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GENERATION OF *G*-TYPE SEISMIC WAVES UNDER INITIAL STRESS

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The propagation of shear waves along the plane surface between two different media under initial stress is discussed. The upper layer is homogeneous while the lower medium is inhomgeneous. It is shown that a large portion of energy can flow along the interface for horizontally polarized shear waves with group velocity lower than the shear wave velocity in the upper mantle. The calculations carried out are valid up to first order of small quantities. The phase velocity curve against wave number have been drawn. The effects of initial stresses are distinctly marked.

INTRODUCTION

The existance of a low-velocity layer in the earth mantle has been established by Gutenberg[1,2]. Lehmann[3] pointed out that the European as well as the north eastern American observations of S waves at small epicentral distances indicate the presence of a low-velocity layer. Bäth[4] studied the shadow zones, travel times and energies of longitudinal seismic waves in the presence of an asthenosphere low-velocity layer. We have represented the low velocity layer by assuming the incremental shear moduli Q_1' and Q_2' in the lower semi-infinite anisotropic medium in the form

$$Q_1' = M(1 - \epsilon \sin sy)$$

$$Q_2' = N(1 - \epsilon_1 \sin sy)$$

where ϵ, ϵ_1, M and N are constants. With this law of variation the equation of motion reduce to Hill's equation with periodic coefficients which has been solved by the method given by Valeev[5]. He considered a certain class of system of linear differential equations with periodic co-efficients which have the property that, by means of Laplace transformation, they may be converted to a system of linear difference equations. Which in turn may be solved by the method of infinite determinants. This

method of solving Hill's differential equation has been successfully employed by Bhattacharya[6], Mal[7] and Chattopadhyay[8,9]. Keeping terms upto first order in ϵ, the Laplace transformation $F(p)$ of the displacement $w_2(y)$ was obtained. It has been established that a large portion of seismic energy is confined near the interface when SH waves propagate with group velocity U satisfying the relation

$$U = \beta_2 \frac{\sqrt{2}\, kH\left\{\left(\frac{N}{M}+\frac{S'_{11}}{2M}\right)-\frac{\epsilon_1}{\epsilon}\frac{N}{M}\left(1+\frac{S'_{22}}{2M}\right)\right\}}{\left[3k^2H^2\left\{\left(\frac{N}{M}+\frac{S'_{11}}{2M}\right)-\frac{\epsilon_1}{\epsilon}\frac{N}{M}\left(1+\frac{S'_{22}}{2M}\right)\right\}+\left(1+\frac{S'_{22}}{2M}\right)S^2H^2\right]^{\frac{1}{2}}}$$

where β_2 is the shear wave velocity in the upper mantle.

The nature of this type of waves may be compared with that of G-waves. G-waves are horizontally polarized surface waves of shear type, which propagate with a group velocity of about 4.4km/s (ref. 10-12). Their periods lie in the range from about 50 to 350 S, corresponding to a flat protion of the group velocity curve for Love waves, which has been extended up to periods of around 700 S (ref. 13). As a consequence of their small dispersion, G-waves have a transient pulse-like character in the records and are followed by a train of dispersed Love waves, especially for continental paths. An outstanding case of well developed G-type waves was provided by the earthquake of January 1960, in Peru. Bath and Arroyo[14] presented the results obtained from this earthquake, especially with regard to absorption and velocity dispersion of G-waves.

SOLUTION OF THE PROBLEM

Let us assume that the layer is of uniform thickness H. The origin is taken on the interface, z-axis horizontal and y-axis vertically downwards. The lower semi-infinite medium is taken to be initially stressed anisotropic and inhomogeneous with the variations of incremental shear moduli as

$$\begin{aligned} Q'_1 &= M(1-\epsilon \sin sy) \\ Q'_2 &= N(1-\epsilon_1 \sin sy) \end{aligned} \qquad \ldots(1)$$

where ϵ and ϵ_1 is small and M, N are constants. The upper semi-infinite medium is taken to anisotropic homogeneous.

