Indian J. pure appl. Math., 17 (8) : 1056-1066. August 1986

RAYLEIGH WAVE SCATTERING BY A PLANE BARRIER IN A SHALLOW OCEAN

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(*Received* 20 *May* 1985; *after revision* 21 *April* 1986)

The paper deals with theoretical study of the problem of scattering of Rayleigh waves due to the presence of a rigid plane barrier in a shallow ocean. A thin and rigid plane barrier of depth H $(0 \leqslant z \leqslant H)$ is held fixed in the surface of the ocean of shallow depth $(0 \leqslant z \leqslant h)$ lying over the solid half space $z \geqslant h$. Fourier transformation and the Wiener-Hopf technique is the method of solution. The waves scattered at the edge of the barrier are found to behave as cylindrical waves decaying rapidly at distant points. Near the scatterer the scattered field has the behaviour of reflected plane waves with large amplitudes.

1. INTRODUCTION

The study of Rayleigh wave scattering due to surface discontinuities has acquired importance as they are responsible for the damages to human beings and buildings on the surface of the earth during an earthquake. As soon as, the seismic waves appear on the surface of the earth they let loose their energies around the inhomogeneities and irregularities in the surface of the earth and display their catastrophic role.

The effect of a vertical barrier, fixed in an infinitely deep sea, on normally incident surface waves was first considered by Ursell[1] for a two dimensional case. Deshwal[2,3] has studied the problem of diffraction of compressional waves due to a rigid barrier in the surface of a deep sea-water and in an ocean super-imposed on a solid halfspace using the technique of Wiener and Hopf[4]. Scattering of Rayleigh waves at the corner of an elastic quarter space has been considered by Momoi[5] using the technique of Fourier transformation. He has approximated the integral transforms to obtain expressions for the energies of Rayleigh waves along the two free surfaces. Deshwal[6] has extended his study to discuss the problem of diffraction of compressional waves by a finite strip in a liquid half space. He has used the Wiener-Hopf technique in the solution of his problems. Mann and Deshwal[7] have used the same technique to study Rayleigh wave scattering by a quarter-plane strip in a liquid half space. The scattered waves are found to behave as dying out cylindrical waves.

The effect of topography and subsurface inhomogeneity on seismic Rayleigh waves has been investigated by Ohtsuki and Yamashara[8]. Several types of topography

such as cliffs are considered and computations are made using a hybrid method combining a particle method with a finite element method. Surface motions with amplitudes as large as 1.5 to 5 times the horizontal surface displacement of the incident Rayleigh waves are produced near the surface inhomogeneity.

We intend to discuss here the problem of diffraction of Rayleigh waves at the edge of a rigid plane barrier. The barrier of depth H is fixed in the surface of a shallow ocean of depth h. The barrier is rigid such that no displacement occurs across it. The model of a shallow ocean over the solid mantle of the earth fits with a liquid layer over a solid half space. The problem is two dimensional in the zx-plane.

2. Statement of the Problem

We take the x-axis in free surface of the ocean and the z-axis pointing vertically downward. The barrier is of depth H and is held fixed in the free surface of water along the z-axis. The water layer and the solid half space are given by $0 \leqslant z \leqslant h$ and $z \geqslant h$ respectively. A time-harmonic two dimensional Rayleigh wave is incident obliquely on the barrier as shown in Fig. 1. When the Rayleigh waves are incident

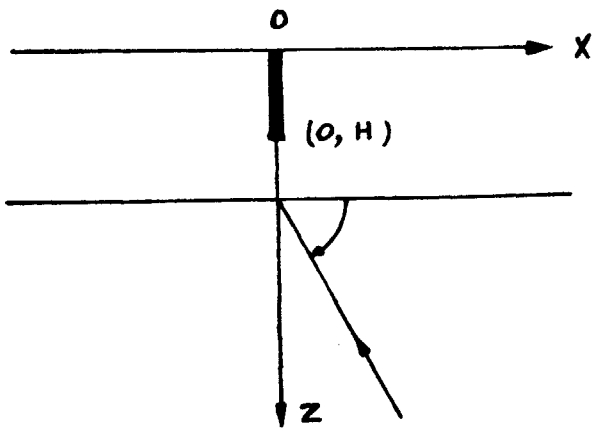

Fig. 1.

