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DEFORMATION OF A SEMI-INFINITE POROELASTIC MEDIUM WITH A TANGENTIAL LOAD ON ITS SURFACE

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The general solutions for the problem of deformation of a semi-infinite poroelastic medium under the action of tangential load applied on its surface are obtained with the help of displacement functions followed by Laplace-Hankel transforms technique. The solutions are then applied to the problem where a tangential force acts uniformly over a circular region of the surface and the initial and final response of the normal displacement, normal and circumferential stresses are shown graphically.

1. INTRODUCTION

Biot's theory[1] of deformation in porous media has drawn the attention to many research workers because of its wide application in many a diversified fields such as soil mechanics, petroleum geology, foundation engineering, ground water hydrology and so on. The theory can be applied to predict the time history of stress and strain in a porous solid in which the fluid seepage occurs and to determine the stresses in a dam the settlement and consolidation of clay in foundations, the flow of oil or water in petroleum reserviors and to solve a great variety of problems relating to poroelastic media.

On the basis of the above theory, Schiffman and Fungaroli[2] solved the problem of consolidation due to tangential loads by using the displacement functions introduced by McNamee and Gibson[2]. The problem was, however, restricted to the case in which the skeleton contains a viscous incompressible fluid. But Biot's theory was developed on the physical assumption that the skeleton was purely elastic and contains a viscous compressible fluid. This is a case of particular interest in the application of theory of soils and natural rock formations.

In the present paper, an attempt has been made to generalize the problem of Schiffman and Fungaroli by considering the deformation of a semi-infinite isotropic poroelastic medium saturated with viscous compressible fiuid and under the action of tangential loads on its surface. The problem is solved by using displacement functions of Verruijt[4] followed by Laplace-Hankel transfroms. As particular cases, the solutions

obtained by Schiffman and Fungaroli and these for the classical elastic case of Muki[5] are noted. The general solutions are applied to the case when a tangential force acts uniformly over a circular region of the surface and the initial and final response of the normal displacement, normal and circumferential stresses are shown graphically.

By thermoelastic analogy[6], the solutions obtained herein may be treated as the solutions of the corresponding problem in coupled, quasi-static thermoelasticity.

2. Basic Equations and the Problem

In the absence of body forces, the equations of equilibrium and Darcy's law of fluid flow for an isotropic poroelastic medium are[1]

$$\mu \nabla^2 u_i + (\lambda + \mu) e_{,i} - \alpha p_{f,,i} = 0 \qquad \ldots(1)$$

$$\nabla^2 p_f = \frac{d}{M_f^2} [\dot{p}_f + \alpha M \dot{e}] \qquad \ldots(2)$$

and the stress-strain relations are

$$\tau_{ij} = 2\mu e_{ij} + (\lambda e - \alpha p_f) \delta_{ij} \qquad \ldots(3)$$

$$\zeta_0 = \frac{1}{M} p_f + \alpha e \qquad \ldots(4)$$

where u_i are the solid displacements, τ_{ij} the total stresses (solid and fiuid), e_{ij} the solid strains, p_f the porepressure, $\zeta_0 = f(e - \epsilon)$ the fluid content, e and ϵ being the solid and fluid dilatations, d the Darcy constant, $f(0 < f < 1)$ is the porosity, λ, μ, α, M the elastic constants and a dot over a function denotes differentiation with respect to time.

Introducing the displacement functions[4], in cylindrical coordinates (r, θ, z) by

$$\begin{aligned} u_r &= -\frac{\partial E}{\partial r} + z \frac{\partial s}{\partial r} + \frac{2}{r} \frac{\partial Q}{\partial \theta} \\ u_\theta &= -\frac{1}{r} \frac{\partial E}{\partial \theta} + \frac{Q}{r} \frac{\partial s}{\partial \theta} - 2 \frac{\partial \theta}{\partial r} \\ u_z &= -\frac{\partial E}{\partial z} + z \frac{\partial s}{\partial z} + (1 - 2\nu) s \end{aligned} \qquad \ldots(5)$$

where $\nu = (\lambda + 2\mu + \alpha^2 M)/(\lambda + \mu + \alpha^2 M)$ it can be shown that the equations (1) and (2) will be satisfied if

