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VARIABLE SAMPLE SIZE AND INFERENCE IN SAMPLES FROM GAMMA POPULATIONS

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In this paper, inference in gamma populations, with known integer shape parameter α is dealt with when sizes of the samples, drawn from these populations, are random variables. For this purpose, distribution of the j-th order statistics is put in the closed form for both the cases, when the sample size is fixed as well as when it is random. Next, the distribution of the ratios in terms of j-th order statistics in series of $(k + 1)$ independent samples is obtained. Through which inferences on θ_t, λ_t $(t = 1,\ldots,k + 1)$ is suggested where θ_t's are the scale parameters in the gamma populations with integer shape parameter α_t's and λ_t's are the parameters in the Poisson distributions taken for $(k + 1)$ sample sizes. Finally, distribution of another statistic, sum of the $(k + 1)$, j-th order statistics is obtained for the case $\alpha = 1$ (α, shape parameter) and from this distribution, series of identities are obtained.

INTRODUCTION

The purpose of this paper is to deal with the inference on the scale parameters of $(k + 1)$ independent gamma populations through series of samples, whose sizes are taken as random variables. Buhrman[1], Consul[2], Gupta and Gupta[3] and Raghunandan and Patil[7], all deal with the analysis of the situation where the sample size is a random variable. Most of these works deal with the distribution of order statistics in a sample of size n drawn from an exponential population, when the sample size is a variable having its pdf as binomial, Poisson, generalised negative binomial, generalised Poisson and generalised log series and the like. In this paper, first, the sample is from a gamma population and the distribution of the j-th order statistics is put in a closed form. Next, series of $(k + 1)$ independent samples are drawn from $(k + 1)$ gamma populations and a statistic in terms of the ratios of j-th order statistics, is developed, which can be used for inference purposes. Along with this, a similar distribution is obtained for the fixed sample size case, which can be compared with the variable sample size case. Finally for $\alpha = 1$ (shape parameter) in the gamma population, distribution of another statistic in terms of sum of order statistics, is obtained and by using this distribution, a series of identities are obtained.

2. Distribution

(a) Variable Sample Size

From Consul[2], the pdf of the j-th order statistics from the population with the distribution function $F(x)$ and when the size of the sample drawn from this population is a random variable, following the Poisson distribution

$$f(x) = e^{-\lambda}\,\lambda^x/x!,\ x = 0, 1, 2,\ldots \qquad \ldots(1)$$

is given as,

$$f(u) = \left[\frac{e^{-\lambda}\,\lambda^j}{(j-1)!\,P_j(\lambda)}\right] \sum_{r=0}^{\infty} \frac{[\lambda(1-F_j)]^r}{r!}\, F_j^{j-1}\, dF_j \qquad \ldots(2)$$

where $u = x_{(j)}$, the j-th order statistics, $F_j = F[x_{(j)}] = F(u)$, $P_j(\lambda) = P(x \geqslant j)$ in (1). Suppose now the population in question in (2) is gamma, given by

$$f(x) = e^{-\theta x}\,(\theta x)^{\alpha-1}\,\theta/\Gamma(\alpha) \qquad \ldots(3)$$

$x > 0, \theta > 0, \alpha > 0$ (positive integer).

Now from (3), we have

$$F(x) = 1 - \sum_{k=0}^{\alpha-1} [e^{-\theta x}\,(\theta x)^k/k!] = 1 - \Sigma. \qquad \ldots(4)$$

Using (4) in (2), we get

$$f(u) = \sum_{r=0}^{\infty} \phi(\lambda, j) \sum_{i=0}^{j-1} (\psi_i)\,\Sigma^{r+i}\,[e^{-\theta u}\,(\theta u)^{\alpha-1}\,\theta/\Gamma(\alpha)] \qquad \ldots(5)$$

where

$$\phi(\lambda, j) = e^{\lambda}\,\lambda^{r+j}/r!\,(j-1)!\,P_j(\lambda)$$

$$(\psi_i) = \binom{j-1}{i}(-1)^i$$

and Σ in (5) corresponds to sum in (4) with u for x.

