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USE OF VARIATIONAL APPROACH IN DETERMINING THE PERIODS OF SMALL ADIABATIC OSCILLATIONS OF ROTATIONALLY AND TIDALLY DISTORTED POLYTROPIC GASEOUS SPHERES

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In this paper we consider the feasibility of using the variational approach to determine the eigen frequencies of pseudoradial and nonradial modes of oscillations of rotationally and tidally distorted gaseous spheres whose undistorted equilibrium structures are polytropic spheres. The variational method developed for this purpose is based on the variational principle of Chandrasekhar and Lebovitz for undistorted gaseous spheres and uses the averaging technique of Kippenhahn and Thomas to incorporate the effects of distortions caused by rotation and tidal forces.

INTRODUCTION

The mathematical problem of determining the effects of rotation and tidal forces on the equilibrium structure, stability and periods of small adiabatic oscillations of gaseous spheres has practical importance in astronomy and astrophysics where it is expected to help in understanding some of the observed features of rotating stars and stars in binary and multiple systems. The exact mathematical problem of determining the effects of rotation and tidal forces on the equilibrium structure, stability and periods of small oscillations of a gaseous sphere representing the inner structure of a realistic star is, however, quite complex. Even though a systematic study of these problems may be said to have been initiated by Chandrasekhar as early as in 1933, the practical difficulties encountered in the theoretical studies of these problems, particularly the problems concerned with determining the effects of rotation and tidal distortions on the periods of small oscillations of the stars, have not as yet been satisfactorily resolved.

Attempts thus far available in literature (for some of the references to earlier works see for instance 'Slettbak[1], Tassoul[2], Cox[3]) have attempted to analyse these problems in approximate ways under different types of simplifying assumptions. In one such attempt Mohan and Saxena[4] (hereafter referred to as paper I) used the

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averaging concept of Kippenhahn and Thomas[5] in conjunction with Kopal's[6] method of evaluating various parameters on the Roche equipotentials to determine the effects of rotation and tidal distortions on the shapes and equilibrium structures of the polytropic models of the stars. Later Mohan and Saxena[7] (hereafter referred to as paper II) further used this approach to compute the eigen frequencies of pseudoradial and nonradial modes of oscillations of rotationally and tidally distorted polytropic models. In this paper they determined the eigen frequencies of the pseudoradial and nonradial modes of oscillations of the distorted polytropic models using the numerical approach usually adopted for determining the radial and nonradial modes of oscillations of gaseous spheres (cf. Ledoux and Walraven[8]).

In an earlier paper Mohan and Saxena[9] (hereafter referred to as paper III) successfully demonstrated the effectiveness of a variational approach in determining the eigen frequencies of the pseudoradial modes of oscillations of rotationally and/or tidally distorted gaseous spheres whose undistorted equilibrium structures were assumed to be based on Roche model. This variational approach was primarily developed on the lines of variational approach of Chandrasekhar and Lebovitz[10] for undistorted gaseous spheres using Kippenhahn and Thomas[5] technique to incorporate distortion effects. Keeping inview the fact that compared to Roche model, polytropic models provide much better insight into the conditions prevailing in the interiors of large varieties of the realistic stars, we consider in this paper the effectiveness of using similar variational techniques in determining the eigen frequencies of pseudoradial as well as nonradial modes of oscillations of rotationally and/or tidally distorted gaseous spheres whose undistorted equilibrium structures are based on polytropic models. In section 2 we have obtained the characteristic equation which determines the eigen frequencies of the first two modes of pseudoradial oscillations of rotationally and tidally distorted polytropic models. The characteristic equation determining the eigen frequencies of three modes of nonradial oscillations of such distorted polytropic models is developed in section 3. Numerical evaluation of the eigen frequencies in case of certain rotationally and tidally distorted polytropic models of indices 1.5 and 3 is then attempted in section 4. Results obtained in section 4 are then critically analysed in section 5 and certain conclusions based on the present study drawn.