$$\frac{\partial}{\partial t} (\nabla_1^2 E) = C \nabla^4 E \qquad \ldots(6)$$

$$\nabla_1^2 \, s = \nabla_1^2 \, Q = 0$$

and

$$\frac{\alpha p_f}{2\mu\eta_1} = -\nabla_1^2 \, E + 2(1-\beta)\frac{\partial s}{\partial z} \qquad \ldots(7)$$

where

$$\nabla_1^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r}\frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

$$\frac{1}{e^2} = \frac{d}{f^2}\left(\frac{1}{M} + \frac{\alpha^2}{2\mu\eta_1}\right) \qquad \ldots(8)$$

$$\eta_1 = \frac{\lambda + 2\mu}{2\mu}, \quad 2\eta_1 \beta = \frac{(\lambda + \mu)\nu}{\mu}.$$

The stress components τ_{ij} and fluid content ζ_0 can be expressed in terms of E, S, Q with the help of (3) to (5).

For convenience, we introduce the following non-dimensional quantities by

$$\rho = \frac{r}{a}, \zeta = \frac{z}{a}, \tau = \frac{ct}{a^2}, \phi(\rho, \theta, \zeta, \tau) = \frac{E(r, \theta\, z, t)}{a^2},$$

$$\psi(\rho, \theta, \zeta, \tau) = \frac{S(r, \theta, z, t)}{a}, \quad \Omega(\rho, \theta, \zeta, \tau) = \frac{\theta(r, \theta, z, t)}{a^2} \qquad \ldots(9)$$

$$P_f(\rho, \theta, \zeta, \tau) = \frac{\alpha p_f}{2\mu\eta_1}, \quad \zeta_n = \frac{\alpha c}{2\mu\eta_1 f^2}\,\zeta_0$$

a being a characteristic length; the equations (6) then reduce to

$$\nabla^4 \phi = \frac{\partial}{\partial t}(\nabla^2\phi), \quad \nabla^2\psi = 0, \quad \nabla^2\Omega = 0 \qquad \ldots(10)$$

where

$$\nabla^2 = \frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\theta^2} + \frac{\partial^2}{\partial\zeta^2}.$$

The non-dimensional displacements are

$$u = \frac{ur}{a} = -\frac{\partial\phi}{\partial\rho} + \zeta\frac{}{\partial\rho} + \frac{2}{\rho}\frac{\partial\Omega}{\partial\theta}$$

(*equation continued on p.* 263)

$$v = \frac{u\theta}{a} = -\frac{1}{\rho}\frac{\partial\phi}{\partial\theta} + \frac{\zeta}{\rho}\frac{\partial\psi}{\partial\theta} - 2\frac{\partial\Omega}{\partial\rho} \qquad \ldots(11)$$

$$\omega = \frac{uz}{a} = -\frac{\partial\phi}{\partial\zeta} + \zeta\frac{\partial\psi}{\partial\zeta} + (1-2\nu)\psi.$$

Taking 2μ as the unit of stress, we can similarly write down the non-dimensional expressions for the stresses τ_{ij}.

Let us now consider a semi-infinite isotropic poroelastic medium bounded by the plane $\zeta = 0$ and choose the ζ—axis normal to this plane and pointing into the medium. The medium is deformed by the action of tangential loads applied to its surface which is supposed to be permeable to the flow of pore fluid. Then, we may write the boundary conditions in the following forms

$$\tau_{\rho\rho} = 0$$

$$\tau_{\theta\rho} = \sum_{m=0}^{\infty} p_m(e) \sin m\theta\, H(\tau)$$

$$\tau_{\zeta} = \sum_{m=0}^{\infty} q_m(e) \cos m\theta\, H(\tau) \qquad \ldots(12)$$

on $\zeta = 0$, where $H(\tau)$ is the Heavyside unit function and $0 \leqslant \rho \leqslant \infty$. The fluid content ζ_n is assumed to be zero initially.

