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MAGNETOHYDRODYNAMIC AND CONSTANT SUCTION/INJECTION EFFECTS OF AXISYMMETRIC STAGNATION POINT FLOW AND MASS TRANSFER FOR POWER-LAW FLUIDS

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The boundary-layer equations for axisymmetric point flow of power-law electrically conducting fluid through a porous medium with transverse magnetic field are solved by a similarity transformation. Theoretical expressions for local and average Sherwood number are derived from the convective diffusion equation for the system with high Schmidt number.

The numerical values of the velocity profiles and shear rate at the wall are discussed.

INTRODUCTION

The experimental and theoretical studies on the diffusion coefficients of solutes in non-Newtonian fluids with well-defined conditions for mass transfer and fluid flow are important from the technological point of view, for they have many practical applications. Hansford and Litt[1] reported on the mass transfer from a rotating disk made of the solute diffusing to non-Newtonian solution. Greif and Paterson[2] obtained more accurate result for mass flux at high Schmidt number. For power-law fluids, Schmidt number effects at rotating cones and disk have been investigated based on a two-term representation of the velocity field[3]. High mass transfer rates may be obtained with this configuration without the need for moving parts. Several distinct flow regimes have been identified in axisymmetric stagnation point flow of Newtonian fluids[4].

The laminar boundary-layer thickness is relatively independent of position for distances within one nozzle radius of the stagnation point and, in this region, the surface is uniformly accessible from a mass transfer standpoint[5,6].

Recently the impinging jet apparatus was propsed as a system for determinating of transport properties or species concentrations in non-Newtonian fluids[7].

The object of the present investigation is to study the effects of transverse magnetic field and suction/injection on the power-law, electrically conducting incompressible fluid in axisymmetric stagnation point flow through porous medium. Also theoretical expressions for local and average mass transfer rate in fluids with high Schmidt numbers have been obtained from the convective diffusion equation. The analysis is restricted to constant concentrations at the reactive surface and to mass transfer rates that are too low to disturb the laminar forced convection flow. If viscous dissipation can be ignored, the same equations would apply to heat transfer in boundary layers with the concentration replaced by temperature and diffusion coefficients replaced by the thermal diffusivity.

Flow Analysis

The stress tensor for a power-law fluid is taken[8]

$$\tau = -\left[k \mid \tfrac{1}{2}(\Delta : \Delta) \mid^{n-1}\right]\Delta \qquad \ldots(1)$$

where Δ is the symmetrical rate of deformation tensor, k is a consistency index for non-Newtonian viscosity which is a non-negative but dimensional quntity and n is the power-law exponent, which is assumed to be constant over the range of shear rates considered. Equation (1) is an empirical relation commonly used to describe the behaviour of generalized Newtonian fluids but neglects transient viscoelastic phenomena. Elongational flow effects can be important in dilute polymer solutions when the relaxation time of the fluid is comparable to the characteristic time of the flow[9,10]. However, for a given fluid, the power-law model is usually valid over some specific range of flow variables[11].

With this two-parameter model, we consider the steady axisymmetric flow of an electrically conducting non-Newtonian fluid. We assume that the magnetic induction B is perpendicular to the surface on which the boundary is formed, the electric field E is perpendicular to B and in the direction of the longitudinal velocity of the boundary layer. In the zero-induction approximation, the system of equation of the motion and continuity become[12]

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \frac{k}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n - \frac{\sigma}{\rho}B(uB + E) \qquad \ldots(2)$$

$$\frac{\partial (xu)}{\partial x} + x\frac{\partial v}{y} = 0 \qquad \ldots(3)$$

provided that the Reynolds number Re is large.

