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HYDROMAGNETIC LAMINAR FLOW OF AN ELASTICO-VISCOUS FLUID PAST AN INFINITE FLAT PLATE

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The effect of a transverse magnetic field on the unsteady laminar flow of an elastico-viscous fluid (Walters liquid B') past an infinite non-conducting and non-magnetic horizontal plate started impulsively has been discussed. Solution to equations governing the flow are derived with the help of Laplace transform technique. The results are discussed with the aid of graphs and tables.

1. INTRODUCTION

Magnetic hydrodynamics has been developed because of the astrophysical phenomena arising out of the discovery of existence of large magnetic fields in some stars and the possibilities of magneto-hydrodynamic generators as a new source of energy. On the other hand non-Newtonian fluids are of increasing importance in modern technology in the case of certain polymers. Different constitutive equations have been proposed for these fluids by different inventors. A system of constitutive equations have been proposed by Walters[4] for elastic-viscous fluids and one of them is known as Walters liquid B'. The flow of viscous incompressible fluid past an impulsively started infinite horizontal plate in its own was first studied by Stokes[3]. Soundalgekar[2] studied the Stokes problem for impulsively started horizontal plate for non-conducting Walters liquid B'. The object of this paper is to study the unsteady flow of an electrically conducting Walters liquid B' past an infinite non-conducting and non-magnetic horizontal flat plate started impulsively from rest in its own plane, in the presence of a transverse magnetic field.

2. FORMULATION OF THE PROBLEM

We take the coordinate origin O at any arbitrary point on thepl ate (which is assumed to be electrically non-conducting) with the axis Ox' and Oy' along and perpendicular to the plate respectively and assumed that,

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(1) All physical quantities are functions of the space coordinate y and time t only.

(2) The velocity field is one dimensional that is $\vec{V} = \vec{V}(u, 0, 0)$.

(3) The applied magnetic field is uniform and perpendicular to the plate, so that in the region considered $\vec{H} = \vec{H}(Hx, Hy, O)$.

(4) The electrical conductivity of the fluid is finite, which is realistic assumption as many of the polymer solutions are finitely electrically conducting.

At time $t' > 0$ the plate starts moving impulsively in its own plane with a velocity U_0. Hence the flow field is governed by the equations

$$\frac{\partial u'}{\partial t'} = \nu \frac{\partial^2 u'}{\partial y'^2} + \frac{\mu_0}{\rho} H_p \frac{\partial H_x}{\partial y} - K' \frac{\partial^3 u'}{\partial y'^2 \partial t'} \qquad \ldots(1)$$

$$\frac{\partial H_x}{\partial t'} = H_y \frac{\partial u'}{\partial y'} + \frac{1}{\sigma\mu_0} \frac{\partial^2 H_x}{\partial y'^2} \qquad \ldots(2)$$

where t' is the the time, ν the kinematic coefficient of viscosity, K' the elastic parameter, σ the conductivity of the fluid, ρ the density of the medium and μ_0 the magnetic permeability.

The initial and the boundary conditions for the velocity field are exactly the same as those in ordinary hydrodynamic case, namely,

$$\left.\begin{aligned} t' \leqslant 0 &: u' = 0 \text{ for all } y' \\ t' > 0 &: u' = U_0 \text{ at } y' = 0 \\ &\quad = 0 \text{ as } y' \to \infty. \end{aligned}\right\} \qquad \ldots(3)$$

Now we shall consider the boundary conditions for the magnetic field. Since we have assumed that the plate is electrically non-conducting, no electric current flows in the plate. Therefore, the magnetic field in the plate should be curlfree, that is curl $\vec{H} = 0$. Thus we have

$$-\frac{dH_x}{dy} = 0 \text{ or } H_x = \text{constant}.$$

This constant must be taken to be zero, since the applied magnetic field is normal to the plate. Also we have

$$\text{div } \vec{H} = 0$$

which gives

$$\frac{dH_y}{dy} = 0 \text{ or } H_y = \text{constant on the plate.}$$

This constant must be identical with the intensity of the applied magnetic field. Therefore, we have $H_x = 0$ and $H_y = H_0$ in the plate, where H_0 is the intensity of the applied magnetic field.