We consider the propagation of horizontal polarized surface waves of shear type. So that the displacement components

$u = v = 0$ and $w = w(x, y, t)$.

In the upper medium the displacement $w(x, y, t)$ satisfy the differential equation

$$\left(Q_1 + \frac{S_{22}}{2}\right)\frac{\partial^2 w}{\partial y^2} + \left(Q_2 + \frac{S_{11}}{2}\right)\frac{\partial^2 w}{\partial x^2} = \rho\,\frac{\partial^2 w}{\partial t^2} \qquad \ldots(2)$$

where S_{11} and S_{22} be the initial stress components acting in x and y-direction respectively. The stress-strain relations given by Biot[15] are

$$S_{11} = 2Q_1\, e_{11}$$

$$S_{22} = 2Q_2\, e_{22}$$

where the elastic moduli Q_1 and Q_2 are incremental shear moduli. If k is the wave-number and C is the velocity than w, can be written as

$$w_1(x, y, t) = w_1(y)\, \exp(ik(x - Ct)). \qquad \ldots(3)$$

Substiuting (3) into (2) we have

$$\frac{d^2 w_1}{dy^2} + T_1^2\, w_1 = 0 \qquad \ldots(4)$$

where,

$$T_1^2 = \frac{k^2}{\left(Q_1 + \frac{S_{22}}{2}\right)}\left\{C^2\rho - \left(\frac{S_{11}}{2} + Q_2\right)\right\}. \qquad \ldots(5)$$

The solution of eqn. (4) is

$$w_1 = A \cos T_1 y + B \sin T_1 y$$

where A and B are constants. As the upper surface is stress free,

so that

$$\Delta f_3 = 0 \text{ at } y = -H. \qquad \ldots(6)$$

Here

$$\Delta f_3 = \left(Q_1 + \tfrac{1}{2} S_{22}\right)\frac{\partial w_1}{\partial y}$$

which gives

$$AT_1 \sin T_1 H + BT_1 \cos T_1 H = 0$$

or,

$$\frac{A}{\cos T_1 H} = -\frac{B}{\sin T_1 H} = w_0.$$

Then the solution for w_1 takes the form

$$w_1 = w_0 \cos (H + y)\, T_1. \qquad ...(7)$$

The lower medium is anisotropic inhomogeneous. Let the displacement $w_2(x, y, t)$ satisfy the differential equation

$$\frac{\partial}{\partial y}\left\{\left(Q_1' + \frac{S_{22}'}{2}\right)\frac{\partial w_2}{\partial y}\right\} + \frac{\partial}{\partial x}\left\{\left(Q_2' + \frac{S_{11}'}{2}\right)\frac{\partial w_2}{\partial x}\right\} = \rho_1 \frac{\partial^2 w_2}{\partial t^2}. \qquad ...(8)$$

Writing

$$w_2(x, y, t) = w_2(y) \exp\,(ik(x - Ct)) \qquad ...(9)$$

and using value of Q_1' and Q_2' from eqn. (1) we have from eqn. (8)

$$e^{isy}\left\{-\frac{\epsilon}{2i}\frac{d^2w_2}{dy^2} - \frac{\epsilon s}{2}\frac{dw_2}{dy} + \frac{NK^2\epsilon_1}{2iM}w_2\right\} + \left(1 + \frac{S_{22}'}{2M}\right)\frac{d^2w_2}{dy^2} + \frac{NK^2}{M}\left\{\frac{\rho_1 C^2}{N} - 1 - \frac{S_{11}'}{2N}\right\}w_2 + e^{-isy}\left\{\frac{\epsilon}{2i}\frac{d^2w_2}{dy^2} - \frac{\epsilon s}{2}\frac{dw_2}{dy} - \frac{Nk^2\epsilon_1}{2iM}\right\}w_2 = 0. \qquad ...(10)$$

