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ON LEFT DUO NEAR-RINGS

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Choudhary[3] proved that if N is a regular left duo distributively generated near-ring with identity, then every left ideal L of N is the intersection of all maximal left ideals of N containing E. In this paper we show that this theorem holds if N is a regular left duo near-ring and also extend some of his results.

The aim of this paper is to extend some theorems of Choudhary[3].

Throughout this paper N stands for a zerosymmetric right near-ring. For basic definitions and results, we refer to Pilz[1]. If A and B are subsets of N, we denote $\{n \in N/nB \subseteq A\}$ by $(A : B)$. N is said to have the IFP if $ab = 0$ implies $anb = 0$ for all $n \in N$.

Recall that a near-ring N is said to be left duo if every left ideal of N is an ideal of N.

Lemma 1—If N is a 2-primitive left duo near-ring, then N has no nonzero proper N-subgroups.

PROOF : By Corollary 4.4 of Pilz[1], there exists a left ideal L of N such that L is 2-modular and $(L : N) = \{0\}$.

Since N is left duo, $L = (L : N) = \{0\}$ by Proposition 3.25 of Pilz[1]. Thus $N = N/L$ is a N-group of type 2. So N has no nonzero proper N-subgroups.

Theorem 2—Suppose $N^2 \neq \{0\}$ and N has a nonzero distributive element d. Then the following are equivalent.

(i) N has no nonzero proper N-subgroups.

(ii) $Na = N$ for all $0 \neq a \in N$.

(iii) N is a near-field.

PROOF : (i) $\Rightarrow$ (ii) Let $a \in N$ with $Na = \{0\}$. Let $A = \{x \in N/dx = 0\}$. A is an additive subgroup of N, since d is a distributive element of N. If $x \in A$ and $n \in N$, then $dx = 0$ and hence by our assumption, $(0 : x) = N$. Therefore $dnx = 0$ and hence $nx \in A$. Thus A is an N-subgroup of N and so, either $A = N$ or $A = \{0\}$. If $A = N$, then $d \in (0 : N)$ and hence by our assumption, $(0 : N) = N$, a contradiction. Thus $A = \{0\}$ and hence $a = 0$. This proves (ii).

(ii) $\Rightarrow$ (iii) follows from Theorem 8.3 of Pilz[1].

(iii) $\Rightarrow$ (i) is obvious.

As an immediate consequence of Theorem 2, we have the following :

Corollary 3 (Ligh[2])—Suppose N has a nonzero distributive element d and for every $0 \neq n \in N$, there exists $m \in N$ such that $mn \neq 0$. If N has no nonzero proper N-subgroups, then N is a near-field.

Since in a 2-primitive near-ring $N^2 \neq \{0\}$, Lemma 1 and Theorem 2 yields the following :

Corollary 4—If N is a 2-primitive left duo near-ring with a nonzero distributive element, then N is a near-field.

Corollary 5 (Choudhary[3], Theorem V. 1. 7)—Let N be a 2-primitive left duo near-ring with a nonzero distributive element d. If for every $0 \neq a \in N$, there exists $b \in N$ such that $ba \neq 0$, then N is a near-field.

Lemma 6—If N has IFP, then for any $x, y \in N$ and $e^2 = e$, $xye = xeye$. The proof of this lemma is easy and hence omitted. Recall that a near-ring N is said to be strongly regular if for each $a \in N$, there exists $b \in N$ such that $a = ba^2$.

For any subset A of N, we write $\sqrt{A} = \{x \in N/x^n \in A$ for some positive integer $n\}$. Clearly $A \subseteq \sqrt{A}$.

Theorem 7—The following are equivalent

(i) N is a regular left duo near-ring

(ii) N is a regular IFP near-ring

(iii) N is a strongly regular near-ring

(iv) $A = \sqrt{A}$ for every N-subgroup A of N.

PROOF : (i) $\Rightarrow$ (ii) follows from Proposition 9.3 of Pilz[1].

(ii) $\Rightarrow$ (iii) Let $a \in N$. Since N is regular, there exists $b \in N$ such that $a = aba$. So, $ba = baba = b(ba)\,a\,(ba) = b^2 a^2$ by Lemma 6. Therefore $a = aba \in Na^2$. Thus N is strongly regular.

(iii) $\Rightarrow$ (iv) let A be an N-subgroup of N. If $a \in \sqrt{A}$, then $a^n \in A$ for some positive integer n. Since N is strongly regular, $a = xa^2 = x^2 a^3 = \ldots = x^{n-1} a^n \in A$. Thus $A = \sqrt{A}$.

(iv) $\Rightarrow$ (i) let $a \in N$. Then $a^3 \in Na^2$ and hence $a \in \sqrt{Na^2} = Na^2$ by (iv). Thus N is strongly regular. Now the result follows from Theorem 3 of Reddy and Murty[4] and Theorem 2.3 of Murty[5].

Corollary 8 (Choudhary[3], Theorem V. 2.5)—If N is a regular left duo near-ring, then for every iedal A of N, $A = \sqrt{A}$.

Recall that an ideal I of N is said to be completely semiprime if $a^2 \in I$ implies $a \in I$. Obviously, in a strongly regular near-ring every ideal is completely semiprime.

We now prove our main theorem.

Theorem 9—A near-ring N is strongly regular if and only if every N-subgroup I of N is the intersection of all maximal ideals of N containing I.

PROOF : Suppose N is strongly regular. Since every ideal of N is completely, semiprime, by Theorem 2.6 of Reddy and Murty[6], it follows that every ideal I of N is the intersection of all completely prime ideals containing I. Therefore every N-subgroup I of N is the intersection of all maximal ideals of N containing I by Theorems 2.3 and 4.1 of Murty[5].

Conversely suppose that every N-subgroup I of N is the intersection of all maximal ideals containing I. Let $x \in N$ and $x \neq ax^2$ for all a. Since Nx^2 is the intersection of all maximal ideals containing Nx^2, there exists a maximal ideal M such that $x \notin M$. But $x^3 \in M$ and M is completely semiprime by Theorem 4.1 of Murty[5], a contradiction. Therefore N is strongly regular.

Combining Theorems 7.9 and 2.3 of Murty[5], we obtain the following:

Corollary 10—A near-ring N is a regular left duo near-ring if and only if every N-subgroup I of N is the intersection of all maximal ideals of N containing I if and only if every N-subgroup I of N is the intersection of all maximal left ideals of N containing I.

The following result is an immediate consequence of Corollary 10.

Corollary 11 (Choudhary[3], Theorem V. 2.6)—Suppose N is regular, left duo and distributively generated near-ring with identity. Then every left ideal L of N is the intersection of all maximal left ideals of N containing L.

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