The boundary conditions for stagnation point flow are :

$$y = 0 \; ; \; u = 0 \; , \; v = \pm c$$

$$y \to \infty \; ; \; u \to U_x \qquad \ldots(4)$$

where c represents the velocity of suction or injection at the wall, and U_x is the bulk streamwise velocity given by the potential flow solution[13] : $U_x = ax$; $U_y = -2ay$. Bernoulli's theorem indicates that, for a vertical jet.

$$-\frac{1}{\rho}\frac{\partial p}{\partial x} = a^2 x \; . \qquad \ldots(5)$$

Making use of

$$E = -BU \; , \text{ and } -\frac{1}{\rho}\frac{dp}{dx} = U\frac{dU}{dx} \qquad \ldots(6)$$

eqn. (2) becomes

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = u\frac{dU}{dx} + \frac{k}{\rho}\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial y}\right)^n + \frac{\sigma}{\rho} + B^2(U - u). \qquad \ldots(7)$$

The induction B is given by the relation[14]

$$B(x) = B. \quad X^{(m-1)/2}, \; B_0 = \text{const.} \quad m \lesssim 0. \qquad \ldots(8)$$

Substitution of eqn. (8) into eqn. (7) and the introduction of similarity variable

$$\eta = a^{(2-n)/(1+n)}(\rho/k)^{1/(+n)} y \; X^{(1-n)/(1+n)} \qquad \ldots(9)$$

gives

$$n(f'')^{n-1} f''' + \frac{(3n+1)}{(n+1)} ff'' - f'^2 + 1 + N(1 - f') = 0 \qquad \ldots(10)$$

with $u = a \times f'$ and, in accordance with eqn. (3)

$$v = -a^{(2n-1)/(n+1)}(k/\rho)^{1/(1+n)}\left(\frac{3n+1}{n+1}\right)\left(f + \frac{(1-n)\eta f'}{(1+3n)}\right). \qquad \ldots(11)$$

The appropriate boundary conditions are

$$f(0) = R \; , f'(0) = 0 \text{ at } y = 0$$

$$f'(\infty) \to 1 \quad \text{as} \quad \eta \to \infty \qquad \ldots(12)$$

where N is the parameter of magnetic field and

$$R = \pm C\left(\frac{n+1}{3n+1}\right)(\rho/k)^{1/(1+n)} a^{((1-2n)/1+n)}.$$

The hydrodynamic parameter a can be evaluated from the height and geometry of the nozzle and the average fluid exit velocity[6].

In this analysis, eqn. (10) is expanded into three simultaneous equations which are solved numerically by a fourth order Rung-Kutta, subject to the boundary conditions (12). Values of $f''(0)$ are not reported for $n > 2$ because the range of Reynolds number for which the laminar boundary layer equations are valid is limited for these fluids[12].

Mass Transfer

The steady-state concentration distribution of a dilute species i in axisymmetric stagnation point flow at high Peclet number and with no homogeneous reactions is described by the convective diffusion equation :

$$u \frac{\partial c_i}{\partial x} + v \frac{\partial c_i}{\partial y} = D_i \frac{\partial^2 c_i}{\partial y^2} \qquad \ldots(13)$$

where D_i, the diffusion coefficient of species i, is assumed to be constant.

At high Schmidt numbers the diffusion layer is generally thin compared to the region of the velocity variation and the velocity components can be approximated by their first terms in Taylor's expansions in the distance y from the surface. With this approximation, the local concentration profiles at different values of x are similar and depend only on the combined variable[16] :

$$\xi = \frac{y\, Sc^{1/2}}{g} = \frac{y\,(x\beta_1)^{1/2}}{(9D_i \int_0^x x'\,(x'\beta_1)^{\frac{1}{2}}\, dx')^{1/3}} \qquad \ldots(14)$$

where $Sc = k\, a^{n-1}/\rho D_i$, being the Schmidt number and

$$\beta_1 = \left(\frac{\partial v}{\partial y}\right)_{y=0} = \left(\frac{\rho a^3 x^2}{k}\right)^{1/(1+n)} f''(0). \qquad \ldots(15)$$

In terms of this similarity variable, the flux of species i normal to the surface for $Sc \to \infty$ is[15]

$$N_i = -D_i \left(\frac{\partial c_i}{\partial y}\right)_{y=0} = \frac{-D_i\,(c_\infty - c_0)\sqrt{x\beta_1}}{\Gamma(4/3)\,(9D_i)^{1/3}\,(\int_0^x x'\,(x'\beta_1)^{\frac{1}{2}}\, dx')^{1/3}}. \qquad \ldots(16)$$

