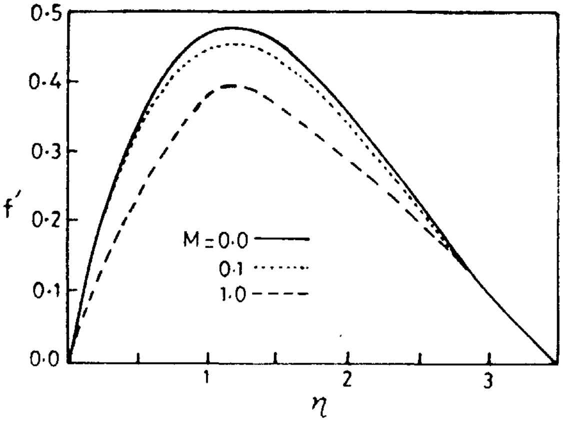

FIG. 1. Velocity profiles for e = 0.1; Pr = 0.73; Sc = 1 at M = 0.0, 0.1, 1.

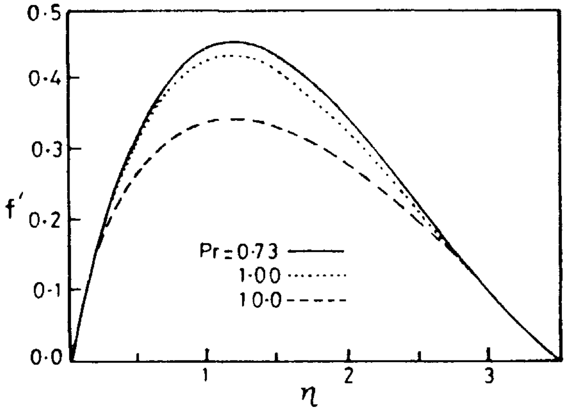

FIG. 2. Velocity profiles for e = 0.0; Pr = 0.73, 1.10; Sc = 1.

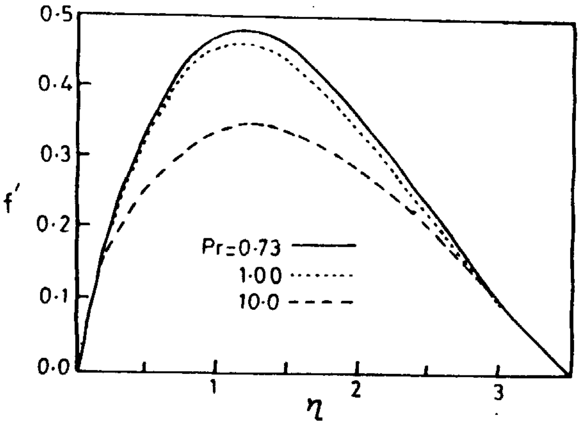

FIG. 3. Velocity profiles for e = 0.1; Pr = 0.73, 1, 10; Sc = 1.

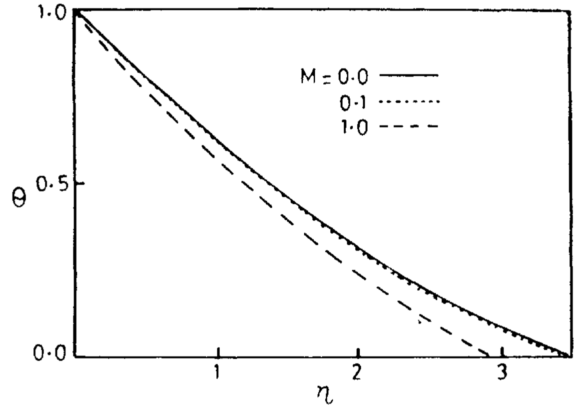

FIG. 4. Temperature profiles for $e = 0.1$; $Pr = 0.73, 1, 10$; $Sc = 1$ at $M = 0.0, 0.1, 1$.

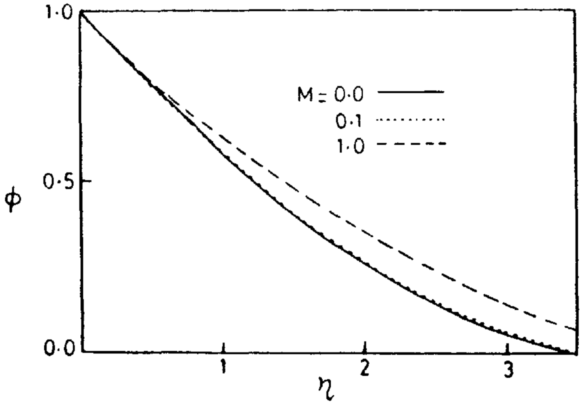

FIG. 5. Concentration profiles for $e = 0.1$; $Pr = 0.73$; $Sc = 1$ at $M = 0.0, 0.1, 1$.