2. Characteristic Equation Determining the Eigen Frequencies of Pseudoradial Modes of Oscillations of Rotationally and Tidally Distorted Polytropic Models

In paper III it was shown that following Chandrasekhar and Lebovitz[10] the characteristic equation determining the eigen frequency σ of small adiabatic pseudoradial modes of oscillations of a rotationally and tidally distorted gaseous sphere can be expressed as :

$$\sigma^2 \int_0^{R_\psi} \rho_\psi \frac{\phi^2}{r_\psi^2}\, dr_\psi = \int_0^{R_\psi} \left[\gamma P_\psi \left(\frac{d\phi}{dr_\psi}\right) + 4\frac{dP_{r\psi}}{dr_\psi} \cdot \frac{\phi^2}{r_\psi}\right] \frac{1}{r_\psi^2}\, dr_\psi \qquad \ldots(1)$$

where P_ψ and ρ_ψ are the values of pressure and density on the equipotential surface of the distorted model to which there corresponds an equivalent spherical surface of radius r_ψ defined on the basis of averaging approach of Kippenhahn and Thomas[5]. R_ψ is the value of r_ψ on the outer most surface, ϕ an unknown function of r_ψ and γ the ratio of specific heats.

On assuming trial function ϕ to be the form

$$\phi = a r_\psi^\lambda + b r_\psi^{\lambda+2}$$

it was shown that the above characteristic equation yields

$$\begin{vmatrix} \sigma^2\rho_{2\lambda-2} - \{\gamma\lambda^2 - 4(2\lambda-3)\}\, p_{2\lambda-4},\ \sigma^2\rho_{2\lambda} - \{\gamma\lambda(\lambda+2) - 4(2\lambda-1)\}\, p_{2\lambda-2} \\ \sigma^2\rho_{2\lambda} - \{\gamma\lambda(\lambda+2) - 4(2\lambda-1)\}\, p_{2\lambda-2},\ \sigma^2\rho_{2\lambda+2} - \{\gamma(\lambda+2)^2 - 4(2\lambda+1)\} p_{2\lambda} \end{vmatrix} = 0 \qquad \ldots(2)$$

as the equation determining the squares of eigen frequency σ of two pseudoradial modes of oscillations of a rotationally and tidally distorted gaseous sphere. In the above equation

$$\rho_l = \int_0^{R_\psi} \rho_\psi\, r_\psi^l\, dr_\psi \text{ and } p_l = \int_0^{R_\psi} p_\psi\, r_\psi^l\, dr_\psi,\ l = 2\lambda - 2, 2\lambda, 2\lambda + 2$$

which infact in-corporate the rotational and tidal distortion effects.

On setting $\lambda = 3$ (a value used by Chandrasekhar and Leboviz[10] for the case of undistorted polytropic models) and using expressions for ρ_ψ, p_ψ and r_ψ for rotationally and tidally distorted polytropic models as obtained by us in paper I, the characteristic equation determining the eigen frequencies of small adiabatic pseudo-radial oscillations of rotationally and tidally distorted polytropic models can be expressed in nondimensional form as

$$\begin{vmatrix} w^2 A_1 - B_1 & w^2 A_2 - B_2 \\ w^2 A_2 - B_2 & w^2 A_3 - B_3 \end{vmatrix} = 0, \qquad \ldots(3)$$

where

$$w^2 = \frac{D^3 r_{os}^3}{GM_0}\sigma^2$$

w being the nondimensional form of the eigen frequency σ, and

$$A_1 = \frac{(N+1)\,\xi_u^2\,k}{3\,r_{os}^3}\left(\frac{\bar{\rho}}{\rho_c}\right)\rho_4^*, \qquad B_1 = (9\gamma - 12)\,p_2^*$$

$$A_2 = \frac{(N+1)\,\xi_u^2\,k}{3\,r_{os}^3}\left(\frac{\bar{\rho}}{\rho_c}\right)\rho_6^*, \qquad B_2 = (15\gamma - 20)\,p_4^*$$

$$A_3 = \frac{(N+1)\,\xi_u^2\,k}{3\,r_{os}^3}\left(\frac{\bar{\rho}}{\rho_c}\right)\rho_8^*, \qquad B_3 = (25\gamma - 28)\,p_6^*.$$