The dimensionless local mass transfer rate to the surface can be defined as :

$$Sh = r\left(\frac{\partial\theta}{\partial y}\right)_{y=0} = \frac{1}{\Gamma(4/3)}\left[\frac{f''(0)(5n+7)}{18(n+1)}\right]^{1/3}\left(\frac{x}{r}\right)^{(1-n)/3(1+n)}$$

$$\mathrm{Re}^{(n+2)/3(1+n)}\, Sc^{1/3} \qquad \ldots(17)$$

where Sh is Sherwood number, $\theta = (c_i - c_0)/(c_\infty - c_0)$ and $\mathrm{Re} = \dfrac{\rho\, a^{2-n} r^2}{k}$. The average mass transfer rate across a region of radius r from the stagnation point is obtained by integration of eqn. (17).

$$Sh_{avg} = \frac{6(n+1)}{\Gamma(4/3)}\left[\frac{f''(0)(5n+7)}{18(n+1)}\right]^{1/3} \mathrm{Re}^{(n+2)/3(n+1)}\, Sc^{1/3}. \qquad \ldots(18)$$

The determination of local or overall mass transfer rates allows to diffusion coefficient of a species to be estimated from eqns. (15) or (16), provided that the density and power-law characteristics of the fluid are known.

Discussion

Velocity profiles are presented in Figs. 1 and 2 for pseudoplastic and dilatent fluids with parameter of magnetic field $N = 0.5$. The velocity approaches its bulk value closer to the surface with larger values of the power-law exponent. However, the shear rate surface $f''(0)$ decreases as n increases (see Fig. 3). The velocity component f' of the fluid increases as the magnetic field parameter is increased for constant power n, as can be seen from Figure 4.

Table I gives the shear rate surface $f''(0)$, for different values of magnetic field parameter N with $n = 0.8$ and $R = 0.0$. The values of skin friction increase as N increases.

Table I

$f''(0)$ *for various* N *with* $n = 0.8$

N	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5
$f''(0)$	1.42330	1.642125	1.840503	2.023522	2.194449	2.355524	2.508368	2.654201

Fig. 5 are shows the variation of f' with respect to η for several values of R (suction/injection) with $N = 0.5$ and $n = 0.8$. The velocity increases as the positive values of R increase. On the other hand, the velocity decreases as the negative values of R increase. The numerical values of skin-friction are entered in Table II against R for $N = 0.5$ and $n = 0.8$. We conclude from Table II that the skin-friction decreases with increases in the positive values of the parameter R, while for the case of negative values of R the skin-friction decreases as R increases.