at the interface $z = h$, some are transmitted to the liquid layer and some of the transmitted waves strike at the barrier and give rise to the scattered waves. We have to find the form of the waves scattered from the barrier in the liquid layer. The wave equations in the liquid and solid media are

$$\partial^2 \phi/\partial x^2 + \partial^2 \phi/\partial z^2 + k^2\phi = 0\,,\ 0 \leqslant z \leqslant h \qquad \ldots(1)$$

$$\partial^2 \phi_1/\partial x^2 + \partial^2 \phi_1/\partial z^2 + (k')^2 \phi_1 = 0\,,\ z \geqslant h \qquad \ldots(2)$$

$$\partial^2 \psi_1/\partial x^2 + \partial^2 \psi_1/\partial z^2 + (k'')^2 \psi_1 = 0\,,\ z \geqslant h \qquad \ldots(3)$$

where ks are complex with positive and small imaginary parts. Let the incident potential distribution in the solid and liquid be

$$\phi_{1i} = \left[D \left(2\alpha_N^2 - k''^2 \right) Y_N \cos Y_N h \exp\left(-i\alpha_N x - Y_{1N}(z-h)\right) \right] \Big/ Y_{1N}(k'')^2,\ z \geqslant h \quad \ldots(4)$$

$$\psi_{1i} = [2Di\alpha_N Y_N \cos Y_N h \exp(-i\alpha_N x - \delta_{1N}(z-h))]/(k'')^2,\ z > h \quad \ldots(5)$$

and

$$\phi_i = D \sin Y_N z \exp(-i\alpha_N x),\ 0 \leqslant z < h \quad \ldots(6)$$

where α_N is a root of frequency equation

$$\tan Y_N h = P_1 Y_N \left[4\alpha_N^2 Y_{1N} \delta_{1N} - \left(2\alpha_N^2 - k''^2 \right)^2 \right] \Big/ P Y_{1N} (k'')^4 \quad \ldots(7)$$

and

$$Y_N^2 = k^2 - \alpha_N^2,\ Y_{1N}^2 = \alpha_N^2 - (k)^2,\ \delta_{1N}^2 = \alpha_N^2 - (k'')^2 \quad \ldots(8)$$

P_1 and P are densities of two media and D is a constant. Let the total potentials in the liquid layer and solid half space be

$$\phi_t = \phi_i + \phi,\ \phi_{1t} = \phi_{1i} + \phi_1,\ \psi_{1t} = \psi_{1i} + \psi_1. \quad \ldots(9)$$

3. Boundary Conditions

The conditions on the boundaries are

(i) $\tau_{zz} = 0$, on $z = 0$, for all x ...(10)

(ii) $u = 0$, on $x = 0$, for $0 \leqslant z \leqslant H$ ...(11)

(iii) $w = w_1,\ \tau_{zz} = (\tau_{zz})_1,\ (\tau_{zx})_1 = 0$, on $z = h$, for all x ...(12)

(iv) $\phi_1(x, z)$ and $\psi_1(x, z)$ are bounded when $z \to \infty$. ...(13)

τ_{zz}, τ_{zx} are the normal and shear stresses. The displacement components in terms of potentials in the solid medium are

$$u_1 = \partial\phi_{1t}/\partial x - \partial\psi_{1t}/\partial z,\ w_1 = \partial\phi_{1t}/\partial z + \partial\psi_{1t}/\partial x. \quad \ldots(14)$$

Similar relations hold in the liquid layer. We define Fourier transforms as

$$\bar{\phi}(\alpha, z) = \int_{-\infty}^{\infty} \phi(x, z) e^{i\alpha x} dx = \int_{-\infty}^{0} \phi(x, z) e^{i\alpha x} dx + \int_{0}^{\infty} \phi(x, z) e^{i\alpha x} dx$$

$$= \bar{\phi}_-(\alpha, z) + \bar{\phi}_+(\alpha, z) \quad \ldots(15)$$

with similar transforms for $\phi_1(x, z)$ and $\psi_1(x, z)$. We assume that for given z, $|\phi|$ $|\phi_1|$ and $|\psi_1|$ are bounded by $M \exp(-k_2 |x|)$, for large x, $M > 0$. It can also be seen that $\bar{\phi}_+(\alpha, z)$, $\bar{\phi}_-(\alpha, z)$ and $\bar{\phi}(\alpha, z)$ (with their derivatives) are regular in the regions $T > -k_2$, $T < k_2$ and $-k_2 < T < k_2$ ($\alpha = \sigma + iT$) respectively of the complex α-plane.