3. Solution of the Problem

Denoting the Laplace transform of a function by bar with p as parameter we take (Muki[5], Sanyal[7])

$$\bar{\phi}(\rho, \theta, \zeta, p) = \sum_{m=0}^{\infty} \bar{\phi}_m(\rho, \zeta, p) \cos m\theta$$

$$\bar{\psi}(\rho, \theta, \zeta, p) = \sum_{m=0}^{\infty} \bar{\psi}_m(\rho, \zeta, p) \cos m\theta \qquad \ldots(13)$$

$$\Omega(\rho, \theta, \zeta, p) = \sum_{m=0}^{\infty} \bar{\Omega}_m(\rho, \zeta, p) \sin m\theta.$$

Let the Hankel transform of a function be denoted by dash with ξ as parameter. For convenience, we shall take only a single value of m (Muki[5], Sanyal[7]). Taking Laplace-

Hankel transforms of the differential equations (10) and then solving for $\bar{\phi}'_m$, $\bar{\psi}'_m$, $\bar{\Omega}'_m$, we have,

$$\bar{\phi}'_m = A_m(\xi, p)\, e^{-\xi\zeta} + B_m(\xi, p)\, e^{-(\xi^2+p)^{1/2}\zeta}$$

$$\bar{\psi}'_m = C_m(\xi, p)\, e^{-\xi\zeta}, \quad \bar{\Omega}'_m = D_m(\xi, p) e^{-\xi\zeta} \qquad \ldots(14)$$

where $\bar{\phi}'_m$ etc. are bounded as $\zeta \to \infty$. Evaluating the fuctions $A_m(\xi, p)$, $B_m(\xi, p)$ etc. by using the boundary conditions (12), we can obtain the Laplace-Hankel transformed expressions for the displacements, stresses etc. The inversions of these transforms lead to

$$u(\rho, \theta, \zeta, \tau) = \frac{1}{4}\sum_{m=0}^{\infty} \cos m\theta \int_0^{\infty} \Big[(X_m + Y_m)\Big\{\frac{2\eta\nu-(2\eta-1)\xi\zeta}{2\eta-1}\, e^{-\xi\zeta}$$

$$-\Big\langle \eta(\nu-\xi\zeta)\psi_1 + \psi_2 - \psi_3 \Big\rangle\Big\}\{J_{m+1}(\xi\rho) - J_{m-1}(\xi\rho)\}$$

$$- 2e^{-\xi\zeta}(X_m - Y_m)\{J_{m+1}(\xi\rho) + J_{m-1}(\xi\rho)\} H(\tau)\Big]\, d\xi \qquad \ldots(15)$$

$$v(\rho, \theta, \zeta, \tau) = \frac{1}{4}\sum_{m=0}^{\infty} \sin m\theta \int_0^{\infty} \Big[(X_m + Y_m)\Big\{\frac{2\eta\nu-(2\eta-1)\xi\rho\, e^{-\xi\zeta}}{2\eta-1}$$

$$-\Big\langle \eta(\nu-\xi\zeta)\psi_1 + \psi_2 - \psi_3 \Big\rangle\Big\}\Big\{J_{m+1}(\xi\rho) + J_{m-1}(\xi\rho)\Big\}$$

$$- 2e^{-\xi\zeta}(X_m - Y_m)\{J_{m+1}(\xi\rho) - J_{m-1}(\xi\rho)\} H(\tau)\Big]\, d\xi \qquad \ldots(16)$$

$$w(\rho, \theta, \zeta, \tau) = \frac{1}{2}\sum_{m=0}^{\infty} \cos m\theta \int_0^{\infty} \Big[\Big(1 - \xi\rho - \frac{2\eta\nu}{2\eta-1}\Big) e^{-\xi\zeta}$$

$$-\psi_4 + \eta(\nu+\xi\zeta)\psi_1 + \psi_5\Big](X_m + Y_m) J_m(\xi\rho)\, d\xi \qquad \ldots(17)$$

$$\tau_{\rho\rho}(\rho, \theta, \zeta, \tau) = \frac{1}{2}\sum_{m=0}^{\infty} \cos m\theta \int_0^{\infty} \xi \Big[\xi \zeta e^{-\xi\zeta} + \psi_2 + \psi_4 - \eta(1 + \xi\zeta)\psi_1 \Big] (X_m + Y_m) J_m(\xi\rho)\, d\xi \quad \ldots(18)$$

