Indian J. pure appl. Math., 27(7) : 701-710, July 1996

THERMAL STRESSES IN A TRANSVERSELY ISOTROPIC ELASTIC MEDIUM DUE TO INSTANTANEOUS HEAT SOURCES

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(*Received* 31 *August* 1995; *accepted* 29 *January* 1996)

The theory of coupled thermoelasticity is applied to determine the distribution of temperature and stresses in an infinite medium having instantaneous heat sources. A double integral transform (Laplace-Fourier) has been applied to the basic equations and the resulting equations are written in the form of a vector-matrix differential equation which is solved by eigenvalue approach. Finally numerical computations of the stresses have been made and represented graphically.

1. INTRODUCTION

The coupling between the strain and temperature fields was first studied by Duhamel[1] who derived the equations for the distribution of strains in an elastic medium subjected to temperature gradient. Biot[2] justified and derived, on the basis of irreversible thermodynamics, the fundamental relations of the equations of thermoelasticity and stated its variational principles. For static problems this coupling vanished and the thermal field becomes independent of the strain field. Several papers have been published by various authors taking into account the coupling vide, Nowaki[3], Dhaliwal and Singh[4]. Owing to mathematical difficulties encountered in coupled thermoelasticity, namely due to inertial and coupling terms in the governing equations, this kind of problems are mostly confined to one-dimensional problems of isotropic material, vide, Hetnarski[5-7], Bahar and Hetnarski[8]. It has been pointed out by Boley and Barber[9] that the assumption that the inertia may be disregarded does not seem reasonable for structural elements exposed to rapid heating, as may be encountered for example, in high speed propulsion unit or in problems of thermal shock, it is necessary to examine the importance of the role of inertia. It is also noticed that, in the recent years, there is a considerable interest in the usage of anisotropic materials in the engineering applications[10].

This paper is concerned with a two-dimensional coupled problem of thermoelasticity by taking into account both the dynamic effect and the influence of coupling terms. A line instantaneous heat source has been considered in the infinite transversely isotropic medium. The new eigenvalue approach of Das *et al.*[11, 12] has been applied for the solution of the problem.

2. Basic Equations

We consider a transversely isotropic infinite elastic slab subject to plane strain parallel to yz-plane, as such

$$U = 0,\ V = V(y, z, t),\quad W = W(y, z, t) \qquad \text{... (2.1)}$$

where U, V and W are displacement components in the x, y, z directions respectively. In this case the stress components are related to the displacement components as follows[14].

$$\tau_{xx} = A_{12}\frac{\partial V}{\partial y} + A_{13}\frac{\partial W}{\partial z} - \beta_1 T$$

$$\tau_{yy} = A_{22}\frac{\partial V}{\partial y} + A_{23}\frac{\partial W}{\partial z} - \beta_2 T$$

$$\tau_{zz} = A_{23}\frac{\partial V}{\partial y} + A_{33}\frac{\partial W}{\partial z} - \beta_3 T$$

$$\tau_{yz} = A_{44}\left(\frac{\partial V}{\partial z} + \frac{\partial W}{\partial y}\right). \qquad \text{... (2.2)}$$

The corresponding displacement equations of motion are :

$$A_{22}\frac{\partial^2 V}{\partial y^2} + A_{44}\frac{\partial^2 V}{\partial z^2} + (A_{23} + A_{44})\frac{\partial^2 W}{\partial y\,\partial z} - \beta_2\frac{\partial T}{\partial y} = \rho\frac{\partial^2 V}{\partial t^2}$$

$$A_{44}\frac{\partial^2 W}{\partial y^2} + A_{33}\frac{\partial^2 W}{\partial z^2} + (A_{23} + A_{44})\frac{\partial^2 V}{\partial y\,\partial z} - \beta_3\frac{\partial T}{\partial z} = \rho\frac{\partial^2 W}{\partial t^2}\,. \qquad \text{... (2.3)}$$