At far distance from the plate, the magnetic field tends to the applied transverse field, since the x'-wise velocity component u' tends to the constant value U_0. Thus we have the boundary conditions for the magnetic field,

$$\left.\begin{aligned} &H_x(0, t) = 0, \; H_y(0, t) = H_0 \\ &H_x(\infty, t) = 0, \; H_y(\infty, t) = H_0. \end{aligned}\right\} \qquad \ldots(4)$$

If we now introduce the dimensionless quantities,

$$y = \frac{U_0 y'}{\nu}, \; t = \frac{U_0^2 t'}{4\nu}, \; u = \frac{u'}{U_0}$$

$$H = \left(\frac{\mu_0}{\rho}\right)^{\frac{1}{2}} \frac{H_x}{U_0}, \; P_m = \nu\sigma\mu_0, \; M = \left(\frac{\mu_0}{\rho}\right)^{\frac{1}{2}} \frac{H_0}{U_0}$$

$$K = \frac{K' U_0}{4\nu^2} \qquad \ldots(5)$$

in eqns (1) and (2), we have

$$\frac{\partial^2 u}{\partial y^2} - \frac{1}{4}\frac{\partial u}{\partial t} = K \frac{\partial^3 u}{\partial y^2 \partial t} - M \frac{\partial H}{\partial y} \qquad \ldots(6)$$

$$\frac{1}{P_m}\frac{\partial^2 H}{\partial y^2} - \frac{1}{4}\frac{\partial H}{\partial t} = -M \frac{\partial u}{\partial y} \qquad \ldots(7)$$

where M is the magnetic field parameter and P_m the magnetic Prandtl number. Also the boundary conditions (3) and (4) in non-dimensional form

$$\left.\begin{aligned} &t \leqslant 0, \; u = 0, \; H = 0 \text{ for all } y \\ &t > 0, \; u = 1 \; H = 0 \text{ at } y = 0 \\ &\qquad = 0, \; = 0 \text{ as } y \to \infty. \end{aligned}\right\} \qquad \ldots(8)$$

Equation (6) shows that the order of the differential equation for elastico-viscous fluid ($K \neq 0$) is three, whereas that for the Newtonian case ($K = 0$) is only two. Therefore for a unique solution, three boundary conditions are necessary. Therefore to overcome this difficulty following Beard and Walters[1] we assume that,

$$u = u_0 + Ku_1 \text{ and } H = H_0 + K H_1 \qquad \ldots(9)$$

which is valid for small values K ($K \ll 1$).

Substituting (9) in (6) and (7) equating the coefficients of different powers of K and neglecting the powers of K^2, we get

$$\left.\begin{aligned}
&\frac{\partial^2 u_0}{\partial y^2} + M\frac{\partial H_0}{\partial y} - \frac{1}{4}\frac{\partial u_0}{\partial t} = 0,\\
&\frac{\partial^2 H_0}{\partial y^2} + M\frac{\partial u_0}{\partial y} - \frac{1}{4}\frac{\partial H_0}{\partial t} = 0,\\
&\frac{\partial^2 u_1}{\partial y^2} + M\frac{\partial H_1}{\partial y} - \frac{1}{4}\frac{\partial u_1}{\partial t} = \frac{\partial^2 u_0}{\partial y^2 \partial t},\\
&\frac{\partial^2 H_1}{\partial y^2} + M\frac{\partial u_1}{\partial y} - \frac{1}{4}\frac{\partial H_1}{\partial t} = 0.
\end{aligned}\right\} \qquad \ldots(10)$$

The boundary conditions (8) become to

$$\begin{aligned}
&u_0 = 1,\ H_0 = 0,\ u_1 = 0,\ H_1 = 0 \text{ at } y = 0\\
&\quad = 0, \quad = 0, \quad = 0, \quad = 0 \text{ as } y \to \infty.
\end{aligned} \qquad \ldots(11)$$