$$\tau_{\theta\xi}(\rho, \theta, \zeta, \tau) = -\frac{1}{4}\sum_{m=0}^{\infty} \sin m\theta \int_0^{\infty} \xi \Big[\Big\{ (1 - \xi\zeta) e^{-\xi\zeta} + \psi_5 - \eta(1 - \xi\zeta)\psi_1 + \psi_4 \Big\} (X_m + Y_m) \Big\{ J_{m+1}(\xi\rho) + J_{m-1}(\xi\rho) \Big\} - e^{-\xi\zeta} (X_m - Y_m) \Big\{ J_{m+1}(\xi\rho) - J_{m-1}(\xi e) \Big\} H(\tau) \Big] d\xi \quad \ldots(19)$$

$$\tau_{\rho\xi}(\rho, \theta, \zeta, \tau) = -\frac{1}{4}\sum_{m=0}^{\infty} \cos m\theta \int_0^{\infty} \xi \Big[\Big\{ (1 - \xi\zeta) e^{-\xi\zeta} + \psi_5 - \eta(1 - \xi\zeta)\psi_1 + \psi_4 \Big\} (X_m + Y_m) \Big\{ J_{m+1}(\xi\rho) - J_{m-1}(\xi\rho) \Big\} - e^{-\xi\zeta} (X_m - Y_m) \Big\{ J_{m+1}(\xi\rho) + J_{m-1}(\xi\rho) \Big\} H(\tau) \Big] d\xi. \quad \ldots(20)$$

$$\tau_{\rho\rho}(\rho, \theta, \zeta, \tau) = -\frac{1}{2}\sum_{m=0}^{\infty} \cos m\theta \Big[\int_0^{\infty} \xi \Big\{ (2 - \xi\zeta) e^{-\xi\zeta} + \psi_3 + \psi_6 - \psi_2 - (2 - \xi\zeta)\eta\psi_1 \Big\} (X_m + Y_m) J_m(\xi\rho)\, d\xi + \frac{1}{2\rho} \int_0^{\infty} \Big\{ \frac{2\eta\nu - (2\eta - 1)\xi\zeta}{2\eta - 1} e^{-\xi\zeta} - (\eta(\nu - \xi\zeta)\psi_1$$

(*equation continued on p.* 266)

$$+ \psi_2 - \psi_3) \Big\} (X_m + Y_m) \Big\{ (m+1) J_{m+1}(\xi\rho)$$

$$+ (m-1) J_{m-1}(\xi\rho) \Big\} - 2e^{-\xi\zeta} (X_m - Y_m)$$

$$\Big\{ (m+1) J_{m+1}(\xi\rho) - (m-1) J_{m-1}(\xi\rho) \Big\} H(\tau) \Big\rangle$$

$$d\xi \Big] \qquad \ldots(21)$$

$$\tau_{\rho\zeta}(\rho, \theta, \zeta, \tau) + \tau_{\theta\theta}(\rho, \theta, \zeta, \tau) = \frac{1}{2} \sum_{m=0}^{\infty} \cos m\theta \int_0^{\infty}$$

$$\xi \Big[\frac{(4 - \xi\zeta)(2\eta - 1) - 2\eta\nu}{2\eta - 1} e^{-\xi\zeta}$$

$$+ \psi_3 - \psi_2 - \eta(4 - \nu - \xi\zeta)\psi_1$$

$$+ 2\psi_6 \Big] (X_m + Y_m) J_m(\xi\rho)\, d\xi \qquad \ldots(22)$$

$$\tau_{\rho\theta}(\rho, \theta, \zeta, \tau) = \frac{1}{2} \sum_{m=0}^{\infty} \sin m\theta \Big[- \int_0^{\infty} \xi (X_m - Y_m) e^{-\xi\zeta} H(\tau) J_m(\xi\rho)\, d\xi$$

$$+ \frac{1}{2\rho} \int_0^{\infty} \Big\langle \Big\{ \frac{2\eta\nu - (2\eta - 1)\xi\zeta}{2\eta - 1} e^{-\xi\zeta} - \Big(\eta(\nu - \xi\zeta)$$