4. Solution of the Problem

Taking the Fourier transform of (2) and noting that the potentials are bounded for large x, we obtain

$$\left(d^2/dz^2 - Y_1^2\right) \bar{\phi}_1(\alpha, z) = 0, \; Y_1 = \pm \sqrt{[\alpha^2 - (k')^2]}. \qquad ...(16)$$

The solution of (16) is

$$\bar{\phi}_1(\alpha, z) = A_1(\alpha) \exp(-Y_1 z) \qquad ...(17)$$

because $\bar{\phi}_1(\alpha, z)$ is bounded as $z \to \infty$. Similarly

$$\psi_1(\alpha, z) = B_1(\alpha) \exp(-\delta_1 z), \; \delta_1 = \pm \sqrt{[\alpha^2 - (k')^2]} \qquad ...(18)$$

$$\bar{\phi}(\alpha, z) = A(\alpha) \exp(Yz) + B(\alpha) \exp(-Yz), \; Y = \pm \sqrt{[\alpha^2 - k^2]}. \qquad ...(19)$$

The signs before the radicals in (16), (18) and (19) are chosen so that the real parts of Y_1 δ_1 and $Y \geqslant 0$, for all α. Taking the Fourier transforms of (12) and using (17-19), we get

$$A(\alpha) \exp(Yh) + B(\alpha) \exp(-Yh) = (C_1 + C_2) A_1 \exp(-Y_1 h) \qquad ...(20)$$

$$A(\alpha) \exp(Yh) - B(\alpha) \exp(-Yh) = C A_1 \exp(-Y_1 h) \qquad ...(21)$$

where

$$C_1 = [(\lambda_1 + 2\mu_1)(k')^2 - 2\mu_1 \alpha^2]/\lambda k^2 \qquad ...(22)$$

$$C_2 = 4 \mu_1 \alpha^2 Y_1 \delta_1/\lambda k^2 (\alpha^2 + \delta_1^2), \; C = Y_1 (k'')^2/Y(\alpha^2 + \delta_1^2). \qquad ...(23)$$

Solving (20) and (21) for $A(\alpha)$ and $B(\alpha)$ and using their values in (19) to find

$$\bar{\phi}(\alpha, z) = [(C_1 + C_2) \cosh Y(z - h) + C \sinh Y(z - h)] A_1 \exp(-Y_1 h). \qquad ...(24)$$

Putting $z = H$ in (24) and in its derivative w.r.t. z and using the notations $\bar{\phi}_+(\alpha)$, $\bar{\phi}_-(\alpha)$, $\bar{\phi}'_+(\alpha)$, and $\bar{\phi}'_-(\alpha)$ for $\bar{\phi}_+(\alpha, H)$, $\bar{\phi}_-(\alpha, H)$ $\bar{\phi}'_+(\alpha, H)$ and $\bar{\phi}'_-(\alpha, H)$ respectively, we have after eliminating $A_1(\alpha)$

$$\bar{\phi}_+(\alpha) + \bar{\phi}_-(\alpha) + \bar{\phi}_+(-\alpha) + \bar{\phi}_-(-\alpha)$$

$$= Y^{-1}\left[\frac{(C_1 + C_2)\cosh Y(H-h) + C\sinh Y(H-h)}{(C_1 + C_2)\sinh Y(H-h) + C\cosh Y(H-h)}\right]$$

$$\times\left[\bar{\phi}'_+(\alpha) + \bar{\phi}'_-(\alpha) + \bar{\phi}'_+(-\alpha) + \bar{\phi}'_-(-\alpha)\right]. \qquad ...(25)$$