The solution of the set of eqns. (10) satisfying boundary conditions (11) obtained with the help of Laplace transform technique is

$$u_0 = \frac{1}{4}\left(e^{-My/2} + e^{My/2}\right)\left[e^{mY/2}\operatorname{erfc}\left(\frac{y}{4\sqrt{t}} + M\sqrt{t}\right)\right.$$
$$\left. + e^{-My/2}\operatorname{erfc}\left(\frac{y}{4\sqrt{t}} - M\sqrt{t}\right)\right]$$

$$H_0 = \frac{1}{4}\left(e^{-My/2} - e^{My/2}\right)\left[e^{My/2}\operatorname{erfc}\left(\frac{y}{4\sqrt{t}} + M\sqrt{t}\right)\right.$$
$$\left. + e^{-My/2}\operatorname{erfc}\left(\frac{y}{4\sqrt{t}} - M\sqrt{t}\right)\right]$$

$$u_1 = \frac{\exp\left(-\left(\frac{y^2}{16t} + M^2 t\right)\right)}{128\sqrt{\pi}t^{5/2}}\left[\frac{8yt(1 - 2M^2 t) - y^3}{2}\right.$$

(equation continued on p. 78)

$$(e^{My/2} + e^{-My/2})$$

$$+ \frac{(4M^2 t - 1)\, y^2 + 8t}{M} \left(e^{My/2} - e^{-My/2}\right)\Big]$$

$$H_1 = \frac{y \exp\left(-\left(\frac{y^2}{16t} + M^2 t\right)\right)}{32 t^{3/2}\sqrt{\pi}} \Big[\frac{y^2 + 16M^2 t^2}{8t}\left(e^{My/2} - e^{-My/2}\right)$$

$$- My\left(e^{-My/2} + e^{My/2}\right)\Big]$$

and

$$u = u_0 + Ku_1, \quad H = H_0 + KH_1.$$

Knowing the velocity field we now calculate the shearing stress at the plate neglecting the coefficient of K^2, is given by

$$\tau = -\left[\frac{\partial u}{\partial y} - K\frac{\partial^2 u}{\partial y\, \partial t}\right]_{y=0}$$

$$= \frac{1}{2}\left[M \operatorname{erf}(M\sqrt{t}) + \frac{e^{-M^2 t}}{\sqrt{\pi t}}\right] + \frac{K(1 + M^2 t)\, e^{-M^2 t}}{8\sqrt{\pi} t^{3/2}}.$$

Finally the expression for the electric current density is given by

$$Z = -\left(\frac{\partial H}{\partial y}\right)$$

$$= \frac{M}{4}\Big[\exp(My)\operatorname{erfc}\left(\frac{y}{4\sqrt{t}} + M\sqrt{t}\right) + \exp(-My)$$

$$\times \operatorname{erfc}\left(\frac{y}{4\sqrt{t}} - M\sqrt{t}\right)\Big] + \left[\frac{\exp\left(-\frac{My}{2}\right) - \exp\left(\frac{My}{2}\right)}{4\sqrt{\pi t}}\right]$$

$$\times \exp\left(-\left(\frac{y^2}{16t} + M^2 t\right)\right) + \frac{K \exp\left(-\left(\frac{y^2}{16t} + M^2 t\right)\right)}{32\sqrt{\pi} t^{3/2}}$$

(equation continued on p. 79)

$$\times \left\{\left[\exp\left(-\frac{My}{2}\right) - \exp\left(\frac{My}{2}\right)\right]\right.$$

$$\times \left[\left(1 - \frac{y^2}{8t}\right)\left(\frac{y^2 + 16M^2 t^2}{8t}\right) + \frac{y^2}{2}\left(\frac{1}{2t} - M^2\right)\right]$$

$$+ My\left[\exp\left(\frac{My}{2}\right) - \exp\left(-\frac{My}{2}\right)\right]$$

$$\left.\left(2 - M^2 t - \frac{3y^2}{16t}\right)\right\}.$$

3. Conclusions

In this paper we have studied the unsteady hydromagnetic flow of an electrically conducting Walters liquid B′ past an infinite horizontal flat plate started impulsively from rest in its own plane and exact solutions have been obtained for the velocity, the induced magnetic field, the shearing stress and the electric current density, when the magnetic Prandtl number $P_m = 1$.