Now, (5) can be written in the form

$$f(u) = [\Gamma^{-1}(\alpha)] \sum_r \phi(\lambda, j) \sum_i (\psi_i) \sum_{s=0}^{(r+i)(\alpha-1)} A_s(r+i, \alpha) e^{-\theta u(r+i+1)}\,\theta.\,(\theta u)^{\alpha+s-1}$$

$$u > 0 \qquad \ldots(5a)$$

where $A_s(r, \alpha)$ is the coefficient of x^s in the expansion of $\left(\sum_{k=0}^{\alpha-1} x^k/k!\right)^r$ and the $A_s(r,\alpha)$ satisfy the recurrence relation given in Lingappaiah[4]. From (5a), we get

$$\int_0^\infty f(u)\,du = \sum_r \phi(\lambda, j) \sum_i (\psi_i) \sum_s \left[\frac{A_s(r+i, \alpha)(\alpha+s-1)!}{\Gamma(\alpha)(r+i+1)^{s+\alpha}}\right]. \quad \ldots(6)$$

But for a positive integer b,

$$\sum_{t=0}^{(b-1)(\alpha-1)} \left[\frac{A_t(b, \alpha)(\alpha+t-1)!}{\Gamma(\alpha) b^{\alpha+t-1}}\right] = 1. \quad \ldots(7)$$

Using (7) in (6), we get (6) as,

$$\sum_r \phi(\lambda, j) \sum_{i=0}^{j-1} \binom{j-1}{i} \frac{(-1)^i}{(r+i+1)}. \quad \ldots(8)$$

Now,

$$\sum_{i=0}^{n} \binom{n}{j} (-1)^i \left(\frac{1}{i+1}\right) = \frac{n!}{\prod_{i=0}^{n}(i+1)}. \quad \ldots(8a)$$

Using (8a), we get (8) as,

$$\sum_{r=0}^{\infty} \phi(\lambda, j)\left[\frac{(j-1)!\,r!}{(r+j)!}\right] \quad \ldots(8b)$$

and (8b) is equal to 1.

From (5), one can evaluate the probabilities, $P(u \geqslant u_0)$ again using (4).

(b) Fixed Sample Size

Now, suppose the sample size is a fixed quantity, then it is well known,

$$f(u) = CF_j^{j-1}(1 - F_j)^{n-j}\,dF_j \quad \ldots(9)$$

where $C = n!(n-j)!(j-1)!$ and F_j as before in (2). Now for the case of gamma distribution, (9) becomes

$$f(u) = [\Gamma^{-1}(\alpha)] \sum_{i=0}^{j-1} (\psi_i)\, \Sigma^{n+1-i} [e^{-\theta u} (\theta u)^{\alpha-1} \theta]\,(C) \qquad \ldots(10)$$

where Σ is the sum in (4) with u for x. Now (10) can be put in the form, (with $\epsilon' = n + i - j + 1$),

$$f(u) = C\,[\Gamma^{-1}(\alpha)] \sum_{i} (\psi_i) \sum_{t=0}^{(\epsilon'-1)(\alpha-1)} A_t(\epsilon' - 1, \alpha)\, \theta e^{-\theta u \epsilon'} (\theta u)^{\alpha+t-1}. \qquad \ldots(11)$$

From (11), we get

$$\int_0^\infty f(u)\, du = C \sum_i (\psi_i) \sum_t \frac{A_t(\epsilon' - 1, \alpha)(\alpha + t - 1)!}{\Gamma(\alpha)(\epsilon')^{\alpha+t}} \qquad \ldots(12)$$

$$= \left[C \sum_i (\psi_i) \left(\frac{1}{\epsilon'}\right)\right] \left[\sum_t \frac{A_t(\epsilon' - 1, \alpha)(\alpha + t - 1)!}{\Gamma(\alpha)(\epsilon')^{\alpha+t-1}}\right]. \qquad \ldots(12a)$$

In (12a), second sum is 1 due to (7) and (12a) now is

$$C \sum_{i=0}^{j-1} \binom{j-1}{i} \frac{(-1)^i}{(n + i - j + 1)} \qquad \ldots(12b)$$

and (12b) is equal to 1 using (8a).