$$\psi_1 + \psi_2 - \psi_3 \Big) \Big\} (X_m + Y_m) \Big\{ (m+1) J_{m+1}(\xi\rho)$$

$$- (m-1) J_{m-1}(\xi\rho) \Big\} - 2\rho^{-\xi\zeta} (X_m - Y_m)$$

$$\Big\{ (m+1) J_{m+1}(\xi\rho) + (m-1) J_{m-1}(\xi\rho) \Big\} H(\tau) \Big\rangle$$

$$d\xi \Big]. \qquad \ldots(23)$$

$$P_f(\rho, \theta, \zeta, \tau) = \tfrac{1}{2} \sum_{m=0}^{\infty} \cos m\theta \int_0^{\infty} \xi (\psi_1 - \psi_6)(X_m + Y_m) J_m(\xi \rho)\, d\xi \quad \ldots(24)$$

$$\zeta_n(\rho, \theta, \zeta, \tau) = \frac{1}{2} \sum_{m=0}^{\infty} \cos m\theta \int_0^{\infty} \xi \left(\frac{2e^{-\xi\zeta}}{2\eta - 1} - \psi_6 \right)(X_m + Y_m) J_m (\xi\rho)\, d\xi \quad \ldots(25)$$

where,

$$\psi_1 = e^{-\xi\zeta} \left[\frac{1}{2\eta - 1} \operatorname{erfc}(\xi\sqrt{\tau}) - \frac{\eta - 1}{\eta(2\eta - 1)} \operatorname{erfc}\left(\frac{\eta - 1}{\eta} \xi\sqrt{\tau} \right) \exp\left(- \frac{2\eta - 1}{\eta^2} \xi^2 \tau \right) \right] \quad \ldots(26)$$

$$\psi_2 = - \frac{\eta(\eta - 1)}{(2\eta - 1)^2} \operatorname{erfc}\left(\frac{\eta - 1)}{\eta} \xi\sqrt{\tau} + \frac{\zeta}{2\sqrt{\tau}} \right) \exp\left(- \frac{2\eta - 1}{\eta^2} \xi^2\tau + \frac{\eta - 1}{\eta} \xi\zeta \right) + \frac{1}{2\eta - 1}$$

$$\left\{ \frac{1}{2} \xi\zeta - \xi^2\tau + \frac{1}{4(2\eta - 1)} \right\} \operatorname{erfc}\left(\xi\sqrt{\tau} - \frac{\zeta}{2\sqrt{\tau}} \right) e^{-\xi\zeta} + \frac{1}{4} \operatorname{erfc}\left(\xi\sqrt{\tau} + \frac{\zeta}{2\sqrt{\tau}} \right) e^{\xi\zeta} + \frac{\xi}{2\eta - 1} \sqrt{\frac{\tau}{\eta}}\, e^{-\zeta^2/4\tau - \xi^2\tau} \quad \ldots(27)$$

$$\psi_3 = e^{-\xi\zeta} \left[- \frac{\eta(\eta - 1)}{(2\eta - 1)^2} \operatorname{erfc}\left(\frac{\eta - 1}{\eta} \xi\sqrt{\tau} \right) \exp\left(- \frac{2\eta - 1}{\eta^2} \xi^2\tau \right) + \frac{1}{2\eta - 1} \left\{ - \xi^2\tau + \frac{2\eta^2 - 2\eta + 1}{2(2\eta - 1)} \right\} \operatorname{erfc}(\xi\sqrt{\tau}) + \frac{\xi}{2\eta - 1} \left(\frac{\tau}{\eta} \right)^{1/2} e^{-\xi^2} \right] \quad \ldots(28)$$

$$\psi_4 = e^{-\xi\zeta} \left[- \left(\frac{\eta - 1}{2\eta - 1} \right)^2 \operatorname{erfc}\left(\frac{\eta - 1}{\eta} \xi\sqrt{\tau} \right) \exp\left(- \frac{2\eta - 1}{\eta^2} \xi^2\tau \right) \right.$$