In order to study the influence of various parameters on the non-dimensional velocity, we have shown velocity profiles in Fig. 1. It is seen from figure that the fluid velocity decreases as the magnetic field parameter M increases, whereas it increases in time t. It is also seen that the velocity increases with the increase in elastic parameter K, for a fixed value of M and t.

The variation of the induced magnetic field H are shown in Fig. 2. The induced magnetic field H have negative values for all values of M, t and K. This means that there is a reverse type of the magnetic field. From figure it is observed that for a fixed values of K, the induced magnetic field decreases with the increase in M and t. It is also seen that the induced magnetic field decreases with the increase in elastic parameter K.

The numerical values of the electric current density Z are given in Table I. From table it is observed that for fixed values of t, the current density Z increases with the increase in elastic parameter K, and the magnetic field parameter M. But for a fixed values of M and K, it decreases with the increase in time t.

The variation of shearing stress τ is shown in Fig. 3. From figure it is observed that for a fixed values of t, the shearing stress at the plate increases with the increasing values of M and K. But for a fixed values of M and K the shearing stress decreases as the time advances.

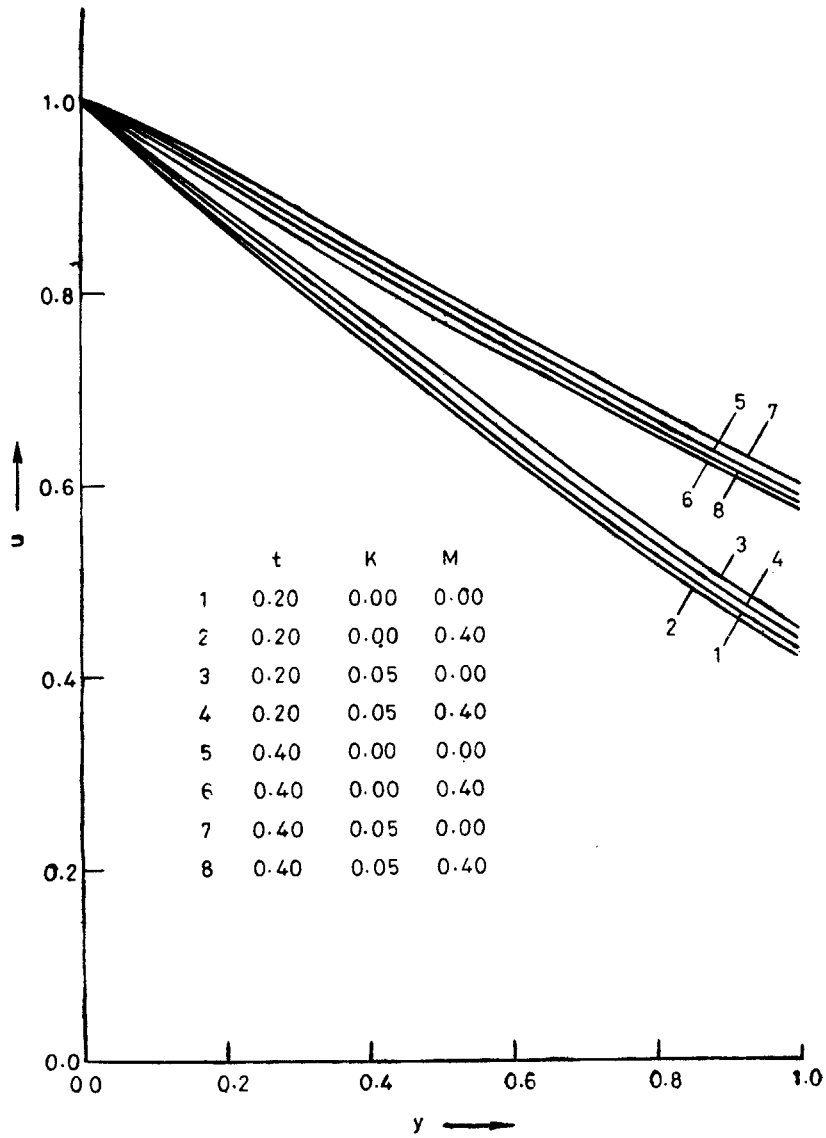

FIG. 1. Variation of the velocity profile (u).

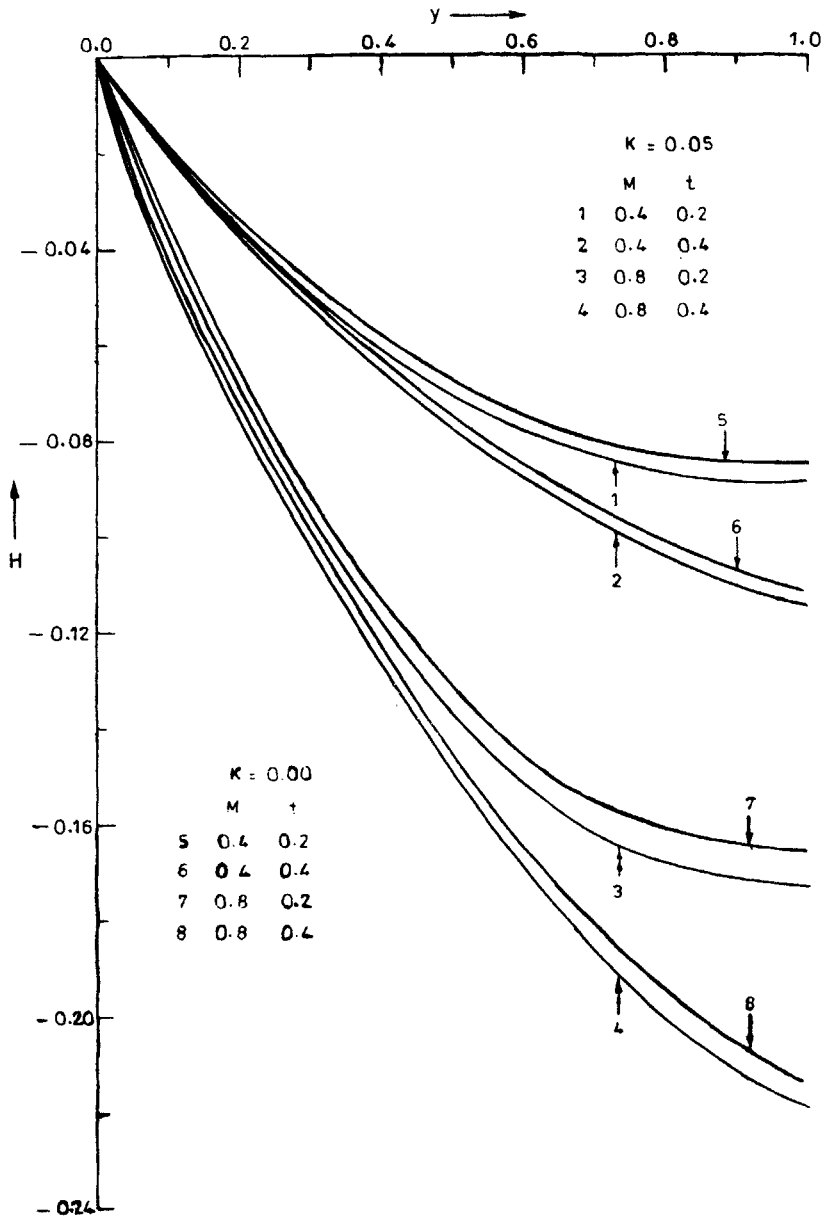

FIG. 2. Variation of the induced magnetic field.