The temperature field $T(y, z, t)$ in the slab is assumed to satisfy the heat conduction equation

$$K_y\frac{\partial^2 T}{\partial y^2} + K_z\frac{\partial^2 T}{\partial z^2} = \rho c\frac{\partial T}{\partial t} + T_0\frac{\partial}{\partial t}\left(\beta_2\frac{\partial V}{\partial y} + \beta_3\frac{\partial W}{\partial z}\right) - Q(y, z, t) \qquad \text{... (2.4)}$$

where A_{jk} are elastic moduli of the material, β_j are the stress temperature coefficients, ρ is the mass density; K_y, K_z are the coefficients of the thermal conductivity in the y and z directions respectively. T_0 is the reference temperature, c the specific heat per unit mass, and Q the heat source.

We assume that the heat sources are instantaneous and act on the line $y = 0$; $z = 0$ and then we can write :

$$Q(y, z, t) = q_0\,\delta(y)\,\delta(z)\,\delta(t) \qquad \text{... (2.5)}$$

where q_0 is the strength of the heat source and $\delta(t)$ is the Dirac delta function of t.

3. METHOD OF SOLUTION : FORMULATION OF A VECTOR MATRIX DIFFERENTIAL EQUATION WITH TRANSFORMED VARIABLES

We apply the Laplace-Fourier double integral transform of the form

$$\overline{T}(y, z, p) = \int_0^{\alpha} T(y, z, t) \exp(-pt)\, dt \qquad \text{... (3.1)}$$

$$\overline{T}_1 (\xi, z, p) = \frac{1}{\sqrt{2\pi}} \int_{-\alpha}^{\alpha} \overline{T}(y, z, p) \exp(i\xi y)\, dy \qquad \text{... (3.2)}$$

(where p and ξ are transform parameters)
to the equations (2.3)-(2.5) and as such we get

$$-K_y \xi^2 \overline{T}_1 + K_z \frac{d^2 \overline{T}_1}{dz^2} - \rho pc\, \overline{T}_1 - T_0 p \left(-i\xi \beta_2 \overline{V}_1 + \beta_3 \frac{d \overline{W}_1}{dz} \right) + \frac{q_0}{\sqrt{2\pi}} \delta(z) = 0 \qquad \text{... (3.3)}$$

$$-(\xi^2 A_{22} + \rho p^2) \overline{V}_1 + A_{44} \frac{d^2 \overline{V}_1}{dz^2} - i\xi (A_{23} + A_{44}) \frac{d \overline{W}_1}{dz} + i\xi \beta_2 \overline{T}_1 = 0 \qquad \text{... (3.4)}$$

$$-(\xi^2 A_{44} + \rho p^2) \overline{W}_1 + A_{33} \frac{d^2 \overline{W}_1}{dz^2} - i\xi (A_{23} + A_{44}) \frac{d \overline{V}_1}{dz} - \beta_3 \frac{d \overline{T}_1}{dz} = 0. \qquad \text{... (3.5)}$$

Where we have used

$$\int_0^{\alpha} \delta(t) \exp(-pt) dt = 1 \text{ and } \frac{1}{\sqrt{2\pi}} \int_{-\alpha}^{\alpha} \delta(y) \exp(i\xi y)\, dy = \frac{1}{\sqrt{2\pi}}. \qquad \text{... (3.6)}$$

We also assume that at time $t = 0$, the body is at rest in an undeformed and unstressed state and is maintained at the reference temperature, then the following initial conditions hold :

$$\left.\begin{aligned} V(y, z, 0) &= \frac{\partial V(y, z, 0)}{\partial t} = 0 \\ W(y, z, 0) &= \frac{\partial W(y, z, 0)}{\partial t} = 0 \\ T(y, z, 0) &= \frac{\partial T(y, z, 0)}{\partial t} = 0. \end{aligned}\right\} \qquad \text{... (3.7)}$$

We have further assumed that V, W as well as their derivatives with respective to y vanish at infinity.