Using (11) again, we can find the tail the probabilities $P(u \geqslant u_0)$ utilising (4). Now compare $P(u \geqslant u_0)$ under the case when n is a variable [from (5a)] and when n is fixed [from (11)]. This comparison gives the effect on tail probabilities under two situations.

(c) *Distribution of Ratios*

Now suppose, we have $(k+1)$ independent populations given by (3) with θ_t, λ_t, $(t = 1, 2, \ldots, k+1)$, then we have from (5a)

$$f(u_1, u_2, \ldots, u_{k+1}) = \sum_{r_1=0}^{\infty} \cdots \sum_{r_{k+1}=0}^{\infty} \left(\prod_{t=1}^{k+1} \phi(\lambda_t, j_t)\right) \sum_{t_1=0}^{j_1-1} \cdots \sum_{t_{k+1}=0}^{j_{k+1}-1}$$
$$\left(\prod_{t=1}^{k+1} \psi(i_t)\right) \cdot \sum_{s_1=0}^{(\alpha-1)(\epsilon_1-1)} \cdots \sum_{s_{k+1}=0}^{(\alpha-1)(\epsilon_{k+1}-1)} \left[\prod_{t=1}^{k+1} A(s_t, \epsilon_t - 1, \alpha)\right]$$

(equation continued on p. 749)

$$\times \left[e^{-\sum_{t=1}^{k+1} \theta_t u_t \epsilon_t} \prod_{t=1}^{k+1} (\theta_t (\theta_t u_t)^{\alpha+s_t-1} \Gamma^{-1}(\alpha)) \right] \quad \ldots(13)$$

where $\epsilon_t = i_t + r_t + 1$, $\psi(i_t)$ is ψ_{i_t}, and $A(s_t, \epsilon_t - 1, \alpha_t)$ is $A_{s_t}(\epsilon_t - 1, \alpha)$. Now setting

$$y_t = u_t u_{k+1}, \; t = 1, 2, \ldots, k \quad \ldots(14)$$

$$y_{k+1} = u_{k+1}$$

and noting the Jacobian of transformation is y_k^{k+1} and integrating out y_{k+1}, we get (13) as

$$f(y_1, \ldots, y_k) = \sum_{r_1=0}^{\infty} \ldots \sum_{r_{k+1}=0}^{\infty} \left[\prod_{t=1}^{k+1} \phi(\lambda_t, j_t) \right] \sum_{i_1=0}^{j_1-0} \ldots \sum_{i_{k-1}=0}^{j_{k+1}-1}$$

$$\left(\prod_{t=1}^{k+1} \psi(i_t) \right) \cdot [\Gamma^{-1}(\alpha)]^{k+1} \cdot \sum_{s_1=0}^{(\alpha-1)(\epsilon_1-1)} \ldots \sum_{s_{k+1}=0}^{(\alpha-1)(\epsilon_{k+1}-1)}$$

$$\left[\prod_{t=1}^{k+1} A(s_t, \epsilon_t - 1, \alpha) \right] \left[\frac{\left[\prod_{t=1}^{k} (\delta_t y_t)^{\alpha+1} s_t^{-1} \delta_t \right] \Gamma(S + (k+1)\alpha}{[\epsilon_{k+1} + (\delta_1 \epsilon_1 y_1 + \ldots + \delta_k \epsilon_k y_k)]^{S+(k+1)\alpha}} \right] \quad \ldots(15)$$