TABLE I

Variation of the electric current density

	$t = 0.2$						$t - 0.4$					
y	$M = 0.4$			$M = 0.8$			M 0.4			$M = 0.8$		
	K= 0.00	0.05	0.10	$K = 0.00$	0.05	0.10	$K = 0.00$	0.05	0.10	$K = 0.00$	0.05	0.10
0.0	0.2000	0.2000	0.2000	0.4000	0.4000	0.4000	0.2000	0.2000	0.2000	0.4000	0.4000	0.4000
0.2	0.1972	0.1999	0.2027	0.3796	0.3838	0.3881	0.1961	0.1970	0.1979	0.3711	0.3721	0.3731
0.4	0.1943	0.1988	0.2034	0.3651	0.3721	0.3791	0.1928	0.1945	0.1962	0.3499	0.3538	0.3536
0.6	0.1899	0.1947	0.1995	0.3527	0.3600	0.3672	0.1897	0.1919	0.1940	0.3343	0.3367	0.3390
0.8	0.1833	0.1865	0.1896	0.3394	0.3439	0.3483	0.1862	9.1885	0.1907	0.3224	0.3247	0.3271
1.0	0.1736	0.1738	0.1739	0.3209	0.3221	0.3232	0.1820	0.1839	0.1858	0.3125	0.3143	0.3161

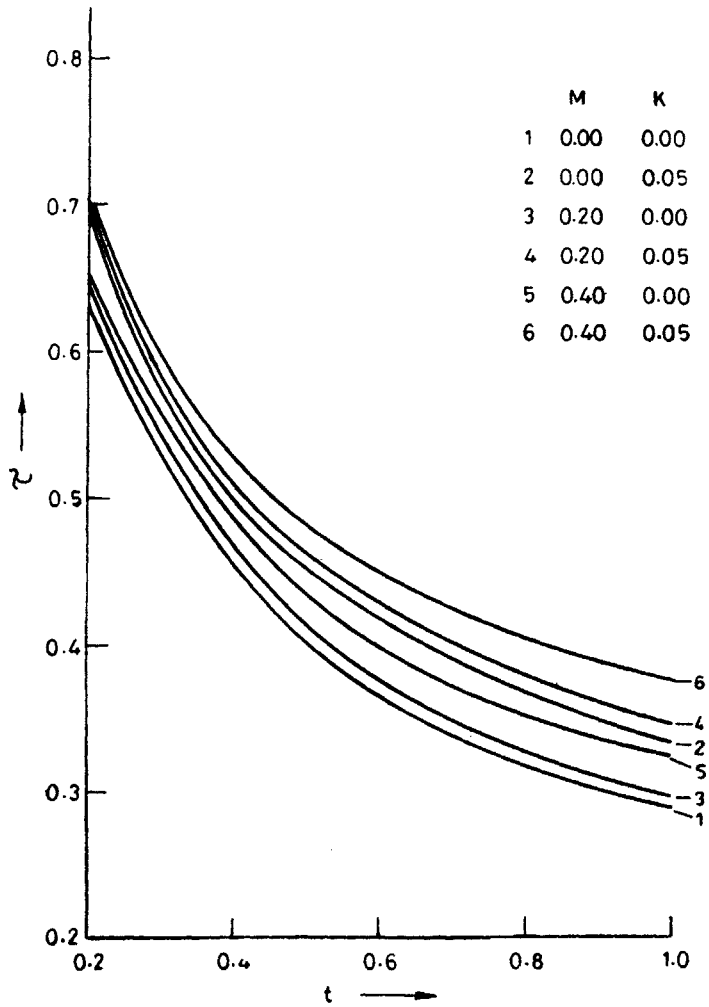

FIG. 3. Variation of the shearing stress (τ).

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