We now multiply (1) by $\exp(i\alpha x)$ and integrate as x varies from 0 to ∞, to find out the differential equation

$$d^2\,[\bar{\phi}_+(\alpha, z) + \bar{\phi}_+(-\alpha, z)]/dz^2 - Y^2\,[\bar{\phi}_+(\alpha, z) + \bar{\phi}_+(-\alpha, z)]$$

$$= 2iD\alpha_N \sin Y_N z \qquad ...(26)$$

whose complete solution

$$\bar{\phi}_+(\alpha, z) + \bar{\phi}_+(-\alpha, z) = 2D_1 \sinh Yz - 2iD\alpha_N \sin Y_N z/(\alpha^2 - \alpha_N^2) \qquad ...(27)$$

is obtained by the condition (10) i.e. $\phi_t = 0$ on $z = 0$. Putting $z = H$ in (27), in its derivative and eliminating D_1 from the resulting equations, we obtain

$$\bar{\phi}\neq(\alpha) \neq \bar{\phi}_+(-\alpha) = Y^{-1}\tan hYH\left[\bar{\phi}'_+(\alpha) + \bar{\phi}'_+(-\alpha)\right]$$

$$+ 2iD\alpha_N Y_N \cos Y_N H \tanh YH/Y\left(\alpha^2 - \alpha_N^2\right)$$

$$- 2iD\alpha_N \sin Y_N H/\left(\alpha^2 - \alpha_N^2\right). \qquad ...(28)$$

If (1) is integrated between the limits $x = -\infty$ to $x = 0$, an equation in $\bar{\phi}_-(\alpha)$ and $\bar{\phi}_-(-\alpha)$ is obtained as in (28). Adding the resulting equation to (28) to find

$$\bar{\phi}_+(\alpha) + \bar{\phi}_+(-\alpha) + \bar{\phi}_-(\alpha) + \bar{\phi}_-(-\alpha)$$

$$= Y^{-1}\tanh YH\left[\bar{\phi}'_+(\alpha) + \bar{\phi}'_+(-\alpha) + \bar{\phi}'_-(\alpha) + \bar{\phi}'_-(-\alpha)\right]. \qquad ...(29)$$

From (25) and (29), it is easily deduced that

$$\bar{\phi}_+(\alpha) + \bar{\phi}_-(-\alpha) = -\bar{\phi}_+(-\alpha) - \bar{\phi}_-(\alpha) = R\,\alpha) \qquad ...(30)$$

$\bar{\phi}_+(\alpha) + \bar{\phi}_-(-\alpha)$ is analytic in the region $T > -k_2$ and $-\bar{\phi}_+(-\alpha) - \bar{\phi}_-(\alpha)$ is analytic in the region $T < k_2$. Therefore, by analytic continuation, they represent an entire function $R(\alpha)$. Since each member of $R(\alpha)$ approaches zero as $|\alpha| \to \infty$, hence

by Liouville's theorem, $R(\alpha)$ is zero at all points of the α-plane. Equating to zero each member of (30), we have

$$\bar{\phi}_+(\alpha) = -\bar{\phi}_-(-\alpha),\ \bar{\phi}_-(\alpha) = -\bar{\phi}_+(-\alpha). \qquad \ldots(31)$$

The relations (31) hold for their derivatives w.r.t. z also. When A_1 in (24) is eliminated between $\bar{\phi}(\alpha)$ and $\bar{\phi}'(\alpha)$ and (28) is used, we get a Wiener-Hopf type differential equation

$$2\bar{\phi}_-(-\alpha) = L(\alpha)\,\bar{\phi}'_+(\alpha) + T(\alpha)\,\bar{\phi}'_-(\alpha) + iD\alpha_N$$

$$[[L(\alpha) - T(\alpha)]\,Y_N \cos Y_N H + 2\sin Y_N H]/\left(\alpha^2 - \alpha_N^2\right) \qquad \ldots(32)$$

where

$$L(\alpha) = L_+(\alpha)\,L_-(\alpha) = \frac{M(\alpha)}{N(\alpha)\,H(\alpha)}$$

$$= \left[\frac{(C_1 + C_2)\cosh Yh - C\sinh Yh}{(C_1 + C_2)\sinh Y(H-h) + C\cosh Y(H-h)}\right]\frac{1}{Y\cosh YH}$$

$$= \frac{P_+(\alpha)\,P_-(\alpha)}{H_+(\alpha)\,H_-(\alpha)}\sum_{N=1}^{\infty}\left(\frac{\alpha^2 - \alpha_N^2}{\alpha^2 - \alpha_{2N}^2}\right). \qquad \ldots(33)$$