This is Hill's differential equation. Which will be solved by the method given by Valeev[5]. We apply Laplace transformation with respect to y i. e. we multiply eqn. (10) by e^{-py} and integrate with respect to y from 0 to ∞. Then we get

$$\int_0^\infty e^{-(p-is)y}\left\{-\frac{\epsilon}{2i}\frac{d^2w_2}{dy} - \frac{\epsilon s}{2}\frac{dw_2}{dy} + \frac{NK^2\epsilon_1}{2iM}w_2\right\}dy$$

(equation continued on p. 1046)

$$+\int_0^\infty \left[\left(1+\frac{S'_{22}}{2M}\right)\frac{d^2w_2}{dy^2}+\frac{k^2N}{M}\left\{\frac{\rho_1 C^2}{N}-1-\frac{S'_{11}}{2N}\right\}w_2\right]e^{-py}dy$$

$$+\int_0^\infty e^{-(p+is)y}\left\{\frac{\epsilon}{2i}\frac{d^2w_2}{dy^2}-\frac{\epsilon s}{2}\frac{dw_2}{dy}-\frac{NK^2\epsilon_1}{2iM}w_2\right\}dy=0. \qquad ...(11)$$

Now the boundary conditions are

$$w_1 = w_2 \text{ at } y = 0 \qquad ...(12)$$

and

$$\Delta f_2 = \Delta f'_2 \quad \text{at } y = 0.$$

Using (3), (9) and boundary condition (12), we have

$$w_2(0) = w_0 \cos HT_1$$

$$q(0) = -\frac{A}{B}T_1 w_0 \sin HT_1 \qquad ...(13)$$

where

$$q(0) = \left[\frac{dw_2}{dy}\right]_{y=0} \qquad ...(14)$$

$$A = Q_1 \qquad ...(15)$$

and

$$B = M\left(1+\frac{S'_{22}}{2M}\right). \qquad ...(16)$$

Denoting the Laplace transformation $w_2(y)$ by $F(p)$

$$F(p) = \int_0^\infty e^{-py} w_2(y)\, dy \qquad ...(17)$$

and using the boundary condition (13) we obtain the following system of difference equation for $F(p)$:

$$\left[-\frac{\epsilon}{2i}(p-is)^2-\frac{\epsilon s}{2}(p-is)+L\right]F(p-is)$$

(*equation continued on p.* 1047)

$$+\left[\left(1+\frac{S'_{22}}{2M}\right)p^2 - D^2\right]F(p) + \left[\frac{\epsilon}{2i}(p+is)^2 - \frac{\epsilon s}{2}\right.$$

$$\left.\times (p+is) - L\right]\ F(p+is) = pA_1 + A_2 \qquad ...(18)$$

where

$$A_1 = \left(1+\frac{S'_{22}}{2M}\right)w_2(0) \qquad ...(19)$$

$$A_2 = \left(L + \frac{S'_{22}}{2M}\right)q(0) \qquad ...(20)$$

$$L = \frac{NK^2\epsilon_1}{2iM} \qquad ...(21)$$

$$D^2 = \frac{k^2N}{M}\left\{\left(1+\frac{S'_{11}}{2N}\right) - \frac{\rho_1 C^2}{M}\right\}. \qquad ...(22)$$

To find the value of $F(p)$ from above eqn. we multiply throughout by $(ism)^{-n}$ $(m \neq 0)$ and replace p by $(p + ism)$. Then we obtain the following infinite system of linear algebraic equations in the quantities $F(p + ism)$, $(m = 0, \pm 1, \pm 2 \ldots)$

$$(ism)^{-n}\left[-\frac{\epsilon}{2i}\{p+is(m-1)\}^2 - \frac{\epsilon s}{2}\{p+is(m-1)\} + L\right]$$

$$\times\ F\{p+is(m-1)\} + (ism)^{-n}\left[\left(1+\frac{S'_{22}}{2M}\right)(p+ism)^2 - D^2\right]$$