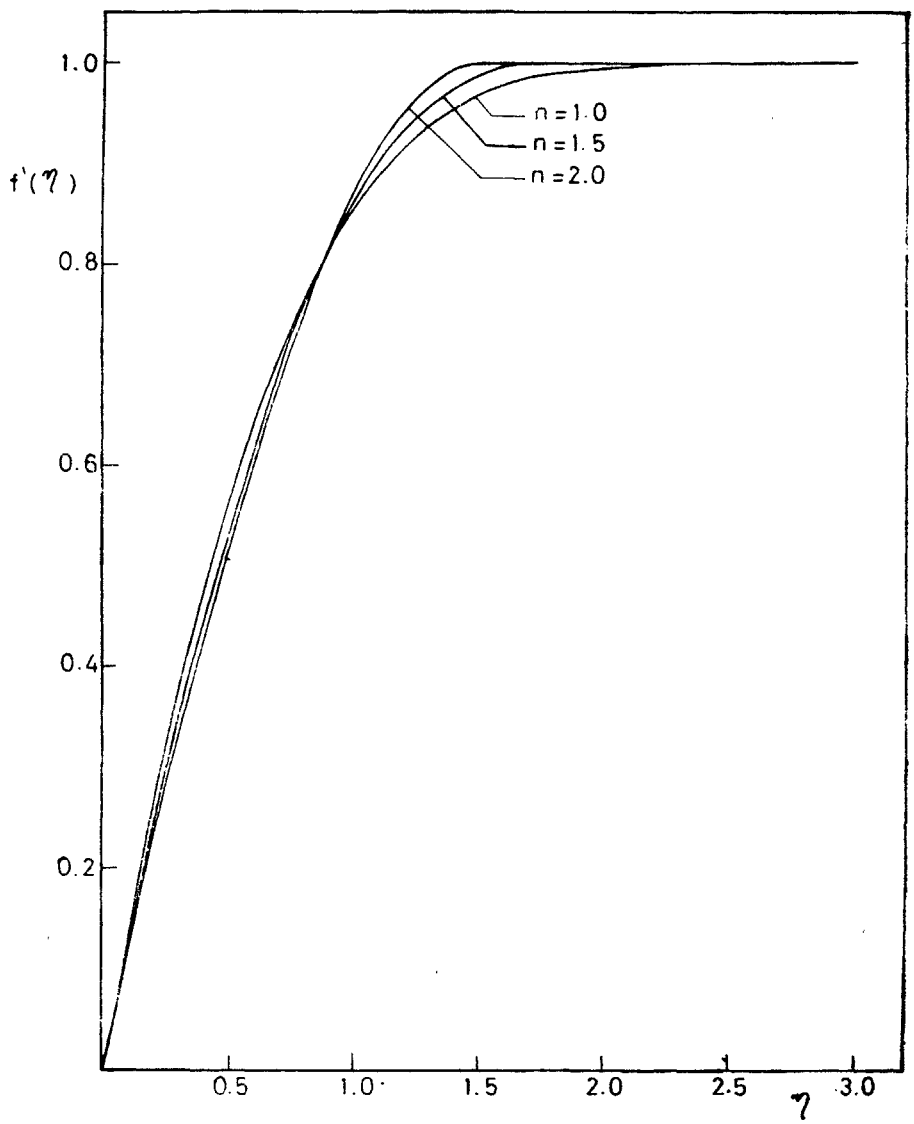

FIG. 1 Velocity profiles for dilatant fluids with $N=0.5$ and $R=0.0$.