Explicit expressions determining ρ_m^* and p_m^* are given by

$$\rho_m^* = r_{os}^{m+1}\int_0^1 \theta_\psi^N\, x^m\left[1 + \frac{2n}{3}(m+4)\,x^m\,r_{os}^3 + \left(\frac{4}{5}\,q^2 + \frac{8}{15}\,nq + \frac{10m+76}{45}\,n^2\right)(m+7)\,x^6\,r_{os}^6 + \frac{5}{7}\,q^2\,(m+9)\,x^8\,r_{os}^8 + \frac{2}{3}\,q^2\,(m+11)\,x^{10}\,x_{os}^{10}\right]dx \quad (m = 4, 6, 8)$$

$$p_m^* = r_{os}^{m+1}\int_0^1 \phi_\psi^{N+1}\, x^m\left[1 + \frac{2n}{3}(m+4)\,x^3\,r_{os}^3 + \left(\frac{4}{5}\,q^2 + \frac{8}{15}\,nq + \frac{10m+76}{45}\,n^2\right)(m+7)\,x^6\,r_{os}^6 + \frac{5}{7}\,q^2\,(m+9)\,x^8\,r_{os}^8 + \frac{2}{3}\,q^2\,(m+11)\,x^{10}\,r_{os}^{10}\right]dx$$

$(m = 2, 4 \text{ and } 6)$

when terms up to second order of smallness in the distortion parameters n and q are retained. Also $2n$ is the square of the normalized angular velocity of rotation and q the ratio of the mass of the secondary component causing tidal distortions to the mass of primary under investigation. Other symbols used in the above expressions have

their meanings as assigned to them in paper I. In particular ξ_u is the nondimensional Emden radius of the undistorted polytropic model, k the ratio of the radius of the undistorted primary model to the distance between the centres of the primary and secondary, ρ_c the density at the centre and $\bar{\rho}$ the mean density of the undistorted primary.

The characteristic equation (3) can be used to derermine the eigen frequencies of the fundamental mode and the first mode of pseudoradial oscillations of a rotationally and or tidally distorted polytropic model of index N for the specified values of the distortion parameters n and q (both assumed less than one).

On setting $q = 0$ or $n = 0$ separately in the above expressions we can determine the eigen frequencies of the modes of pseudo-radial oscillations of polytropic models distorted by rotation alone or by tidal forces alone. Also by setting $n = (q + 1)/2$ we can study the pseudo-radial oscillations of the primary component (having its equilibrium structure as that of a rotationally and tidally distorted polytropic model) of a synchronously rotating binary system.

3. Characteristic Equation Determining the Eigen Frequencies of Nonradial Modes of Oscillations of Rotationally and Tidally Distorted Polytropic Models

In order to derive the characteristic equations which may be used to determine the effects of rotation and or tidal distortions on the eigen frequencies of nonradial modes of oscillations of a gaseous sphere, we again use the concepts of topologically equivalent spherical models which were used in Paper III to determine the eigen frequencies of pseudo radial modes of oscillations of rotationally and tidally distorted gaseous spheres. Using these concepts and following essentially Chaudrasekhar and Lebovitz[10], the equation providing variational base for determining the effects of rotation and tidal distortions on the eigen frequencies of nonradial modes of oscillations of gaseous spheres can be expressed as :

$$\sigma^2 \int_0^{R_\psi} \rho_\psi \left[\frac{\Phi^2}{r_\psi^2} + \frac{1}{l(l+1)}\left(\frac{d\chi}{dr_\psi}\right)^2\right] dr_\psi = \gamma \int_0^{R_\psi} p_\psi \left[\frac{d}{dr_\psi}(\Phi - \lambda)\right]^2 \frac{dr_\psi}{r_\psi^2}$$