Equations (3.3), (3.4) and (3.5) can be written in the form of a vector-matrix differential equation as follows : vide, Das *et al.*[11, 12].

$$\frac{d\tilde{\mathbf{V}}}{dz} = \tilde{\mathbf{A}}\,\tilde{\mathbf{V}} + f(z) \qquad \text{... (3.8)}$$

where $\tilde{\mathbf{V}} = [\,\overline{V}_1, \overline{W}_1, \overline{T}_1, \overline{V}'_1, \overline{W}'_1, \overline{T}'_1\,]^T$

$$f(z) = \left[\,0, 0, 0, 0, 0, -\frac{q_0\,\delta(z)}{\sqrt{2\pi}\,K_z}\right]^T \qquad \text{... (3.9)}$$

where primes indicate differentiation with respect to z. The matrix $\tilde{\mathbf{A}}$ is

$$\tilde{\mathbf{A}} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ C_{41} & 0 & C_{43} & 0 & C_{45} & 0 \\ 0 & C_{52} & 0 & C_{54} & 0 & C_{56} \\ C_{61} & 0 & C_{63} & 0 & C_{65} & 0 \end{bmatrix} \qquad \text{... (3.10)}$$

where

$$C_{41} = \frac{\xi^2 A_{22} + \rho p^2}{A_{44}}, \quad C_{43} = \frac{-i\xi\,\beta_2}{A_{44}}, \quad C_{45} = \frac{i\xi\,(A_{23} + A_{44})}{A_{44}}$$

$$C_{52} = \frac{\xi^2 A_{44} + \rho p^2}{A_{33}}, \quad C_{54} = \frac{i\xi\,(A_{23} + A_{44})}{A_{33}}, \quad C_{56} = \frac{\beta_3}{A_{33}}$$

$$C_{61} = \frac{-i\xi\,\beta_2 T_0 p}{K_z}, \quad C_{63} = \frac{K_y \xi^2 + \rho p c}{K_z}, \quad C_{65} = \frac{T_0 \beta_3 p}{K_z}. \qquad \text{... (3.11)}$$

Solution of the Vector-Matrix Equations

As for the solution of the equation, we follow the method as in Das *et al.*[11]. The characteristic equation of the matrix $\tilde{\mathbf{A}}$ takes the form

$$\lambda^6 - \lambda^4 (C_{41} + C_{52} + C_{63} + C_{45} C_{54} + C_{56} C_{65}) + \lambda^2 (C_{63} C_{45} C_{54} - C_{43} C_{54} C_{65} + C_{41} C_{56} C_{65} - C_{45} C_{61} C_{56} + C_{52} C_{63} + C_{41} C_{52} + C_{41} C_{63} - C_{43} C_{61}) - (C_{41} C_{52} C_{63} - C_{43} C_{61} C_{52}) = 0. \qquad \text{... (3.12)}$$

The roots of eqn. (3.12), which are also the eigenvalues of the matrix $\tilde{\mathbf{A}}$ are of the form :

$$\lambda = \pm\lambda_1, \quad \lambda = \pm\lambda_2, \quad \lambda = \pm\lambda_3. \qquad \text{... (3.13)}$$

The right eigenvector $\mathbf{X} = [x_1, x_2, x_3, x_4, x_5, x_6]^T$ corresponding to the eigenvalue λ can be calculated as :

$$\underset{\sim}{X} = \begin{bmatrix} \lambda^2 (C_{45} C_{56} + C_{43}) - C_{43} C_{52} \\ \lambda[\lambda^2 C_{56} + (C_{54} C_{43} - C_{41} C_{56})] \\ \lambda^4 - \lambda^2 (C_{41} + C_{52} + C_{45} C_{54}) + C_{41} C_{52} \\ \lambda[\lambda^2 (C_{45} C_{56} + C_{43}) - C_{43} C_{52}] \\ \lambda^2 [\lambda^2 C_{56} + (C_{54} C_{43} - C_{41} C_{56})] \\ \lambda[\lambda^4 - \lambda^2 (C_{41} + C_{52} + C_{45} C_{54}) + C_{41} C_{52}] \end{bmatrix}. \quad \dots (3.14)$$