Each of these functions is an even functions of α. $\alpha = \pm\alpha_N$ and $\pm\alpha_{2N}$ $(N=1,2,3,\ldots)$ are the roots of $M(\alpha) = 0$ and $N(\alpha) = 0$ respectively. $P_\pm(\alpha)$ and $H_\pm(\alpha)$ are obtained in the appendices of Deshwal[3] and are asymptotic to constants as $|\alpha| \to \infty$. Similary if

$$T_+(\alpha)\,T_-(\alpha) = T(\alpha) = \frac{T_1(\alpha)}{N(\alpha)\,H(\alpha)}$$

$$= \left[\frac{(C_1+C_2)\cosh Y(2H-h)+C\sinh Y(2H-h)}{(C_1+C_2)\sinh Y(H-h)+C\cosh Y(H-h)}\right]\frac{1}{Y\cosh YH} \qquad \ldots(34)$$

then $T_\pm(\alpha)$ are asymptotic to constants as $|\alpha| \to \infty$. We use the notations of (33) and (34) in (32) to have

$$\frac{2\bar{\phi}_-(\alpha)}{T_+(\alpha)\,L_-(\alpha)} - \frac{T_-(\alpha)\,\bar{\phi}'_-(\alpha)}{L_-(\alpha)} + \frac{iD\alpha_N Y_N \cos Y_N H T_-(\alpha)}{\left(\alpha^2 - \alpha_N^2\right)L_-(\alpha)}$$

(equation continued on p. 1062)

$$- \frac{2iD\alpha_N \sin Y_N H}{\left(\alpha^2 - \alpha_N^2\right) L_-(\alpha)\, T_+(\alpha)}$$

$$= \frac{\bar{\phi}'_+(\alpha)\, L_+(\alpha)}{T_+(\alpha)} + \frac{iD\alpha_N Y_N \cos Y_N H L_+(\alpha)}{\left(\alpha^2 - \alpha_N^2\right) T_+(\alpha)}. \qquad ...(35)$$

Decomposing (35) as in case of (30) and by the arguments after it, we find

$$\bar{\phi}'_+(\alpha) = \frac{D'_1\ T_+(\alpha)}{L_+(\alpha)} - \frac{D'_1}{L_+(\alpha)} - \frac{iDY_N \cos Y_N H\, T_-(-\alpha_N)\, T_+(\alpha)}{2(\alpha + \alpha_N) L_-(-\alpha_N) L_+(\alpha)}$$

$$- \frac{iD\alpha_N Y_N \cos Y_N H}{\left(\alpha^2 - \alpha_N^2\right)}$$

$$+ \frac{iDY_N \cos Y_N H\, T_+(\alpha)\, L_+(\alpha_N)}{2(\alpha - \alpha_N) L_+(\alpha)\, T_+(\alpha_N)}$$

$$- \frac{2iD\alpha_N \sin Y_N H}{(\alpha + \alpha_N)(p_l - \alpha_N) L_+(\alpha) L_-(p_l)} \qquad ...(36)$$

where $p_l = -\alpha_N, p_i$; and p_l are the roots of $T_+(\alpha) = 0$. The constant $D'_1 = -2\bar{\phi}'_-(p_l)/L_-(p_l)$ is independend of α. $\bar{\phi}'_-(\alpha)$ can be obtained from $\bar{\phi}'_+(\alpha)$ by changing α to $-\alpha$ and using $\bar{\phi}'_-(\alpha) = -\bar{\phi}'_+(\alpha)$. Taking the inverse Fourier transform of (15) and using (24), we obtain

$$\phi(x, z) = \frac{1}{2\pi} \int_{-\infty + iT}^{\infty + iT} [(C_1 + C_2) \cosh Y(z - h) + C \sinh Y(z - h)]$$

$$A_1 \exp(-Y_1 h - i\alpha x)\, d\alpha \qquad ...(37)$$

where A_1 is obtained by differentiating (24) and using (36). Further $-k_2 < T < k_2$.