where $S = s_1 + \ldots + s_{k+1}$, $\delta_t = (\theta_t / \theta_{k+1})$, $t = 1, 2, \ldots, k$. Setting $(\epsilon_t y_t / \epsilon_{k+1}) = w_t$, $t = 1, 2, \ldots, k$, we get (15) as

$$f(w_1, \ldots, w_k) = [\Gamma^{-1}(\alpha)]^{k+1} \sum_{r_1=0}^{\infty} \ldots \sum_{r_{k+1}=0}^{\infty} \left[\prod_{t=1}^{k+1} \phi(\lambda_t, j_t) \right] \sum_{i_1=0}^{j_1-1} \ldots \sum_{i_{k+1}=0}^{j_{k+1}-1}$$

$$\left[\prod_{t=1}^{k+1} \psi(i_t) \right] \cdot \sum_{s_1=0}^{(\alpha-1)(\epsilon_1-1)} \ldots \sum_{s_{k+1}=0}^{(\alpha_1-1)(\epsilon_{k+1}-1)} \prod_{t=1}^{k+1}$$

$$\left(\frac{A(s., \epsilon_t - 1, \alpha)}{\epsilon_t^{(\alpha+s_t)}} \right) \cdot \left[\frac{\Gamma(S + (k+1)\alpha) \left(\prod_{t=1}^{k} w_t^{\alpha+s_t-1} \right)}{(1 + w_1 + \ldots + w_k)^{S+(k+1)\alpha}} \right] \cdot$$

$$\ldots(16)$$

The last braces in (16) refer to inverted Dirichlet distribution given in Lingappaiah[5,6] which is

$$f(x_1,\ldots,x_k) = x_1^{a_1-1} \ldots x_k^{a_k-1} \Big/ (1 + x_1 + \ldots + x_k)^a \,.\, B(a_1,\ldots a_k;\, a_{k+1})$$

$$x_t > 0,\ t = 1, 2, \quad , k \qquad \ldots(17)$$

where

$$B(a_1, a_2, \ldots, a_k;\, a_{k+1}) = \left[\prod_{t=1}^{k+1} \Gamma(a_t)/\Gamma(a)\right] \qquad \ldots(17a)$$

with

$$a = a_1 + \ldots + a_{k+1}.$$

Now using (17) in (16), we get,

$$\int_0^\infty \ldots \int_0^\infty f(w_1,\ldots, w_k)\, dw_1 \ldots dw_k \ldots = [\Gamma^{-1}(\alpha)]^{k+1} \sum_{r_1=0}^{\infty} \ldots \sum_{r_{k+1}=0}^{\infty}$$

$$\left(\prod_{t=1}^{k-1} \phi(\lambda_t, j_t)\right) \cdot \sum_{i_1=0}^{j_1-1} \ldots \sum_{i_{k-1}=0}^{j_{k+1}-1} \left[\prod_{t=1}^{k+1} \frac{\psi(i_t)}{\epsilon_t}\right]$$

$$\sum_{s_1=0}^{(\alpha-1)(\epsilon_1-1)} \ldots \sum_{s_{k+1}=0}^{(\alpha-1)(\epsilon_{k+1}-1)} \left(\prod_{t=1}^{k+1} A(s_t, \epsilon_t - 1, \alpha)\right) \cdot \left[\prod_{t=1}^{k+1} \left(\frac{(\alpha+s_t-1)!}{\epsilon_t^{\alpha+s_t-1}}\right)\right]$$

$$\ldots(18)$$

Now, $(k+1)$ sums on s_t $(t = 1, 2, \ldots, k+1)$ are each equal to 1 from (7) and in succession each $(k+1)$ sums on i_t $(t = 1, \ldots, k+1)$ is equal to $(j_t - 1)!\, r_t!/(r_t + j_t)!$, $(t = 1, 2, \ldots, k+1)$ due to (8a) and now (18) reduces to

$$\prod_{t=1}^{k+1} \left[\sum_{r_t} \phi(\lambda_t, j_t) \frac{(j_t + 1)!\, r_t!}{(r_t + j_t)!}\right] \qquad \ldots(19)$$

and (19) is equal to 1 similar to (8b).