$$+ 2 \int_0^{R_\psi} \frac{dp_\psi}{dr_\psi} \cdot \frac{\Phi}{r_\psi^2}\left(2\frac{\Phi}{r_\psi} - \frac{d\chi}{dr_\psi}\right) dr_\psi$$

$$+ \frac{8\pi G}{2l+1} \int_0^{R_\psi} \rho_\psi k_l r_\psi^l \left(l\frac{\Phi}{r_\psi} + \frac{d\chi}{dr_\psi}\right) dr_\psi \qquad \ldots(4)$$

where

$$K_l(r_\psi) = \int_{r_\psi}^{R_\psi} \frac{\rho_\psi(s)}{s^{l+1}} \left[(l+1)\frac{\Phi(s)}{s} - \frac{d\chi(s)}{ds} \right] ds$$

Φ and χ are certain unknown functions of r_ψ and other symbols have their usual meanings as assigned to them in papers I and III. Following Chandrasekhar and Lebovitz[10] we assume the dependence of trial functions Φ and χ on r_ψ of the form

$$\Phi = a\, r_\psi^\lambda + b\, r_\psi^{\lambda+2} \text{ and } \chi = a\, r_\psi^\lambda + c\, r_\psi^{\lambda+2}$$

where a, b and c are variational parameters and λ is some parameter depending upon the nature of the equilibrium structure of the model. On substituting the values of the trial functions from (3) in (2) and using the variational principle, the characteristic equation determining the eigen frequencies of nonradial modes of oscillations of a rotationally and tidally distorted gaseous sphere can be can expressed as

$$\begin{vmatrix} \sigma^2 W_{11} - Z_{11} & \sigma^2 W_{12} - Z_{12} & \sigma^2 W_{13} - Z_{13} \\ \sigma^2 W_{12} - Z_{12} & \sigma^2 W_{22} - Z_{22} & Z_{23} \\ \sigma^2 W_{13} - Z_{13} & Z_{23} & \sigma^2 W_{33} - Z_{33} \end{vmatrix} = 0 \qquad \ldots(5)$$

where

$$W_{11} = \left\{ 1 + \frac{\lambda^2}{l(l+1)} \right\} \rho_{2\lambda+2},\ W_{12} = \rho_{2\lambda},\ W_{13} = \frac{\lambda(\lambda+2)}{l(l+1)} \rho_{\lambda 2}$$

$$W_{22} = \rho_{2\lambda+2},\ W_{33} = \frac{(\lambda+2)^2}{l(l+1)} \rho_{2\lambda+2},\ Z_{11} = 2(\lambda-2)(2\lambda-3)\rho_{2\lambda-4}$$

$$Z_{12} = (\lambda-4)(2\lambda-1)\, p_{2\lambda-2} + \frac{4\pi G}{2l+1} \lambda(2\lambda-1)\, g_{l+\lambda-1}$$

$$Z_{13} = (\lambda+2)(2\lambda-1)\, p_{2\lambda-2} - \frac{4\pi G}{2l+1} (\lambda+2)(2\lambda-1)\, g_{l+\lambda-1}$$

$$Z_{22} = \{\gamma(\lambda+2)^2 - 4(2\lambda+1)\}\, p_{2\lambda} + \frac{8\pi G}{2l+1} \lambda(\lambda-1)\, g_{l+\lambda+1}$$

$$Z_{23} = \{\gamma(\lambda+2)^2 - (\lambda+2)(2\lambda+1)\}\, p_{2\lambda} - \frac{4\pi G}{2l+1} (\lambda+2)\, g_{l+\lambda+1}$$

$$Z_{33} = \gamma(\lambda+2)^2\, p_{2\lambda} - \frac{8\pi G}{2l+1} (\lambda+2)^2\, g_{l+\lambda+1}.$$