5. Various Waves

We evaluate the integral (37) along a closed contour in the α-plane. $\alpha = \pm \alpha_N$, $\pm \alpha_{2N}$ $(N = 1, 2, 3, ...)$ are the poles inside the contour and $\alpha = \pm k, \pm k', \pm k''$ are the branch points. The conditions Re $(Y) = 0$, Re $(Y_1) = 0$ and Re $(\delta_1) = 0$ discussed by Ewing and Press[9] give the parts of hyperbolae to be used as the branch cuts. Let the coutour of integration be in the lower part of the complex α-plane as shown in Figure 2. Using Cauchy residue theorem, we have

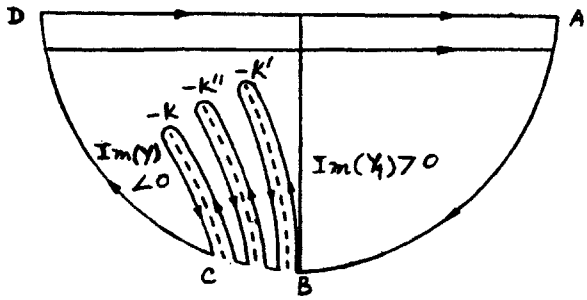

FIG. 2.

$$\int_{-\infty+iT}^{\infty+iT} + \int_{A}^{B} + \int_{C}^{D} + \int_{L_k} + \int_{L_{k'}} + \int_{L_{k''}} = 2\pi i \text{ (sum of residues)} \quad \ldots(38)$$

where L_k, L_k' and $L_{k'}'$ are the branch cuts corresponding to the branch points $-k$, $-k'$, $-k''$ respectively. The presence of the factor $\exp(-i\alpha x) = \exp(-i\sigma x)$. $\exp(-Tx)$ ($\alpha = \sigma - iT$) makes the integrals along the infinite circular arcs AB and CD vanish in the region $x > 0$. The contribution at poles $\alpha = \pm \alpha_N$ are

$$-\frac{L_1}{2}\Big[\Big(C_1' + C_2' - C''\Big)\exp[iY_N(z-h)] + \Big(C_1' + C_2' + C''\Big)\exp[-iY_N z) - h)]\Big] e^{-i\alpha_N x} \quad \ldots(39)$$

and

$$\frac{L_1}{2}\Big[\Big(C_1' + C_2' - C''\Big)\exp(iY_N(z-h)) + \Big(C_1' + C_2' + C''\Big)\exp(-iY_N(z-h))\Big] e^{i\alpha_N x} \quad \ldots(40)$$

respectively, where

$$L_1 = \Big[D_1'(T_+(-\alpha_N) - 1) - \frac{iDY_N \cos Y_N H L_+(\alpha_N) T_+(-\alpha_N)}{4\alpha_N T_+(\alpha_N)} + \frac{iDY_N M'(\pm\alpha_N)}{2L_-(-\alpha_N)}\Big]\frac{Y_N L_-(-\alpha_N)\cos Y_N H}{M'(\pm\alpha_N)} \quad \ldots(41)$$

and C_1', C_2', C'' are the values of C_1, C_2, and C at $\alpha = \pm\alpha_N$. In (40), we have the waves reflected from the barrier and from the free surface after its reflection from the barrier and in (39), the transmitted waves in the region $x < 0$. They have equal but opposite amplitudes.

We now consider the contribution of the integral along the branch cuts. On L_k, Re $(Y) = 0$ and it can be verified that Im (Y) has a positive sign on the upper bank of L_k (which lies on the side of lower bank of L_k'' in Fig. 2) and a negative sign on the lower bank. Im (Y_1) and Im (δ_1) have signs on L_k' and L_k'' respectively as stated for Im (Y) on L_k. We find out that the contribution of integral vanishes along the branch cut L_k. On $L_{k'}$, the main contribution comes from the neighbourhood of the branch point $\alpha = -k'$. To find this, we substitute $\alpha = -k' - iu$, u being small. Along the cut, limits of u vary from zero to infinity. Integrating (37) along the two sides of $L_{k'}$, it is obtained that

$$I_1(x, z) = \frac{i}{2\pi} \int_0^\infty \Big[[(C_1 + C_2) \cosh Y(z - h) + C \sinh Y(z - h)]$$

$$A_1 e^{-Y_1 h} \mid Y_1 = iY_1'$$

$$- [(C_1 + C_2) \cosh Y(z-h) + C \sinh Y(z-h)]$$

$$A_1 e^{-Y_1 h} \mid Y_1 = -iY_1' \Big] e^{-ux}\, du \qquad \ldots(42)$$

where it is used that Y_1^2 is negative and so

$$Y_1 = \pm [(k' + iu)^2 - (k')^2]^{1/2} = \pm i \left(2k_2' \; u \right)^{1/2} = \pm iY_1'. \qquad (43)$$

The integral in (42) is evaluated using a result due to Ewing and Press[9] i.e.

$$\int_0^\infty u^{1/2} G(u) e^{-ux}\, du = \frac{\Gamma(3/2)\, G(0)}{x^{3/2}} + \frac{\Gamma(5/2)\, G'(0)}{x^{5/2}} + \ldots$$

$\Gamma(x)$ is the Gamma function.