(*equation continued on p.* 268)

$$+\frac{1}{2\eta-1}\Big)\Big\{\xi^2\tau+\frac{2\eta^2-1}{2(2\eta-1)}\Big\}\operatorname{erfc}(\xi\sqrt{\tau})$$

$$-\frac{\xi}{2\eta-1}\left(\frac{\tau}{\eta}\right)^{1/2}e^{-\xi^2/4\tau}\Big]. \qquad \ldots(29)$$

$$\psi_5=-\left(\frac{\eta-1}{2\eta-1}\right)^2\operatorname{erfc}\left(\frac{\eta-1}{\eta}\xi\sqrt{\tau}+\frac{\xi}{2\sqrt{\tau}}\right)$$

$$\exp\left(-\frac{2\eta-1}{\eta^2}\xi^2\tau+\frac{\eta-1}{\eta}\xi\zeta\right)+\frac{1}{2\eta-1}\Big\{\xi^2\tau-\frac{1}{2}\xi\zeta$$

$$+\frac{4\eta-3}{4(2\eta-1)}\Big\}\operatorname{erfc}\left(\xi\sqrt{\tau}-\frac{\zeta}{2\sqrt{\tau}}\right)e^{\xi\zeta}+\frac{1}{4}\operatorname{erfc}\left(\xi\sqrt{\tau}\right.$$

$$\left.+\frac{\zeta}{2\sqrt{\tau}}\right)e^{\xi\zeta}-\frac{\xi}{2\eta-1}\left(\frac{\tau}{\pi}\right)^{1/2}\exp\left(-\frac{\zeta^2}{4\tau}-\xi^2\tau\right) \qquad \ldots(30)$$

$$\psi_6=\frac{1}{2\eta-1}\operatorname{erfc}\left(\xi\sqrt{\tau}-\frac{\zeta}{2\sqrt{\tau}}\right)e^{-\xi\zeta}-\frac{\eta-1}{\eta(2\eta-1)}\operatorname{erfc}$$

$$\times\left(\frac{\eta-1}{\eta}\xi\sqrt{\tau}+\frac{\zeta}{2\sqrt{\tau}}\right)\exp\left(-\frac{2\eta-1}{\eta^2}\xi^2\tau+\frac{\eta-1}{\eta}\xi\zeta\right)$$

$$\eta=\frac{1}{2(1-\beta)} \qquad \ldots(31)$$

and

$$X_m(\xi)=-\int_0^\infty \rho\,[p_m(\rho)-q_m(\rho)]\,J_{m-1}(\rho\xi)\,d\rho.$$

$$Y_m(\xi)=-\int_0^\infty \rho\,[p_m(\rho)+q_m(\rho)]\,J_{m+1}(\xi\rho)\,d\rho. \qquad \ldots(32)$$

Making $\alpha \to 1$, $M \to \infty$ i. e. $\nu \to 1$, it can be see that the above solution coincide with those of Schiffman and Fungaroli[2].

Again if we consider the case when $\alpha \to 0$, $f \to 0$, so that $\zeta_0 \to 0$, $p_f \to 0$ then the Darcy law (2) and the relation (4) vanish so that the equilibrium (1) and the stress-strain relations (3), reduce to these for classical elastic case. Also, the vanishing of α and f show from (S) that

$$\tau \to \infty,\ \beta \to 1,\ \eta \to \infty,\ \nu \to 2(1-\sigma)$$

where $\sigma = \dfrac{\lambda}{2(\lambda + \mu)}$ is the Poisson's ratio. Using these limits it is easy to verify that the solutions (15) to (23) coincide with the corresponding solutions of classical elasticity[5].