In the above expressions

$$\rho_m = \int_0^{R_\psi} \rho_\psi \, r_\psi^m \, dr_\psi \; (m = 2\lambda - 2, 2\lambda, 2\lambda + 2)$$

$$p_m = \int_0^{R_\psi} p_\psi \, r_\psi^m \, dr_\psi \; (m = 2\lambda - 4, 2\lambda - 2, 2\lambda)$$

$$g_m = \int_0^{R_\psi} g(r_\psi) \, \rho_\psi \, r_\psi^m \, dr_\psi \; (m = l + \lambda - 1, l + \lambda + 1),$$

with

$$g(r_\psi) = \int_{r_\psi}^{R_\psi} \rho_\psi(s) \, sds.$$

Characteristic equation (5) is in general form and can be used to determine the effects of rotation and or tidal distortions on the eigen frequencies of nonradial modes of oscillations of a distorted gaseous sphere provided the values of rotational and tidal distortion parameters n and q are both small so that terms beyond second order of smallness in n and q can be neglected. In the absence of any distortion (i.e., $n = q = 0$) and setting $\lambda = l + 1$, eqn. (5) reduces to the characteristic equation obtained by Chandrasekhar and Lebovitz[10], to determine the eigen frequencies of non-radial modes of oscillations of an undistorted gaseous sphere.

Equation (5) is expressed in a form in which it can be directly used to determine the effects of rotation and or tidal distortions on the eigen frequencies of nonradial modes of oscillations of a gaseous sphere. In order to determine the effects of rotation and tidal distortions on the eigen frequencies of nonradial modes of oscillations of polytropic models, we set $\lambda = l + 1$ (a value used by Chandrasekhar and Lebovitz'[0] for undistorted polytropic models) and use the values of r_ψ, ρ_ψ and p_ψ as obtained by us in paper I for polytropic models. On using these values the nondimensional form of the characteristic equation (4) determining the eigen frequencies of nonradial modes of oscillations of a rotationally and tidally distorted polytropic model can be expressed as :

$$\begin{vmatrix} A_{11} w^2 - B_{11} & A_{12} w^2 - B_{12} & A_{13} w^2 - B_{13} \\ A_{12} w^2 - B_{12} & A_{22} w^2 - B_{22} & B_{23} \\ A_{13} w^2 - B_{13} & B_{23} & A_{33} w^2 - B_{33} \end{vmatrix} = 0 \qquad \ldots(6)$$

where

$$w^2 = \frac{D^3 r_{os}^3}{G M_0} \sigma_2$$

w being the nondimensional form of the eigen frequency σ, and

$$A_{11} = \frac{(N+1)\,\xi_u^2\,k}{3\,r_{os}^3}\left(\frac{\bar{\rho}}{\rho_c}\right)\frac{2l+1}{l}\;\rho_{2l}^*,\; B_{11} = 2\,(l-1)\,(2l-1)\;p_{2l-2}^*,$$

$$A_{12} = \frac{(N+1)\,\xi_u^2\,k}{3\,r_{os}^3}\left(\frac{\bar{\rho}}{\rho_c}\right)\;\rho_{2l+2}^*,$$

$$B_{12} = (l+1)\,(N+1)\,\frac{\xi_u^2}{k^2}\,g_{2l}^* - (3-l)\,(2l+1)\;p_{2l}^*$$

$$A_{13} = \frac{(N+1)\,\xi_u^2\,k}{3\,r_{os}^3}\left(\frac{\bar{\rho}}{\rho_c}\right)\frac{l+3}{l}\;\rho_{2l+2}^*,$$

$$B_{13} = (2l+1)\,(l+1)\;p_{2l}^* - (l+3)\,(N+1)\,\frac{\xi_u^2}{k^2}\;g_{2l}^*$$

$$A_{22} = \frac{(N+1)\,\xi_u^2\,k}{3\,r_{os}^3}\left(\frac{\bar{\rho}}{\rho_c}\right)\;\rho_{2l+4}^*,$$