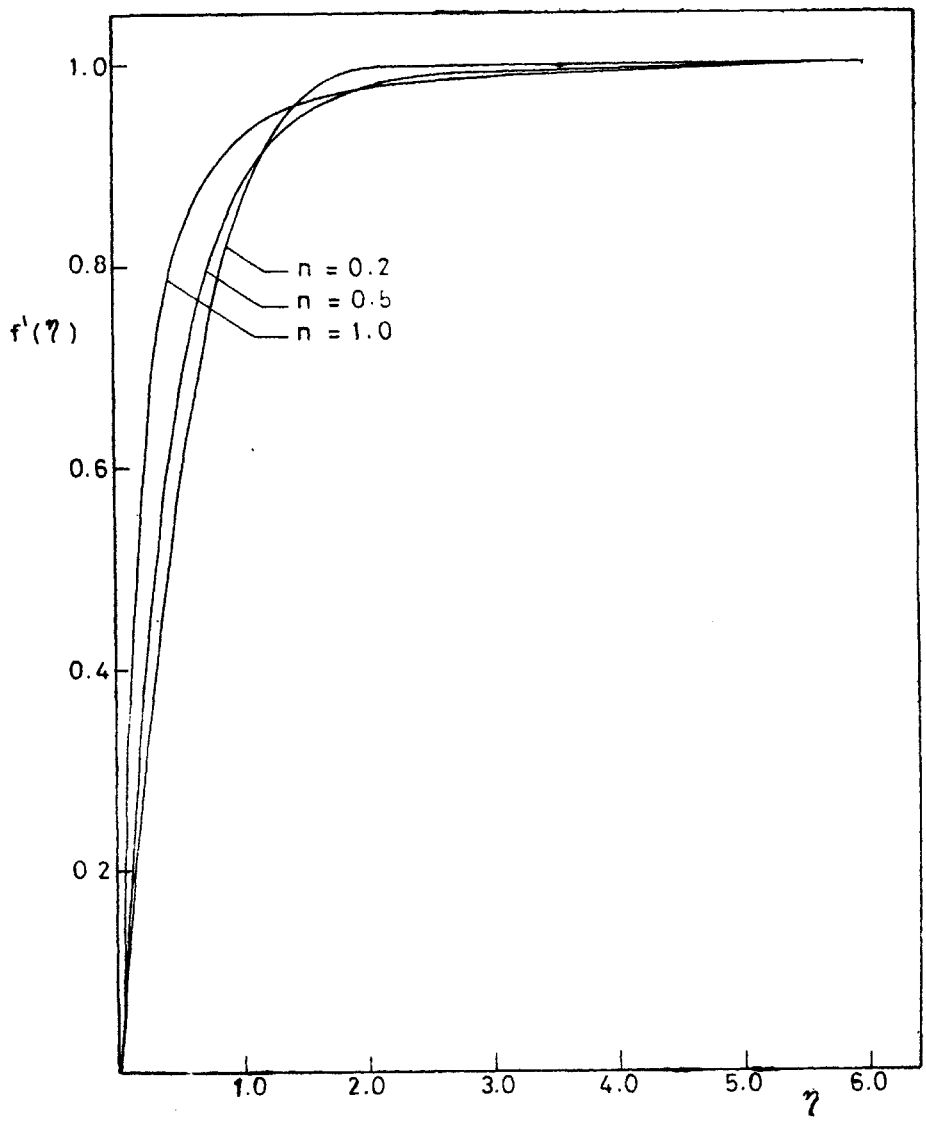

FIG. 2. Velocity profiles for pseudoplastic fluids with $N=0.5$ and $R=00$.

