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# A NOTE ON THE DUAL OF A PBIB DESIGN

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Expressions for the variances of the elementary contrasts of treatments for different associate classes of a PBIB design having 5 associate classes have been computed.

## 1. INTRODUCTION

By interchanging the role of blocks and treatments in a block design, a new design is obtained. The design thus obtained is known as the dual design of the former design. The dual of a BIB design was given the name Linked Block Design by Youden[1]. Roy and Laha[2] have made exhaustive study of the classification and enumeration of all linked block designs with $r, k \leqslant 10$. Ramakrishnan[3] and Mohan[4] have obtained PBIB designs having 5 associate classes by dualising a suitably chosen GD design. Mohan[4] has not given the variances of the elementary contrasts of treatments belonging to different associate classes, without which the discussion is incomplete.

In the present note, we have obtained a relation between $M^*$ and $M$, the Pseudo Variance-Covariance matrix of the dual design and original design respectively.

## 2. MAIN RESULTS

Let $D$ be a block design with parameters $(v, b, r, k)$ and incidence matrix $N$. Its Pseudo Variance-Covariance matrix will be denoted by $M$, i.e.

$$M_{v\times v} = \left(r I_v - \frac{1}{k} N N'\right)^-$$

$$= \text{the Pseudo inverse of } \left(r I_v - \frac{1}{k} N N'\right).$$

Let $D^*$ be the dual design of $D$ with parameters $(b, v, k, r)$ and the incidence matrix $N'$. Let $M^*$ stand for the Pseudo Variance-Covariance matrix of the design $D^*$, i.e.,

$$M^*_{b\times b} = \left(k\, I_b - \frac{1}{r}\, N'\, N\right)^-$$

$$= \text{the Pseudo inverse of } \left(k\, I_b - \frac{1}{r}\, N'\, N\right),$$

Now, we shall show that

$$M^*_{b\times b} = \frac{1}{k}\, I_b + \frac{1}{k^2}\, N'\, MN.$$

PROOF : By definition, we consider

$$\left(k\, I_b - \frac{1}{r}\, N'\, N\right)\left(\frac{1}{k}\, I_b + \frac{1}{k^2}\, N'\, M\, N\right)\left(k\, I_b - \frac{1}{r} N'\, N\right)$$

$$= \left(k\, I_b - \frac{1}{r}\, N'\, N\right)\left(I_b + \frac{1}{k}\, N'\, M\, N - \frac{1}{rk}\, N'\, N\right.$$

$$\left. - \frac{1}{rk^2}\, N'\, M\, N\, N'\, N\right)$$

$$= \left(k\, I_b + N'\, M\, N - \frac{1}{r}\, N'\, N - \frac{1}{r\,k} N'\, M\, N\, N'\, N\right.$$

$$- \frac{1}{r}\, N'\, N$$

$$- \frac{1}{rk}\, N'\, N\, N'\, M\, N + \frac{1}{r^2 k}\, N'\, N\, N'\, N$$

$$\left. + \frac{1}{r^2 k^2}\, N'\, N\, N'\, M\, N\, N'\, N\right)$$

$$= \left(k\, I_b - \frac{1}{r}\, N'\, N\right) - \frac{1}{r^2}\, N'\left(r\, I_b - \frac{1}{k} N\, N'\right) N$$

$$+ \frac{1}{r^2}\, N'\left(r\, I_b - \frac{1}{k}\, N\, N'\right) M \left(r\, I_b - \frac{1}{k}\, N\, N'\right) N$$

$$= \left(k\, I_b - \frac{1}{r}\, N'\, N\right) - \frac{1}{r^2}\, N'\left(r\, I_b - \frac{1}{k}\, N\, N'\right) N$$

*(equation continued on p. 140)*

$$+ \frac{1}{r^2} N' \left( r I_b - \frac{1}{k} N N' \right) N$$

$$= \left( k I_b - \frac{1}{r} N' N \right).$$

This shows that

$$M^* = \frac{1}{k} I_b + \frac{1}{k^2} N' M N.$$

This result holds for all equireplicate and proper (not necessarily Binary) designs. In case of unequal replications and block sizes, we have

$$M^* = k^{-\delta} + k^{-\delta} N' M N k^{-\delta}$$

where $k^{-\delta}$ is a $b \times b$ diagonal matrix having $1/k_j$ as the $j$th element, $k_j$ being the size of the $j$th block.

## 3. Variances of the Elementary Contrasts of Treatments

The GD design used by Mohan[4] for dualization has the parameters :

$$v = am, \qquad b = \frac{a^2 m(m-1)}{2}$$

$$r = a(m-1), \; k = 2$$

$$\lambda_1 = 0, \; \lambda_2 = 1, \; n = a, \; m = m$$

$$n_1 = a - 1, \; n_2 = a(m-1)$$

and the dual $D^*$ of design $D$ has the parameters :

$$v^* = \frac{a^2 m(m-1)}{2}, \quad b^* = am, \; r^* = 2, \quad k^* = a(m-1)$$

$$\lambda_1^* = \lambda_2^* = 1, \; \lambda_3^* = \lambda_4^* = \lambda_5^* = 0$$

$$n_1^* = 2(a-1), \; n_2^* = 2a(m-2), \; n_3^* = (a-1)$$

$$n_4^* = 2a(a-1)(m-2), \; n_5^* = \frac{a^2(m-2)(m-3)}{2}.$$

It is worth noticing that (*i*) for $a = 2$ this dual design is a Lattice design; (ii) for $a = 3$ this design is a 4-associate-class PBIB design and (iii) this design is a 5-asso-

ciate-class PBIB design for $a > 3$. Mohan[4] has not specifically investigated the cases (i) and (ii).

Using (1) we can very easily obtain the variances and covariances of the dual design $D^*$ for different associate classes, the scheme of association being as given by Mohan. Thus the variances of the elementary contrasts of treatments for different associate classes are :

$$\text{Var}\left(\hat{t}_l - \hat{t}_{l'}\right) = \frac{a^2m(m-1)+(am-1)}{a^2m(m-1)} \text{ for first associates,}$$

$$= \frac{a(m-1)+1}{a(m-1)} \text{ for second associates,}$$

$$= \frac{a(m-1)+2}{a(m-1)} \text{ for third associates,}$$

$$= \frac{a^2m(m-1)+(2am-1)}{a^2m(m-1)} \text{ for fourth associates,}$$

$$= \frac{a^2m(m-1)+(2am-2)}{a^2m(m-1)} \text{ for fifth associates.}$$

Once the variances are obtained the compution of Efficiency Factor is straight forward.

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