$$\times F(p+ism) + (ism)^{-n}\left[\frac{\epsilon}{2i}\{p+is(m+1)\}^2\right.$$

$$\left. - \frac{\epsilon s}{2}\{p+is(m+1)\} - L\right]F\{p+is(m+1)\}$$

$$= \{(p+ism)\ A_1 + A_2\}(ism)^{-n} \qquad ...(23)$$

where p is considered as a parameter in the coefficient. It should be noted that in order not to have to consider the special case $m = 0$. When $m = 0$, $(ism)^{-n} = 1$. Solving the system of difference equations (23) we obtain $F(p)$ as the ratio of two infinite determinants, viz.,

$$F(p) = \frac{\Delta_1}{\Delta_2}. \qquad ...(24)$$

where,

$$\Delta_1 = \begin{vmatrix} \cdots & \cdots & \cdots \\ \vdots\ (is)^{-n}\left\{\left(1+\frac{S'_{22}}{2M}\right)(p+is)^2 - D^2\right\} & (is)^{-n}\{A_1(p+is)+A_2\} & 0 \quad \vdots \\ \vdots\ \left\{\frac{\epsilon}{2i}(p+is)^2 - \frac{\epsilon s}{2}(p+is) - L\right\} & \{A_1 p + A_2\} & \left\{-\frac{\epsilon}{2i}(p-is)^2 - \frac{\epsilon s}{2}(p-is) + L\right\} \vdots \\ \vdots\ 0 & (-is)^{-n}\{A_1(p-is)+A_2\} & (-is)^{-n}\left\{\left(1+\frac{S'_{22}}{2M}\right)(p-is)^2 - D^2\right\} \vdots \\ \cdots & \cdots & \cdots \end{vmatrix}$$

and,

$$\Delta_2 = \begin{vmatrix} \vdots\ (is)^{-n}\left\{\left(1+\frac{S'_{22}}{2M}\right)(p+is)^2 - D^2\right\} & \left\{-\frac{\epsilon}{2i}p^2 - \frac{\epsilon s}{2}p + L\right\}(is)^{-n} & 0 \\ \vdots\ \left\{\frac{\epsilon}{2i}(p+is)^2 - \frac{\epsilon s}{2}(p+is) - L\right\} & \left\{\left(1+\frac{S'_{22}}{2M}\right)p^2 - D^2\right\} & \left\{-\frac{\epsilon}{2i}(p-is)^2 - \frac{\epsilon s}{2}(p-is) + L\right\} \vdots \\ \vdots\ 0 & (-is)^{-n}\left\{\frac{\epsilon}{2i}p^2 - \frac{\epsilon s}{2}p - L\right\} & (-is)^{-n}\left\{\left(1-\frac{S'_{22}}{2M}\right)(p-is)^2 - D^2\right\} \vdots \\ \cdots & \cdots & \cdots \end{vmatrix}$$

The first approximation of eqn. (24) is $F(p) = \dfrac{A_1 p + A_2}{\left(1+\dfrac{S'_{22}}{2M}\right)p^2 - D^2}$ ...(25)

The second approximation of eqn. (24) is $F(p) = \dfrac{\Delta_3}{\Delta_4}$ ...(26)

where,

$$\Delta_3 = \begin{vmatrix} (is)^{-n}\left\{\left(1+\frac{S'_{22}}{2M}\right)(p+is)^2 - D^2\right\} & (is)^{-n}\left\{A_1(p+is)+A_2\right\} & 0 \\ \left\{\frac{\epsilon}{2i}(p+is)^2 - \frac{\epsilon s}{2}(p+is) - L\right\} & (A_1 p + A_2) & \left\{-\frac{\epsilon}{2i}(p-is)^2 - \frac{\epsilon s}{2}(p-is) + L\right\} \\ 0 & (-is)^{-n}\{A_1(p-is)+A_2\} & (-is)^{-n}\left\{\left(1+\frac{S'_{22}}{2M}\right)(p-is)^2 - D^2\right\} \end{vmatrix}$$