From (3.14) we can easily calculate the eigenvector X_i corresponding to the eigenvalue $\lambda = \lambda_i$. For our further reference we shall use the following notations :

$$X_1 = [X]_{\lambda=\lambda_1},\ X_2 = [X]_{\lambda=-\lambda_1},\ X_3 = [X]_{\lambda=\lambda_2},\ X_4 = [X]_{\lambda=-\lambda_2},$$

$$X_5 = [X]_{\lambda=\lambda_3},\ X_6 = [X]_{\lambda=-\lambda_3}. \quad \dots (3.15)$$

The left eigenvector $\underset{\sim}{Y} = [y_1, y_2, y_3, y_4, y_5, y_6]$ corresponding to the eigenvalue λ can be calculated as :

$$\underset{\sim}{Y} =$$

$$\begin{bmatrix} \lambda^3 C_{61} + \lambda[-C_{41} C_{61} - C_{61} C_{52} - C_{61} C_{45} C_{54} + C_{41} C_{61} C_{52} + C_{41} C_{54} C_{65}] \\ C_{52}[\lambda^2 C_{65} + C_{45} C_{61} - C_{65} C_{41}] \\ \lambda^5 - \lambda^3 [C_{41} + C_{52} + C_{45} C_{54} + C_{56} C_{65}] + \lambda [C_{41} C_{52} - C_{45} C_{61} C_{56} + C_{41} C_{56} C_{65}] \\ \lambda^2 [C_{61} C_{52} + C_{54} C_{65}] - C_{61} C_{52} \\ \lambda[\lambda^2 C_{65} + C_{45} C_{61} - C_{65} C_{41}] \\ \lambda^4 - \lambda^2 (C_{41} + C_{52} + C_{45} C_{54}) + C_{41} C_{52} \end{bmatrix}^T. \quad \dots (3.16)$$

For simplicity, henceforth, we shall denote them as :

$$Y_1 = [Y]_{\lambda=\lambda_1},\ Y_2 = [Y]_{\lambda=-\lambda_1},\ Y_3 = [Y]_{\lambda=\lambda_2},\ Y_4 = [Y]_{\lambda=-\lambda_2},$$

$$Y_5 = [Y]_{\lambda=\lambda_3},\ Y_6 = [Y]_{\lambda=-\lambda_3}. \quad \dots (3.17)$$

Assuming the regularity condition at $z = \alpha$, as in Das *et al.*[11] the solution of eqn. (3.8) can be written as :

$$\tilde{V}(z) = a_2(z) X_2 \exp(-\lambda_1 z) + a_4(z) X_4 \exp(-\lambda_2 z) + a_6(z) X_6 \exp(-\lambda_3 z) \quad \dots (3.18)$$

where

$$a_2(z) = \frac{-1}{Y_2 X_2} \int_{s=z_0}^{z} [\lambda_1^4 - \lambda_1^2 (C_{41} + C_{52} + C_{45} C_{54}) + C_{41} C_{52}] \frac{q_0}{\sqrt{2\pi} K_z} \delta(s) e^{-\lambda_1 s} ds, \quad z_0 < 0 < z.$$

Integrating the expression, we get :

$$a_2(z) = \frac{-q_0}{\sqrt{2\pi}\,Y_2 X_2 K_z}\,[\lambda_1^4 - \lambda_1^2(C_{41} + C_{52} + C_{45}C_{54}) + C_{41}C_{52}],$$

$$z_0 < 0 < z. \quad \ldots (3.19)$$

Similarly

$$a_4(z) = \frac{-q_0}{\sqrt{2\pi}\,Y_4 X_4 K_z}\,[\lambda_2^4 - \lambda_2^2(C_{41} + C_{52} + C_{45}C_{54}) + C_{41}C_{52}],$$

$$z_0 < 0 < z. \quad \ldots (3.20)$$

$$a_6(z) = \frac{-q_0}{\sqrt{2\pi}\,Y_6 X_6 K_z}\,[\lambda_3^4 - \lambda_3^2(C_{41} + C_{52} + C_{45}C_{54}) + C_{41}C_{52}],$$