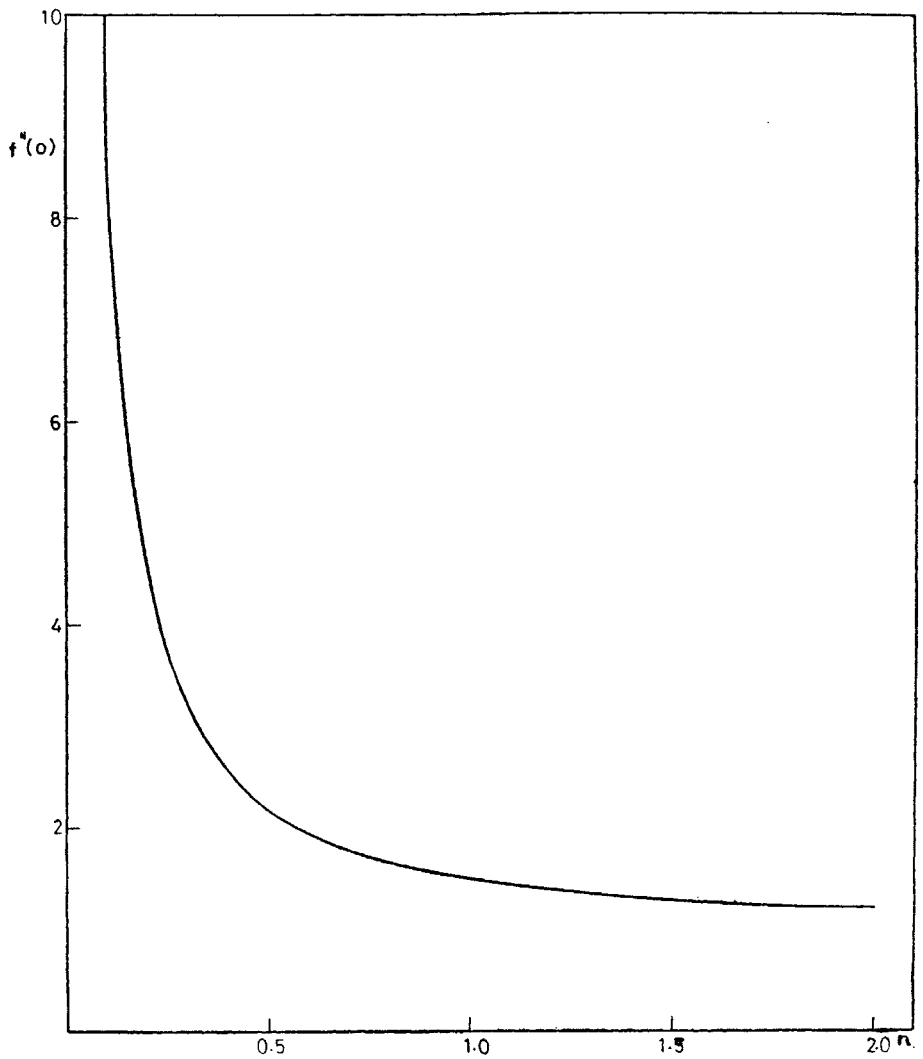

FIG. 3. Dependence of surface shear rate on power-law exponent for $N=0.5$.

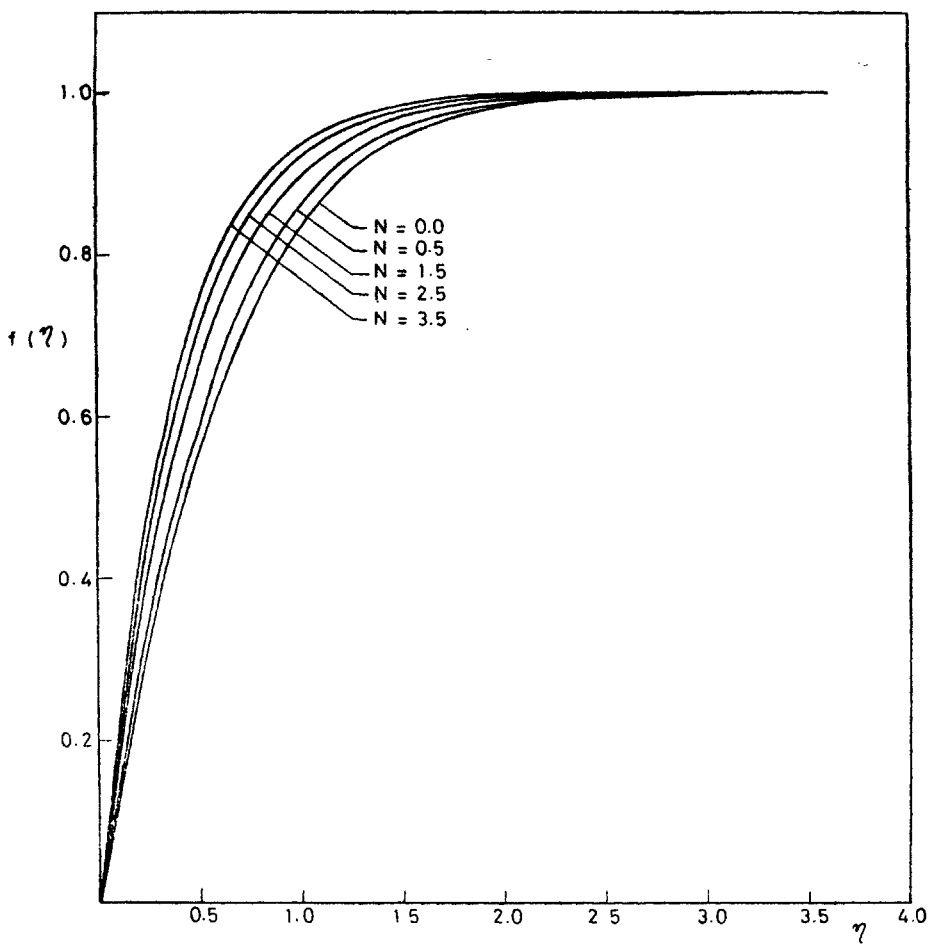

FIG. 4. Velocity distribution for different values of N with $n = 0.8$ and $R = 0.0$.