and

$$\Delta_4 = \begin{vmatrix} (is)^{-n}\left\{\left(1+\frac{S'_{22}}{2M}\right)(p+is)^2 - D^2\right\} & (is)^{-n}\left\{-\frac{\epsilon}{2i}p^2 - \frac{\epsilon s}{2}p + L\right\} & 0 \\ \left\{\frac{\epsilon}{2i}(p+is)^2 - \frac{\epsilon s}{2}(p+is) - L\right\} & \left\{\left(1+\frac{S'_{22}}{2M}\right)p^2 - D^2\right\} & \left\{-\frac{\epsilon}{2i}(p-is)^2 - \frac{\epsilon s}{2}(p-is) + L\right\} \\ 0 & (-is)^{-n}\left\{\frac{\epsilon}{2i}p^2 - \frac{\epsilon s}{2}p - L\right\} & (-is)^{-n}\left\{\left(1+\frac{S'_{22}}{2M}\right)(p-is)^2 - D^2\right\} \end{vmatrix}$$

Neglecting the terms containing ϵ^2 and higher powers, we obtain

$$\Delta_3 S^{2n} = (A_1 p + A_2)\left\{\left(1+\frac{S'_{22}}{2M}\right)(p+is)^2 - D^2\right\}\left\{\left(1+\frac{S'_{22}}{2M}\right)(p-is)^2 - D^2\right\} + \{A_1(p-is)+A_2\}$$
$$\times\left\{\left(1+\frac{S'_{22}}{2M}\right)(p+is)^2 - D^2\right\}\left\{\frac{\epsilon}{2i}(p-is)^2 + \frac{\epsilon s}{2}(p-is) - L\right\} + \left\{\left\{\left(1+\frac{S'_{22}}{2M}\right)(p-is)^2 - D^2\right\}\{A_1(p+is)+A_2\}\left\{\frac{\epsilon}{2i}(p+is)^2 - \frac{\epsilon}{2i}(p+is) - L\right\}\right. \quad \ldots(27)$$

and

$$\Delta_4 S^{2n} = \left\{\left(1+\frac{S'_{22}}{2M}\right)(p+is)^2 - D^2\right\}\left\{\left(1+\frac{S'_{22}}{2M}\right)p^2 - D^2\right\}\left\{\left(1+\frac{S'_{22}}{2M}\right)(p-is)^2 - D^2\right\} \quad \ldots(28)$$

Hence from eqn. (26)

$$F(p) = \frac{A_1 p + A_2}{\left\{\left(1 + \frac{S'^{22}}{2M}\right) p^2 - D^2\right\}} + \frac{\left\{A_1 (p - is) + A_2 \frac{\epsilon}{2i}(p - is)^2 + \frac{\epsilon s}{2}(p - is) - L\right\}}{\left\{\left(1 + \frac{S'_{22}}{2M}\right) p^2 - D^2\right\} \left\{\left(1 + \frac{S'_{22}}{2M}\right)(p - is)^2 - D^2\right\}} + \frac{\left\{A_1 (p + is) + A_2 \frac{\epsilon}{2i}(p + is)^2 - \frac{\epsilon s}{2}(p + is)^2 - L\right\}}{\left\{\left(1 + \frac{S_{22}}{2M}\right) p^2 - D^2\right\} \left\{\left(1 + \frac{S'_{22}}{2M}\right)(p + is)^2 - D^2\right\}}. \quad \ldots(29)$$

Then $w_2(y)$ will be given by the inversion formula

$$w_2(y) = \frac{1}{2\pi i} \int_{\gamma - i\infty}^{\gamma + i\infty} F(p)\, e^{py}\, dy. \quad \ldots(30)$$