Now one can use (15) to make inferences on the hypothesis whether $\theta_1 = \ldots = \theta_{k+1}$ or not and equally on the hypothesis $\lambda_1 = \ldots = \lambda_{k+1}$ or not.

3. Another Statistic (sum) $\alpha = 1$

Now suppose we take the fixed sample size case 2b, and eqn. (11) with $\alpha = 1$ we get (11) as

$$f(u) = C \sum_{i=0}^{j-1} (\psi_i)\, \theta e^{-\theta u(n-i+j+1)}. \qquad \ldots(20)$$

Again taking $(k+1)$ independent populations, we have from (20)

$$f(u_1 \ldots, u_{k+1}) = \left[\prod_{t=1}^{k+1} (C_t)\right] \sum_{i_1=0}^{j_1-1} \cdots \sum_{i_{k+1}=0}^{j_{k-1}-1} \prod_{t=1}^{k+1} \left[\psi(i_t)\, \theta_t e^{-\theta_t u_t \epsilon_t'}\right] \qquad \ldots(21)$$

where $\epsilon_t' = n_t + i_t - j_t + 1$ and C_t refers to C in (9) and $\psi(i_t)$ is ψ_{i_t}. Now, making transformations,

$$z_t = u_1 + \ldots + u_t,\ t = 1, 2 \ldots, k+1 \qquad \ldots(22)$$

and noting the Jacobian of transformation is 1, we get the distribution of $z = u_1 + \ldots + u_{k+1}$, after integrating out $z_1, \ldots, z_k$, as

$$f(z) = \left[\prod_{t=1}^{k+1} C_t\, \theta_t\right] \sum_{i_1=0}^{j_1-1} \cdots \sum_{i_{k+1}=0}^{j_{k+1}-1} \left[\prod_{t=0}^{k+1} \psi(i_t)\right] \left[\sum_{t=1}^{k} \frac{(-1)^{k-1} e^{-zb_{k+1}} - e^{-zb_t}}{\prod_{\substack{j=1 \\ j \neq t}}^{k+1} (Q_{tj})}\right] \qquad \ldots(23)$$

where

$$b_t = \theta_t\ \epsilon_t',\ t = 1, \ldots, k+1,\ Q_{tj} = b_t - b_j. \qquad \ldots(23a)$$

Now evaluating $E(z^r)$ from (23), we get few identities. For example, from (23), we have

$$E(z^r) = (r!)\left[\prod_{t=0}^{k+1} (C_t\, \theta_t)\right] \sum_{i_1=0}^{j_1-1} \cdots \sum_{i_{k+1}=0}^{j_{k+1}-1} \left[\sum_{t=1}^{k+1} \psi(i_t)\right] \left[\sum_{t=1}^{k} \frac{(-1)^{k-1} \left\{b_t^{r+1} - b_{k+1}^{r+1}\right\}}{(b_t\, n_{k+1})^{r+1} \prod_{\substack{j=1 \\ j \neq t}}^{k+1} (Q_{tj})}\right]. \qquad \ldots(24)$$

Using

$$(x^m - y^m)/(x - y) = \sum_{p=0}^{m-1} x^{m-p-1} y^p. \qquad \ldots(24a)$$

We can write the last braces in (24) as

$$\sum_{t=1}^{k} \frac{(-1)^{k-1}}{\prod_{\substack{j=1 \\ j \neq t}}^{k} (Q_{tj})} \left[\frac{1}{b_t\, b_{k+1}^{r+1}} + \frac{1}{b_{k+1}^{r}\, b_t^2} + \ldots + \frac{1}{b_t^{r+1}\, b_{k+1}} \right]. \qquad \ldots(24b)$$