$$B_{22} = 2\,(N+1)\,\frac{l\,(l+1)}{2l+1}\,\frac{\xi_u^2}{k^2}\;g_{2l+2}^* + \{\gamma\,(l+3)^2 - 4\,(2l+3)\}\;p_{2l+2}^*$$

$$B_{23} = \{(l+3)^2\,\gamma - (l+3)\,(2l+3)\}\;p_{2l+2}^* - (N+1)\,\frac{(l+3)}{2l+1}\,\frac{\xi_u^2}{k^2}\;g_{2l+2}^*,$$

$$A_{33} = \frac{(N+1)\,\xi_u^2\,k}{3\,r_{os}^3}\left(\frac{\bar{\rho}}{\rho_c}\right)\frac{(l+3)^2}{l\,(l+1)}\;\rho_{2l+4}^*$$

and

$$B_{33} = \gamma (l+3)^2 p^*_{2l+2} - 2(N+1)\frac{(l+3)^2}{2l+1}\frac{\xi_u^2}{k^2} g^*_{2l+2}.$$

In the above expressions ρ^*_m and p^*_m are same as defined in equation (3) and

$$g^*_m = r_{os}^{m+1}\int_0^1 \theta_\psi^N g^*(x) x^m \left[1 + \frac{2n}{3}(m+4)x^3 r_{os}^3 + \left(\frac{4}{5}q^2 + \frac{8}{15}nq + \frac{10m+76}{45}n^2\right)(m+7)x^6 r_{os}^6 + \frac{5}{7}q^2(m+9)x^8 r_{os}^8 + \frac{2}{3}q^2(m+11)x^{10} r_{os}^{10}\right]dx, \quad (m=2l, 2l+2)$$

where

$$g^*(x) = r_{os}^2\int_x^1 \theta_\psi^N y\left[1 + \frac{10}{3}y^3 r_{os}^3 + \left(\frac{32}{5}q^2 + \frac{64}{15}nq + \frac{688}{45}n^2\right)y^6 r_{os}^6 + \frac{50}{7}q^2 y^8 r_{os}^8 + 8q^2 y^{10} r_{os}^{10}\right]dy.$$

As mentioned earlier, in obtaining the above expressions terms upto second order of smallness in n and q have been retained.

Characteristic equation (6) can be used to determine the effects of rotation and or tidal distortions on the eigenfrequencies of nonradial modes of oscillations of polytropic models provided the distortion parameters n and q are both smaller than one. On setting $n = 0$ or $q = 0$ separately in the above characteristic equation we can investigate the effects of tidal distortions alone or the effects of rotational distortions alone on the eigen frequencies of nonradial modes of oscillations of polytropic models. Also by setting $n = (q+1)/2$ we may study the combined effects of rotation and tidal distortions on the eigen frequencies of nonradial modes of oscillations of a primary component (having its structure as that of a distorted polytropic model) of a synchronously rotating binary system.

4. Numerical Results

Characteristic equation (3) was solved numerically to determine the eigen frequencies of small adiabatic pseudo-radial modes of oscillations of rotationally and or

tidally distorted polytropic models of indices 1.5 and 3.0. Using the values of θ_ψ for the distorted polytropic models as obtained by us earlier in paper I, the integrals ρ_m^* and p_m^* were evaluated numerically using Simpson's rule. Computations have been done for different values of the distortion parameters n and q. The results are presented in Table I.

Characteristic equation (6) which is a cubic in w^2 was solved numerically using Cardon's method to determine the effects of rotation and or tidal distortions on the eigen frequencies of the nonradial modes of oscillations of polytropic models of indices 1.5 and 3.0. The values of θ_ψ needed at various points for the distorted polytropic models of indices 1.5 and 3.0 were again taken from paper I. The values of the integral ρ_m^*, p_m^* and g_m^* for each model were evaluated numerically using Simpson's rule. The computations have been done for certain values of k and distortion parameters n and q for polytropic models of indices 1.5 and 3.0. The results are presented in Table 2.