The paper has its application to scattering of Rayleigh waves due to (i) a dock, (ii) a submerged floating ice and (iii) an artificial plane barrier inserted into the surface of an ocean. Artificial plane barriers can be placed in the surface of the ocean to lessen the effects of the seismic waves due to scattering. If it is assumed that there is no displacement across the submerged ice and it or the dock or the rigid plane barrier is of small depth then $H \ll h$. Expanding sines and cosines for small kH, $k'H$ etc. and taking H to be small in comparison to h, the scattered field is

$$I_1(x, z) = \frac{\Gamma(3/2)\, \lambda\, (k_2 k^*)^2\, e^{-k' \frac{x}{2}}}{\pi Y' \lambda_1 \left(ik_2' \; x \right)^{3/2} \left(\left(k_2^* \right)^2 - \left(2k_2^2 \right) \right)}$$

(equation continued on p. 1065)

$$\left[\left[D_1' - \frac{iD\alpha_N Y_N}{\alpha_N^2 + (k_2')^2} - \frac{D_1'}{L_-(-k')} - \frac{iDY_N H}{\left(\alpha_N - ik_2'\right)L_-(-\alpha_N)}\right]\right.$$

$$\frac{Y'(\cosh Y' z - Y' H \sinh Y' z)}{\sinh^2 Y' h} + \left(D_1' - \frac{iDY_N H}{\left(\alpha_N - ik_2'\right)L_-(-\alpha_N)}\right)$$

$$\left.\frac{(\sinh Y' z)\, L_-(-k')}{\cosh^2 Y' h}\right], \quad y' = i\,(k_2')^2 k^2)^{1/2} \qquad \ldots(44)$$

When $x \gg z$, then the scattered field is of the form $\exp(-k_2' r)/r^{1/2}$, $r = (x^2 + z^2)^{1/2}$. The far-field has the behaviour of a cylindrical wave decaying exponentially for large r. For points close to the scatterer, the form of the field is $\exp(ik' x \pm Y' z)/x^{3/2}$. These are plane waves reflected from the barrier and then from the free surface of the liquid layer. For small x, the amplitude of these waves is of the order of $x^{-3/2}$.

6. Conclusions

The waves reflected from the barrier and then from the free surface of the liquid layer are obtained in (40). The transmitted waves in the region $x < 0$ are given by (39). The contribution due to the residues of the poles $\alpha = \pm \alpha_N$ of the integrand (37) gives multiply reflected waves from the boundaries of the liquid layer. The far-field of the scattered waves, when z is small and r is nearly equal to x, has the behaviour of cylindrical waves. The amplitude decays exponentially as the wave moves away from scatterer. For points close the scatterer, the near-field is of the form $\exp(ik' x \pm Y'z)/x^{3/2}$. These are plane waves reflected from the plane barrier and then from the free surface of the liquid layer after their reflection from the barrier. The scattered waves propagate with a velocity equal to the velocity of the compressional waves of the solid halfspace. The near-field is of the order of $x^{-3/2}$ which is large for small x. The scattered waves are dominant near the scatterer and die out exponentially as they move away from scatterer. In the free surface (where $z = 0$), the scattered field is of the form $\exp(-k_2' x)/x^{3/2}$ both for near and far-off points. When $H = 0$, the scattered field does not vanish for there is a discontinuity of surface at the point (0,0) If the integral (37) is evaluated along the branch cut L_k'', we obtain the scattered waves propagating with a velocity equal to transverse waves of the solid. Waves reflected in the solid can be had by evaluating the inverse Fourier transforms of $\bar{\phi}_1(\alpha, z)$ and $\bar{\psi}_1(\alpha, z)$.

With seismic stations spread all over the world, the results of the paper are valid at distant places like Antarctica.

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