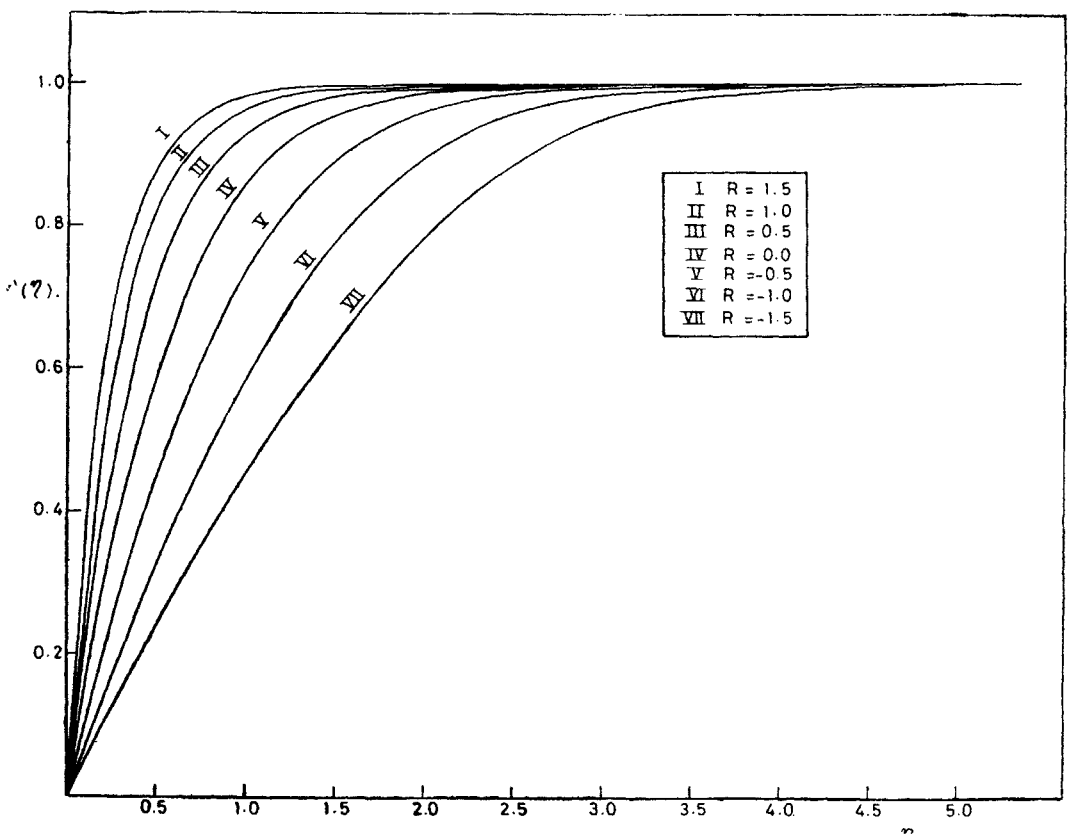

FIG. 5. Velocity profiles for several values of R with $N=0.5$ and $n=0.8$.

TABLE II

f″ (0) for various R with N = 0.5 and n = 0.8

R	1.5	1.0	0.5	0.1	0.01	0.0	−0.01	−0.1	−0.5	−1.0	−1.5
$f''(0)$	4.833969	3.55943	2.48326	1.790087	1.656454	1.642125	1.408987	1.285212	0.844334	0.516792	0.352044