5. Concluding Observations

A comparison of the results shown in Table I with the corresponding results presented in Table I of paper II for the eigen frequencies of the fundamental and the

TABLE I

Eigen frequencies $w^2 \left(= r_{os}^3 D^3 \sigma^2 / GM_0 \right)$ *for the fundamental and the first mode of pseudo-radial oscillations of rotationally and tidally distorted polytropic models* for $\gamma = 5/3$

		$N = 1.5$		$N = 3.0$	
n	q	w_0^2	w_1^2	w_0^2	w_1^2
		Undistorted models			
0.0	0.0	2.7075	13.9954	9.7811	30.4585
		Rotationally distorted models			
0.1	0.0	2.2853	11.2211	8.5615	26.7022
		Tidally distorted models			
0.0	0.1	2.7069	13.9887	9.7801	30.4546
		Rotationally and tidally distorted models			
0.1	0.1	2.6528	13.6318	9.6292	29.9788
0.2	0.2	2.5963	13.2500	9.4738	29.4883
		Synchronous binary system			
0.55	0.1	2.4115	12.0380	8.9386	27.8365

first mode of small adiabatic pseudo-radial oscillations of rotationally and or tidally distorted polytropic models of index 1.5 shows that in general there is a good agreement between the corresponding eigen frequencies of the fundamental mode by the two approaches. The difference between the corresponding values of the squares of the eigen frequencies of the fundamental mode by the two approaches in no case exceeds 0.9%. In the case of the first mode, however, these differences are between 10 to 20%. In the case of rotationally and or tidally distorted polytropic models of index 3.0, whereas the difference between the corresponding values of the squares of the eigen frequencies of the fundamental modes as obtained by the two methods varies between 5 to 12%, there is appreciable difference between the values of the eigen frequencies of the first mode by the two approaches. One of the reasons for this could be that we have used $\lambda = 3$ as the value of λ in the trial function Φ for the polytropic models of index 3.0. Since $\lambda = 3$ is exact only for the homogeneous model and increase in the value of the polytropic index N implies increase in the central condensation, therefore by using a trial function with $\lambda = 3$ we can not expect very accurate results for the first mode of polytropic models with larger values of the polytropic index N. In fact in our earlier paper III we got very accurate results by variational approach for Roche model only by using $\lambda = 3 + \sqrt{9 - 12/\gamma}$ in place of $\lambda = 3$.

A comparison of the results shown in Table II with the corresponding results presented in Table II of paper II for the eigen frequencies of the fundamental mode of nonradial oscillations of rotationally and or tidally distorted polytropic models of indices 1.5 and 3.0 shows that the middle root of characteristic equation (5) corresponds to the fundamental mode of nonradial oscillations of these models. In general there is a good agreement between the corresponding values of the eigen frequencies of the fundamental mode obtained by the two approaches. Whereas the difference between the corresponding values of the squares of the eigen frequencies of the fundamental mode for rotationally and or tidally distorted polytropic models of index 1.5 varies from 0.4 to 7.4%, this difference in the case of distorted polytropic models of index 3.0 varies from 0.7 to 6%. The results of Table II also show that the third root of the characteristic equation for polytropic models of index 1.5 is closer to the corresponding eigen frequency of p_1 mode as obtained by us in paper II. The first root of the characteristic equation for the models of the polytrope of index 1.5 does not seem to have any physical significance as a polytropic model of index 1.5 is not expected to exhibit any g mode of nonradial oscillations. In the case of polytropic models of index 3.0, whereas the third root of the characteristics equation is closer to the corresponding square of the eigen frequency of p_2 mode as obtained by us in paper II, the first root is closer to the corresponding eigen frequency of g_3 mode.