$$z_0 < 0 < z. \quad \ldots (3.21)$$

Writing (a_2, a_4, a_6) as (A_1, A_2, A_3), the deformations $\overline{V}_1(\xi, z, p)$, $\overline{W}_1(\xi, z, p)$ and the temperature field $\overline{T}_1(\xi, z, p)$ can be compactly written from (3.18) as

$$\overline{V}_1(\xi, z, p) = \sum_{i=1}^{3} A_i\,[\lambda_i^2(C_{45}C_{56} + C_{43}) - C_{43}C_{52}]\exp(-\lambda_i z) \quad \ldots (3.22)$$

$$\overline{W}_1(\xi, z, p) = \sum_{i=1}^{3} A_i\,[-\lambda_i\{\lambda_i^2 C_{56} + (C_{54}C_{43} - C_{41}C_{56})\}]\exp(-\lambda_i z)$$

$$\ldots (3.23)$$

$$\overline{T}_1(\xi, z, p) = \sum_{i=1}^{3} A_i\,[\lambda_i^4 - \lambda_i^2(C_{41} + C_{52} + C_{45}C_{54}) + C_{41}C_{52}]\exp(-\lambda_i z).$$

$$\ldots (3.24)$$

Using (2.2) and (3.22)-(3.24) the stresses in the Laplace-Fourier transform domain can now be written as :

$$(\overline{\tau}_{xx})_1 = -i\xi A_{12}\sum_{i=1}^{3} A_i\,[\lambda_i^2(C_{45}C_{56} + C_{43}) - C_{43}C_{52}]\exp(-\lambda_i z)$$

$$+ A_{13}\sum_{i=1}^{3} A_i\,[\lambda_i^2\{\lambda_i^2 C_{56} + (C_{54}C_{43} - C_{41}C_{56})\}]\exp(-\lambda_i z)$$

$$-\beta_1\sum_{i=1}^{3} A_i\,[\lambda_i^4 - \lambda_i^2(C_{41} + C_{52} + C_{45}C_{54}) + C_{41}C_{52}]\exp(-\lambda_i z)$$

$$\ldots (3.25)$$

$$(\bar{\tau}_{yy})_1 = -i\xi A_{22} \sum_{i=1}^{3} A_i [\lambda_i^2 (C_{45} C_{56} + C_{43}) - C_{43} C_{52}] \exp(-\lambda_i z)$$

$$+ A_{23} \sum_{i=1}^{3} A_i [\lambda_i^2 \{\lambda_i^2 C_{56} + (C_{54} C_{43} - C_{41} C_{56})\}] \exp(-\lambda_i z)$$

$$- \beta_2 \sum_{i=1}^{3} A_i [\lambda_i^4 - \lambda_i^2 (C_{41} + C_{52} + C_{45} C_{54}) + C_{41} C_{52}] \exp(-\lambda_i z)$$

... (3.26)

$$(\bar{\tau}_{zz})_1 = -i\xi A_{23} \sum_{i=1}^{3} A_i [\lambda_i^2 (C_{45} C_{56} + C_{43}) - C_{43} C_{52}] \exp(-\lambda_i z)$$

$$+ A_{33} \sum_{i=1}^{3} A_i [\lambda_i^2 \{\lambda_i^2 C_{56} + (C_{54} C_{43} - C_{41} C_{56})\}] \exp(-\lambda_i z)$$

$$- \beta_3 \sum_{i=1}^{3} A_i [\lambda_i^4 - \lambda_i^2 (C_{41} + C_{52} + C_{45} C_{54}) + C_{41} C_{52}] \exp(-\lambda_i z)$$