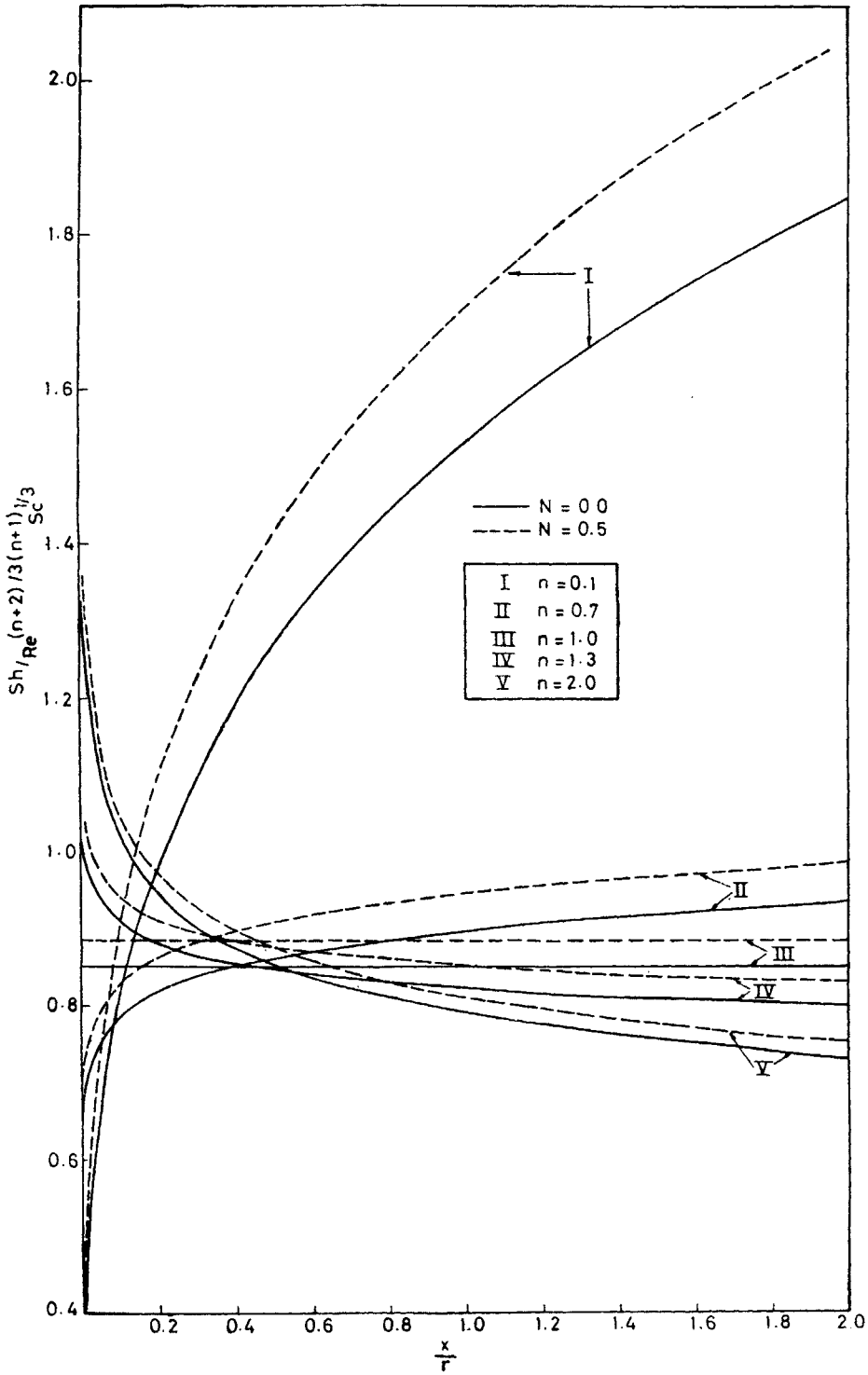

FIG. 6. Graphical representation of eqn. (17) for several values of power-low exponent with $N = 0.0$, $N = 0.5$ and $R = 0.0$

Figure 6 shows this position dependence of the mass transfer rate for several values of the power-law exponent with the $N = 0.0$. and $N = 0.5$.

In the case of pseudoplastic fluid, the rate of transfer increases monotonically with radial distance from the stagnation point, the rate transfer increases as N increases. On the other hand, in dilatant fluids, the opposite behaviour is observed where mass transfer rate approaches an infinite value at the stagnation point, the rate transfer increases as the magnetic field increases. However, the boundary-layer equations presented are not valid in the region near the centre where the diffusion layer is considerably thicker than the hydrodynamic boundary layer.

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