Thus we conclude that the variational approach developed in this paper can be used to determine the eigen frequencies of the fundamental mode of small adiabatic pseudo-radial and nonradial modes of oscillations of rotationally and or tidally

TABLE II

Eigen frequencies $w^2 = \left(r_{os}^3 D^3 \sigma^2 / GM_0\right)$ *for the nonradial modes of oscillations of rotationally and tidally distorted polytropic models for* $l = 2$ *and* $\gamma = 5/3$

n	q	k	$N = 1.5$				$N = 3.0$			
			Roots of the characteristic equation			Fundamental mode	Roots of the characteristic equation			Fundamental mode
			Undistorted models							
0.00	0.00	1.0	−0.1228,	2.1020,	11.4582	2.1020	2.1852,	7.7211,	27.6076	7.7211
			Rotationally distorted models							
0.05	0.00	1.0	0.0110,	1.9428,	10.2284	1.9428	2.0794,	7.3325,	25.8266	7.3325
0.10	0.00	1.0	0.0165,	1.7525,	9.1890	1.7525	1.9675,	6.9487,	24.0758	6.9487
			Tidally distorted models							
0.00	0.10	0.5	−0.1223,	2.1014,	11.4530	2.1014	2.1850,	7.7206,	27.6042	7.7206
0.00	0.20	0.5	−0.1212,	2.0997,	11.4379	2.0997	2.1843,	7.7195,	27.5942	7.7195
			Rotationally and tidally distorted models							
0.10	0.10	0.5	−0.1039,	2.0661,	11.2043	2.0661	2.1592,	7.6235,	27.1582	7.6235
0.20	0.20	0.5	−0.0834,	2.0132,	10.8508	2.0132	2.1320,	7.5247,	26.6991	7.5247
			Synchronous binary system							
0.55	0.10	0.5	−0.0225,	1.8687,	9.8593	1.8687	2.0368,	7.1846,	25.1469	7.1846

distorted polytropic models. However for better accuracies in the values of eigen frequencies of other modes it is suggested that in place of $\lambda = 3$ a more appropriate value of λ be used. (This appropriate value of λ is one for which the eigen frequency of the fundamental mode of the undistorted model determined by the variational approach agrees to a reasonable accuracy with the eigen value of this mode obtained by the numerical solution approach). In our previous paper III it was found that for the Roche model $\lambda = 3 + \alpha$ where $\alpha = \sqrt{9 - 12/\gamma}$ was more appropriate value of λ in place of $\lambda = 3$. For polytropic models this value of λ is expected to vary from 3 to 4 as polytropic index N increase from 0 to 4.5 for the case when γ is taken as 5/3.

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1. P. 1267 line 15 should read "as well as for $n = 1$ for the"

2. P. 1271 line 14 should read "Then $H^n_\phi(\mathrm{g}, v) = 0$, in case (i) for all $n > 0$, in case (2) for $n = 1$."

3. P. 1271 (i) line 22 should read "$a = \sum_{i=1}^{n+1} a(X_i)$. Then $H^n_\phi(\mathrm{g}, V) = 0 \ \forall \ n>0$."

   (ii) Line 25 should read "$H^n_\phi(\mathrm{g}, V) = 0 \quad \forall \ n > 0$."

4. P. 1272 1st line should read "(3.2) also holds for $n = 1$ for the trivial representation".

5. P. 1278 line 21 should read "(1a) $[q]_+ \ \mathbf{\Theta} \ [q - \frac{1}{2}]_+$."

6. (i) P. 1283 line 4 should read "For the representation $[q] \ \mathbf{\Theta} \ [q - \frac{1}{2}]$"

   (ii) P. 1283 line 7 should read "(1b) $V = [q - \frac{1}{2}]_+ \ \mathbf{\Theta} \ [q]_+$"

7. P. 1294 line 1 Ref. 4 should read "Bani Mitra, and K. C. Tripathy, Cohomology and reducibility of finite dimensional representations of semisimple Γ-graded Lie Algebras."

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(See Rules 8)

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