The poles of eqn. (29) are

$$\left(\frac{1 + S'_{22}}{2M}\right) p^2 - D^2 = 0, \text{ i. e., } p = \frac{D}{\left(1 + \frac{S'_{22}}{2M}\right)^{1/2}} = R_1$$

and

$$\left(1 + \frac{S'_{22}}{2M}\right)(p \pm is)^2 - D^2 = 0, \text{ i. e, } p \pm \text{is} = \frac{D}{\left(1 + S'_{22}\right)^{1/2}} = R_1. \quad \ldots(31)$$

The residues C_1, C_2, C_3 at the poles $p = R_1$, $p = R_{1+}$ is, and $p = R_{1-}$ is are given respectively by

$$C_1 = \left\{\frac{A_1 R_1 + A_2}{2R_1}\right\} e^{R_1 y} \left[1 + \frac{\left\{\frac{\epsilon}{2i}(R_1 - is)^2 + \frac{\epsilon s}{2}(R_1 - is) - L\right\}}{\left\{\left(1 + \frac{S'_{22}}{2M}\right) R_1 - is) - D^2\right\}}\right.$$

(*equation continued on p.* 1051)

$$+\frac{\left\{\frac{\epsilon}{2i}(R+is)^2-\frac{\epsilon s}{2}(R_1+is)-L\right\}}{\left\{\left(1+\frac{S'_{22}}{2M}\right)(R_1+is)-D^2\right\}}\Bigg]$$

$$+\frac{A_1 S^2\, ie^{R_1 y}}{2}$$

$$\frac{\left\{-\epsilon\left(1+\frac{S'_{22}}{2M}\right)(R_1^2+s^2)+D^2\right\}+\left(\frac{2NK^2\epsilon_1}{M}\right)\left(1+\frac{S'_{22}}{2M}\right)}{\left\{\left(1+\frac{S'_{22}}{2M}\right)(R_1-is)^2-D^2\right\}\left\{\left(1+\frac{S'_{22}}{2M}\right)(R_1+is)^2-D^2\right\}} \quad ...(32)$$

$$C_2=\frac{(A_1R_1+A_2)\left\{\frac{\epsilon}{2i}R_1^2+\frac{\epsilon s}{2}R_1-L\right\}}{2\left\{\left(1+\frac{S'_{22}}{2M}\right)(R_1+is)^2-D^2\right\}(R_1+is)}e^{(is+K_1)y} \quad ...(33)$$

and

$$C_3=\frac{(A_1R_1+A_2)\left\{\frac{\epsilon}{2i}R_1^2-\frac{\epsilon s}{2}R_1-L\right\}}{2\left\{\left(1+\frac{S'_{22}}{2M}\right)R_1^2-D^2\right\}(R_1-is)}e^{(R_1-is)y}. \quad ...(34)$$

Equation (32), (33) and (34) shows that the condition for large amount of energy to confined near the surface are

$$A_1R_1+A_2=0 \quad ...(35)$$

and

$$\left\{-\epsilon\left(1+\frac{S'_{22}}{2M}\right)\left(R_1^2+s^2\right)+D^2\right\}+\frac{2Nk^2\epsilon_1}{M}\left(1+\frac{S'_{22}}{2M}\right)=0. \quad ...(36)$$

Using values of A_1 and A_2 from eqn. (19), (20) and R_1 from (31), eqn. (35) gives

$$\tan HT_1=\frac{M\left(1+\frac{S'_{22}}{2M}\right)D}{Q_1T_1\left(1+\frac{S'_{22}}{2M}\right)^{1/2}}.$$

Substituting the values of D from eqn. (22) and T_1 from eqn. (5), we have

$$\tan HK\left[\frac{\rho C^2 - \left(Q_2 + \frac{S_{11}}{2}\right)}{\left(Q + \frac{S_{22}}{2}\right)}\right]^{1/2} = \frac{M}{Q_1}\left(1 + \frac{S'_{22}}{2M}\right)^{1/2}$$

$$\times \frac{\left\{\left(1 + \frac{S'_{11}}{2N}\right) - \frac{\rho_1}{N} C^2\right\}^{1/2}}{\left\{\frac{\rho C^2 - \left(Q_2 + \frac{S_{11}}{2}\right)}{\left(Q_1 + \frac{S_{22}}{2}\right)}\right\}^{1/2}} \qquad \ldots(37)$$

which is the dispersion equation of Love type waves.