... (3.27)

$$(\bar{\tau}_{yz})_1 = A_{44} \sum_{i=1}^{3} A_i [-\lambda_i^3 (C_{45} C_{56} + C_{43}) + \lambda_i C_{43} C_{52}] \exp(-\lambda_i z)$$

$$- i\xi A_{44} \sum_{i=1}^{3} A_i [-\lambda_i \{\lambda_i^2 C_{56} + (C_{54} C_{43} - C_{41} C_{56})\}] \exp(-\lambda_i z).$$

... (3.28)

We now write down from equations (3.24)—(3.27), the expression of the temperature field and the stresses from the Laplace-Fourier transform domain to Laplace transform domain as :

$$[\bar{T}, \bar{\tau}_{xx}, \bar{\tau}_{yy}, \bar{\tau}_{zz}] (y, z, p) = \sqrt{2/\pi} \int_0^{\alpha} [\bar{T}_1, (\bar{\tau}_{xx})_1, (\bar{\tau}_{yy})_1, (\bar{\tau}_{zz})_1 \cos \xi y \, d\xi$$

... (3.29)

$$\bar{\tau}_{yz} (y, z, p) = \sqrt{2/\pi} \int_0^{\alpha} (\bar{\tau}_{yz})_1 \sin \xi y \, d\xi \qquad \text{... (3.30)}$$

where, $\bar{T}_1, (\bar{\tau}_{xx})_1, (\bar{\tau}_{yy})_1, (\bar{\tau}_{zz})_1$ are even function of ξ and $(\bar{\tau}_{yz})_1$ is an odd function of ξ.

The evaluation of these infinite integrals when all the terms are written out in full from (3.24)-(3.28), and (3.19)-(3.21) becomes very unwieldy, and moreover, we have to perform the inverse laplace transform of the resulting expressions in order to find the temperature and stresses in the space-time domain.

4. Numerical Solution

Considering the complexity of the problem of Fourier-Laplace inversion of the expressions in (3.24)-(3.28) we prefer to develop an efficient computer software for the purpose. As such for numerical inversion of the Laplace transform, we follow the method of Bellman *et al.*[13] which will provide us the value of the variables at the times $t = t_i$, i = 1, 2, 3, 4, 5, 6, 7 where t_i, are the roots of the Legendre polynomial of degree 7. Simultaneous calculations for the inversion of the Fourier transforms; the infinite integrals in (3.29) and (3.30) have been evaluated numerically by seven point Gaussian-quadrature formula for several prescribed values of y and z.

For numerical computations, the following data in M.K.S. units of Cobalt (considered as transversely isotropic) have been used, vide Dhaliwal and Singh[4].

$$
\begin{aligned}
A_{12} &= 1.650 \times 10^{11} && \mathrm{Nm^{-2}} \\
A_{13} &= 1.027 \times 10^{11} && \mathrm{Nm^{-2}} \\
A_{22} &= 3.071 \times 10^{11} && \mathrm{Nm^{-2}} \\
A_{23} &= 1.027 \times 10^{11} && \mathrm{Nm^{-2}} \\
A_{33} &= 3.581 \times 10^{11} && \mathrm{Nm^{-2}} \\
\beta_1 &= 7.04 \times 10^{6} && \mathrm{Nm^{-2}\ deg^{-1}} \\
\beta_2 &= 7.04 \times 10^{6} && \mathrm{Nm^{-2}\ deg^{-1}} \\
\beta_3 &= 6.90 \times 10^{6} && \mathrm{Nm^{-2}\ deg^{-1}} \\
K_y &= 0.69 \times 10^{2} && \mathrm{Nm^{-1}\ deg^{-1}} \\
K_z &= 0.69 \times 10^{2} && \mathrm{Nm^{-1}\ deg^{-1}} \\
\rho &= 8.836 \times 10^{3} && \mathrm{kg\ m^{-3}} \\
c &= 4.27 \times 10^{2} && \mathrm{J(kg)^{-1}\ deg^{-1}} \\
T_0 &= 293 && {}^{\circ}\mathrm{k}
\end{aligned}
\qquad \ldots (4.1)
$$

In the space time domain the stresses $\tau_{xx}(y, z, t)$, $\tau_{yy}(y, z, t)$, $\tau_{zz}(y, z, t)$, $\tau_{yz}(y, z, t)$ are calculated for different values of y ($1 \le y \le 80$) and z ($.001 \le z \le 1$) in the prescribed time interval t = 0.025775, 0.138382, 0.352509, 0.693147, 1.21376, 2.04612, 3.67119.