4. An Example

To consider an example of the previous analysis, we suppose that $\tau_{x\zeta}$ is the only non-zero stress on the surface $\zeta = 0$ and is uniformly distributed over a circular area. Thus the boundary conditions on $\zeta = 0$ become

$$\tau_{x\zeta} = \frac{Q}{\pi.12} H(\tau) = q\, H(\tau),\ \sqrt{x^2 + y^2} < 1,\ \tau > 0,$$

$$= 0\ ,\ \sqrt{x^2 + y^2} > 1,\ \tau > 0,$$

$$\tau_{\zeta\rho} = \tau_{y\zeta} = 0,\ 0 \leqslant \sqrt{x^2 + y^2} \leqslant \infty,\ \tau > 0$$

where Q is the total force applied to the surface. Introducing cylindrical coordinates, the non-zero stresses on $\zeta = 0$ are

$$\tau_{\theta\zeta} = -\frac{Q}{\pi} \sin\theta\, H(\tau),\quad \rho < 1,$$

$$\tau_{\zeta\rho} = -\frac{Q}{\pi} \cos\theta.\ H(\tau),\ \rho < 1. \qquad \ldots(33)$$

Using (12), (32) and (33)

$$X_1(\xi) = \frac{2Q}{\pi} \int_0^1 \rho\, J_0(\xi\rho)\, d\rho = \frac{2Q}{\pi} \cdot \frac{J_1(\xi)}{\xi} \qquad \ldots(34)$$

$$Y_1(\xi) = 0.$$

Inserting (34) into equations (15) to (25) and putting $m = 1$ we can obtain the displacements, stresses etc. for this problem. The initial and final response of the medium and the solutions for classical elastic problem can also be obtained by using the limits discussed in section 3.

It is found that the expressions for the displacemets, stresses etc. for initial and final response and those for classical elasticity involve integrals of the form

$$\int_0^\infty J_\mu(\xi)\, J_\nu(\xi\rho)\, e^{-\xi\zeta}\, \xi^\rho\, d\xi$$

a number of which was evaluated and tabulated by Eason *et al.*[8] Using these results and the recurrence relations developed by these authors in $x - \zeta$ plane the displacement ω and the stresses $\tau_{\zeta\rho}$ and $\tau_{\theta\theta}$ have been plotted graphically for $\zeta = 0$ and $\zeta = 1.0$ in Fig. 1 to 6. The initial response has been shown by continuous curves, final response by discontinuous curves and the classical elastic response by dotted curves. For numerical calculations we put $\nu = 1.3$, $\eta = 1.5$ and $\sigma = 0.25$.

It follows from Fig. 1 and 2 that the normal displacements for ultimate response are the same and they differ much from those of initial response. For $\zeta=0$ the response is maximum at the circumference of the circle $\rho = 1$, While for $\zeta = 0$ it is maximum near $\rho = 1.2$.

The radial stress, as can be seen from Fig. 3 is discontinuous at $\rho = 1$ for $\zeta = 0$ and the response for both the three cases are the same in nature. They are equal up to near about $\rho = 0.7$ and for other values of ρ (except $\rho = 1$) they differ numeri-

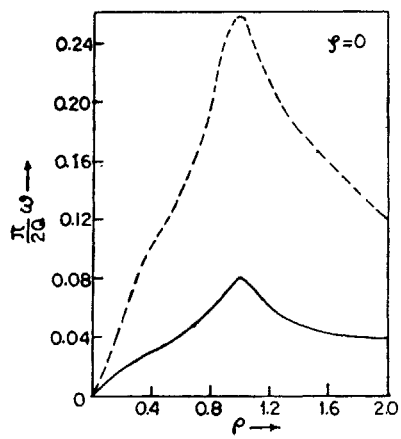

FIG. 1.

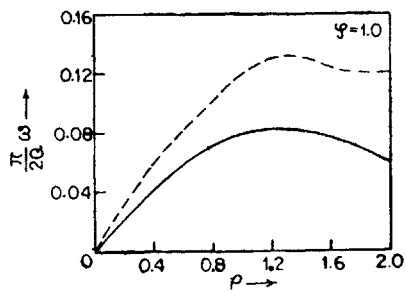

FIG. 2.

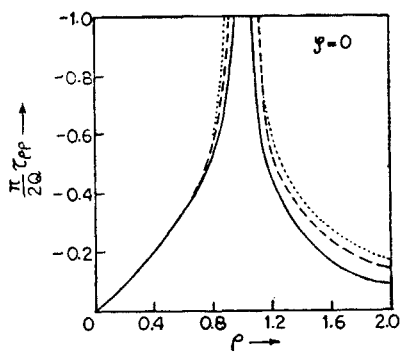

FIG. 3.