Now from eqn. (36)

$$U = \frac{d(kC)}{dk}$$

$$= \beta_1 \frac{\sqrt{2}\, kH \left\{\left(\frac{N}{M} + \frac{S'_{11}}{2M}\right) - \frac{\epsilon_1}{\epsilon}\frac{N}{M}\left(1 + \frac{S'_{22}}{2M}\right)\right\}}{\left[2k^2H^2\left\{\left(\frac{N}{M} + \frac{S'_{11}}{2M}\right) - \frac{\epsilon_1}{\epsilon}\frac{N}{M}\left(1 + \frac{S'_{22}}{2M}\right)\right\} + \left(1 + \frac{S'_{22}}{2M}\right) S^2H^2\right]^{1/2}}$$

where

$$\beta_2 = \frac{\sqrt{M}}{\rho}. \qquad \ldots(38)$$

Equation (38) gives the group velocity U. It is clear from Table I that $U < \beta_2$. Hence it follows that if horizontally polarized S-waves propagate, a large fraction of energy may flow along the surface only when the wave-length and period satisfy the dispersion equation for Love waves in uniform media, with the group velocity U satisfying relation (38). It follows that the group velocity is lower than that of the shear wave velocity in the upper mantle. This type of wave may be compared with that of G-waves.

Numerical Calculation and Discussions

For the purpose of numerical calculations we have used the following values of the elastic constants

$$\frac{M}{Q_1} = 0.5, \quad \frac{Q_2}{Q_1} = 0.8, \quad \frac{N}{M} = 0.4.$$

TABLE 1

η_1, η_2 \ $\frac{2\pi}{kH}$	0.01	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5
0.0	0.576	0.575	0.573	0.570	0.566	0.560	0.552	0.545	0.536	0.527
0.6,0	0.965	0.965	0.963	0.961	0.959	0.955	0.951	0.946	0.940	0.934
$\frac{U}{\beta_2}$ 0.6,.6	0.323	0.321	0.316	0.307	0.298	0.270	0.270	0.256	0.243	0.230
0.2,−0.2	0.738	0.738	0.737	0.734	0.732	0.728	0.723	0.718	0.712	0.706

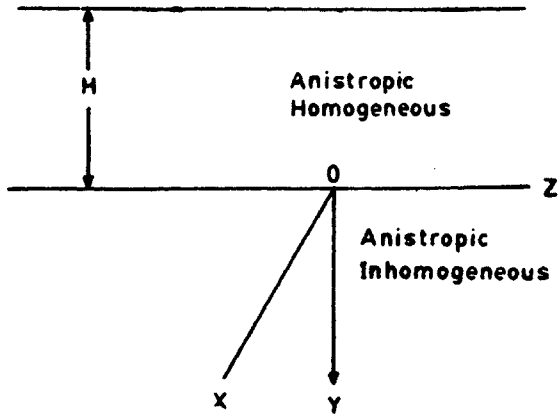
H
Anistropic
Homogeneous
0
Z
Anistropic
Inhomogeneous
X
Y

FIG. 1.