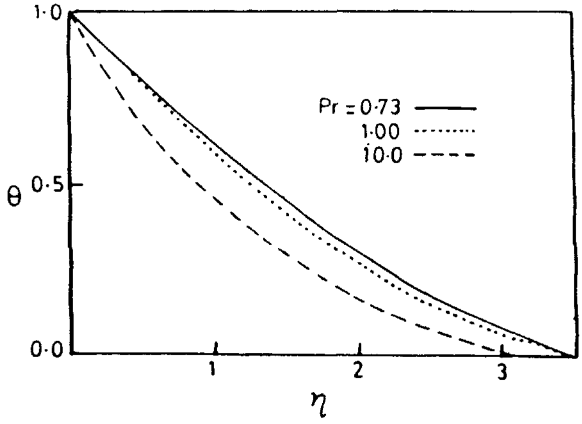

FIG. 6. Temperature profiles for $e = 0.0$; $Sc = 1$; $Pr = 0.73, 1, 10$.

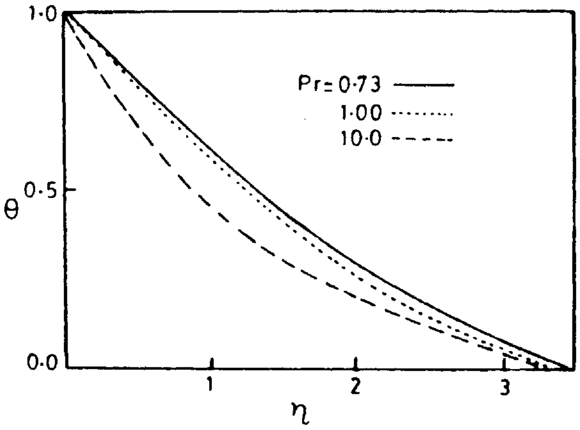

FIG. 7. Temperature profiles for $e = 0.1$; $Sc = 1$; $Pr = 0.73, 1, 10$.

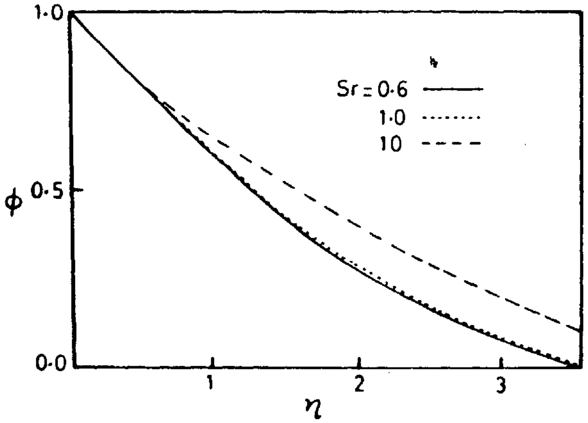

FIG. 8. Concentration profiles for $e = 0.0$; $Sc = 0.6, 1, 10$; $Pr = 1$.

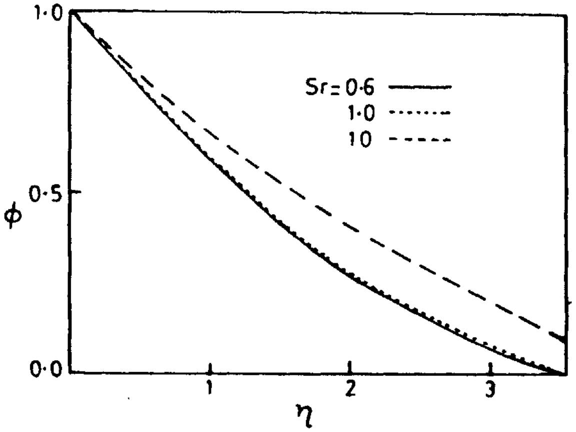

FIG. 9. Concentration profiles for $e = 0.1$; $Sc = 0.6, 1, 10$; $Pr = 1$.

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