CONCLUDING REMARKS

In order to study the stress characteristics we have drawn several graphs of the stresses $\tau_{xx}, \tau_{yy}, \tau_{zz}$, and τ_{yz} for different values of y and z as also for different values of times. As a result it is observed that

(1) the characteristics of the stresses τ_{xx}, τ_{yy} for the material considered [as in (4.1)] are almost same in respect of wave propagations.

(2) it has also been observed that when y assumes relatively larger values than z, the stresses τ_{xx} and τ_{zz} assumes only positive values for the prescribed values of the time t_i as mentioned above.

(3) for fixed values of y and t we notice that as z increases the amplitude of τ_{xx}, τ_{zz} and τ_{yz} decreases much more rapidly than the case when the values of z and t are fixed but y increases.

(4) for fixed values of y and z; the amplitude of $\tau_{xx}, \tau_{zz}, \tau_{yz}$ decreases with gradually greater wave length as t increases.

(5) we now present a typical graph Fig. 1 for the stresses $[\tau_{xx}(y, z, t), \tau_{zz}(y, z, t), \tau_{yz}(y, z, t)]$ for time variable t = 0.025775, 0.138382, 0.352509, 0.693147, 1.21376, 2.04612, 3.67119 as the abscissa and for particular values of the space variables $y = 1$, $z = 0.001$. The graphs have been plotted by using cubic spline formalisms.

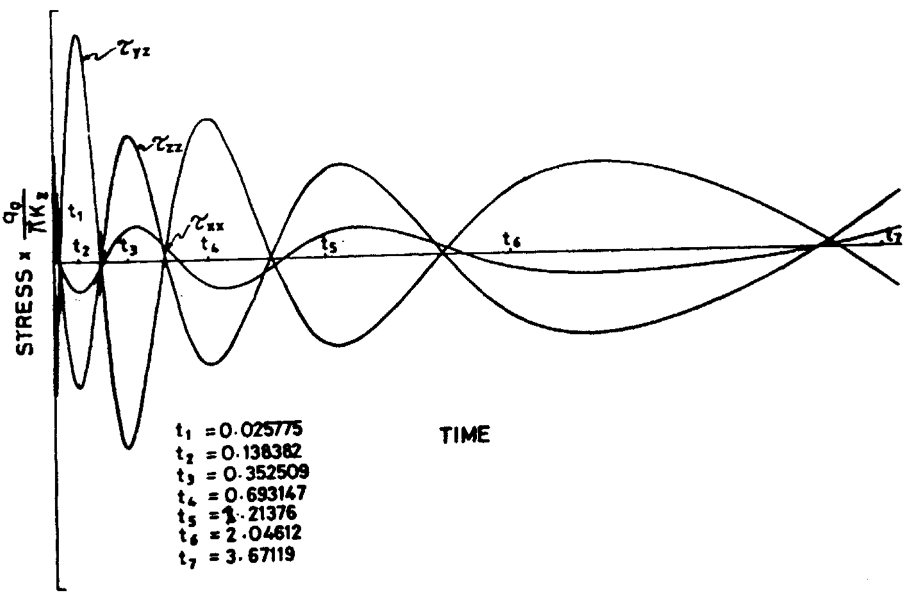

FIG. 1. Distribution of stresses (for $y = 1$ and $z = 0.001$) versus time.

ACKNOWLEDGEMENT

We are thankful to the U.G.C.-D.S.A. programme of the Department of Mathematics, Jadavpur University, Calcutta, for the support of this work. We are also thankful to Dr P. K. Sen, Department of Mathematics, Jadavpur University, Calcutta, for his kind help in the computational works and in drawing several graphs by using computer packages.

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