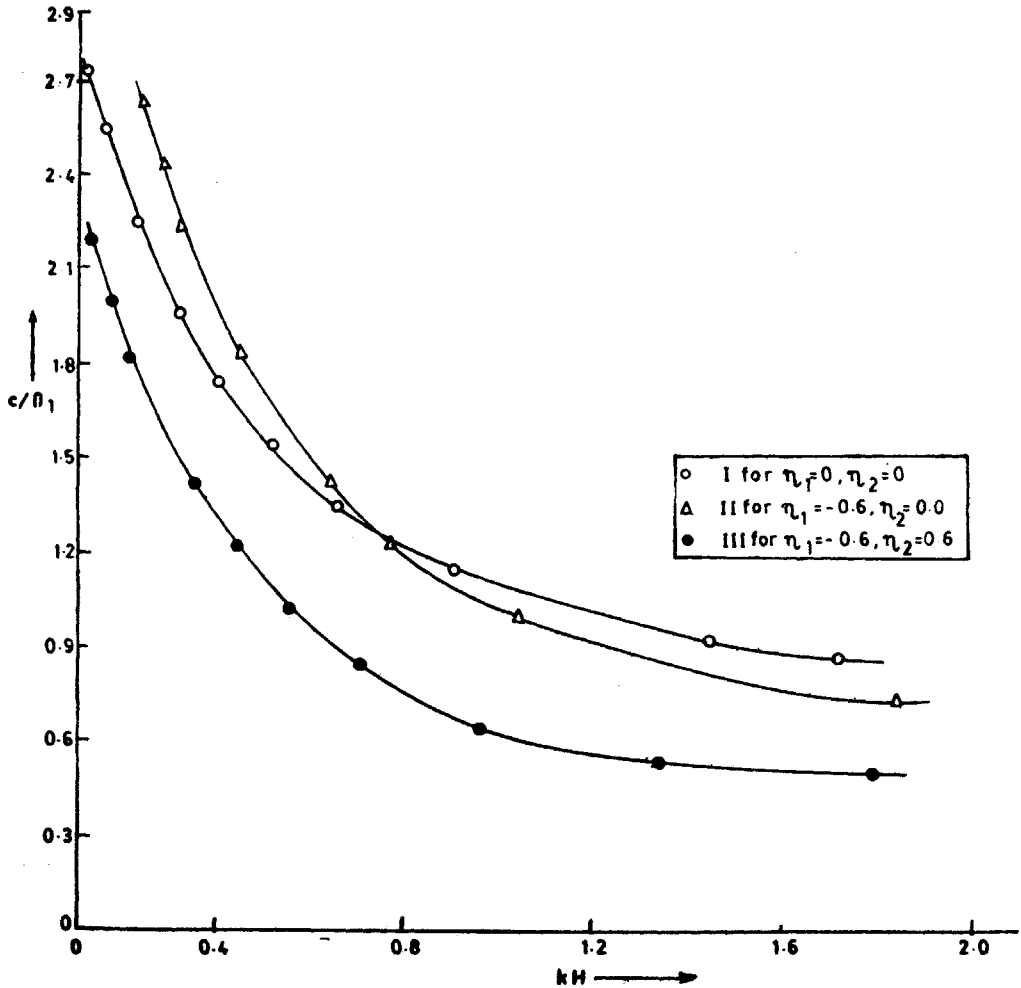
2·9
2·7
2·4
2·1
1·8
1·5
1·2
0·9
0·6
0·3
0
c/β1
0
0·4
0·8
1·2
1·6
2·0
kH
I for η1=0, η2=0
II for η1=-0·6, η2=0·0
III for η1=-0·6, η2=0·6

FIG. 2.

The dispersion curves have been plotted for different values of $\eta_1 = \frac{S'_{11}}{2M}$ and $\eta_2 = \frac{S'_{22}}{2M}$. From the first curve of Fig. 2 it is clear that due to the absence of initial stresses the phase velocity decreases as kH increases.

The same qualitative natures are observed for the other curves. The phase velocity decreases more due to the action of initial stresses in the medium than the stress free medium.

It is interesting to note that these exists an inverse point between the curves I and II at 0.71. Between $kH = 0.05$ and $kH = 0.71$, the phase velocity decreases more in the curve I than II but the reverse effect is observed after 0.71.

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