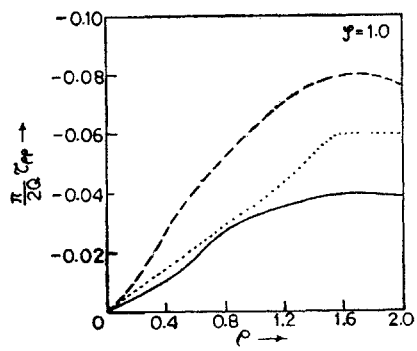

FIG. 4.

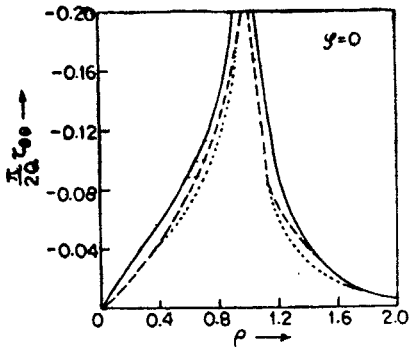

FIG. 5.

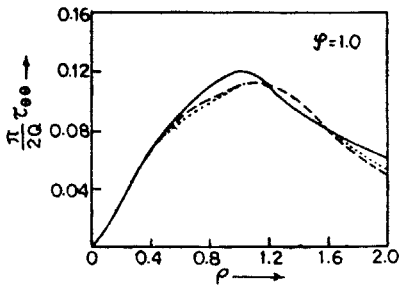

FIG. 6.

cally. For $\rho = 1$ the response is infinite. But for $\zeta=1$ the curves are continuous and the difference is significant (Fig. 4).

The natures of $\tau_{\theta\theta}$ for $\zeta = 0$ (Fig. 5) and $\zeta = 1$ (Fig. 6) are almost the same as those of $\tau_{\rho\rho}$. They are infinite at $\rho = 1$ for $\zeta = 0$. For $\zeta = 1$ is approximately the same for the ultimate response and classical elastic response.

5. THERMOELASTIC ANALOGY

It was pointed out by Biot[6] that the theory of quasistatic poroelastic medium is analogus to that of coupled, quasistatic thermoelastic medium in which the body forces and heat sources are absent. For example, the pore fluid pressure p_f plays the role of increment of temperature θ over the initial stress-free temperature. We may obtain the expressions for temperature, displacements etc. for the corresponding coupled thermoelastic problem by using the correspondence rule given by Biot[6] from (15) to (25). As for example, the expression for the temperature θ is given by

$$\theta = \tfrac{1}{2} \sum_{m=0}^{\infty} \cos m\theta \int_0^{\infty} \xi (\psi_1 - \psi_6)(X_m + Y_m) J_m(\xi\rho)\, d\xi$$

where $\psi_1(\rho, \theta, \zeta, \tau)$ and $\psi_6(\rho, \theta, \zeta, \tau)$ are now given by (26) and (31) respectively in which we have to replace η by $\dfrac{(\lambda + \mu) + \epsilon(\lambda + 2\mu)}{2\epsilon\mu}$ and τ by $\dfrac{\tau}{[a^2 x_0 (1 + \epsilon)]}$, ϵ being the coupling parameter and x_0 is the thermal diffusivity.

In particular, for the uncoupled thermoelastic problem $\epsilon \to 0$ and we shall have $\theta \equiv 0$.

6. CONCLUSIONS

The solutions presented herein are believed to be new and are more general than those of Muki[5] and Schiffman and Fungaroli[2] in the sense that each of them becomes

of function of time due to the presence of fluid in the pores of the medium and the fluid is compressible. In classical elasticity however they are independent of time. Due to complicacy of the results in the present paper it was felt difficult to work out the numerical calculations for all values of time and therefore the initial and ultimate response have been shown graphically. It is evident that these responses and the classical elastic solutions vary numerically. Thus the presence of fluid in an elastic medium affects the solutions. As the displacements, stresses etc. must vary with time from practical point of view, it necessiates us to consider the effect of fluid in problems in soil-like elastic medium.

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