Now, if we take $j_t = 1$ (all minima instead of j_t-th order statistic), then in (24) all sums on i_t vanish and both C_t and ϵ_t' are now n_t $(t = 1, 2, \ldots, k+1)$ and with $e_t = n_t \theta_t$, (24) can be put as, (with $b_t = e_t$ now),

$$E(z^r) = (r!) \left(\prod_{t=1}^{k} e_t \right) \sum_{t=1}^{k} \frac{(-1)^{k-1}}{\left[\prod_{\substack{j=1 \\ j \neq t}}^{k} (e_t - e_j) \right]}$$

$$\left[\frac{1}{e_t e_{k+1}^{r}} + \frac{1}{e_t^2 e_{k+1}^{r-1}} + \ldots + \frac{1}{e_t^{r+1}} \right]. \qquad \ldots(25)$$

If $r = 0$, we get from (25) the first identity, noting $E(z^0) = 1$,

$$\sum_{t=1}^{k} \frac{(-1)^{k-1}}{e_t \prod_{\substack{j=1 \\ j \neq t}}^{k} (e_t - e_j)} = \frac{1}{(e_1\, e_2 \ldots e_k)}. \qquad \ldots(26)$$

From (22), now

$$E\left(z_{k+1}^r\right) = E(u_1 + \ldots + u_{k+1})^r = \sum \frac{r!}{i_1! \ldots i_{k+1}!} \left[\prod_{t=1}^{k+1} E\left(u_t^{i_t}\right) \right] \qquad \ldots(27)$$

$i_t = 0, 1, \ldots r, j = 1, \ldots, k+1$, and where sum Σ in (27) is on all combinations of i_j's such that $i_1 + \ldots + i_{k+1} = r$. Suppose u_t^0 represents the minimum in the sample

of size n_t from t-th population, that is, $u_t^0 = x_{(1)t}$, $t = 1, 2, \ldots, k+1$ then it is well known that for $\alpha = 1$,

$$f\left(u_t^0\right) = (n_t\ \theta_t)\ \exp\left[-\ n_t\ \theta_t\ u_t^0\right] \qquad \ldots(27a)$$

and

$$E\left(u_t^0\right) = 1/n_t\theta_t = 1/e_t. \qquad \ldots(27b)$$

So, from (2) and (27b), it follows, for $r = 1$,

$$E(z_0) = \sum_{i_1, \cdots, i_{k+1}} \left[\prod_{t=1}^{k+1} \left(1 \Big/ e_t^{i_t}\right)\right] \qquad \ldots(27c)$$

where

$$z_0 = u_1^0 + \ldots + u_{k+1}^0\ ,\ i_1 + i_2 + \ldots + i_{k+1} = 1.$$

Now, eqn (27c) is

$$E(z_0) = \sum_{t=1}^{k+1} (1/e_t). \qquad \ldots(27d)$$

If $r = 1$, from (25), (27) and (27d), we get

$$\left(\prod_{t=1}^{k} e_t\right) \sum_{t=1}^{k} \frac{(-1)^{k-1}}{\prod_{\substack{j=1 \\ j \neq t}}^{k} (e_t - e_j)} \left[\frac{1}{e_t^2} + \frac{1}{e_t\ e_{k+1}}\right] = \sum_{t=1}^{k+1} \frac{1}{e_t}. \qquad \ldots(28)$$

Using (26) in (28), we get another identity,

$$\left(\prod_{t=1}^{k} e_t\right) \sum_{t=1}^{k} \frac{(-1)^{k-1}}{\left[\prod_{\substack{j=1 \\ j \neq t}}^{k} (e_t - e_j)\right] \left(e_t^2\right)} = \sum_{t=1}^{k} \frac{1}{e_t}. \qquad \ldots(29)$$

If $r = 2$, we get from (25),

$$\left(\prod_{t=1}^{k} e_t\right) \sum_{t=1}^{k} \frac{(-1)^{k-1}}{\prod_{\substack{j=1 \\ j \neq t}}^{k} (e_t - e_j)} \left[\frac{1}{e_t^3} + \frac{1}{e_t^2\ e_{k+1}} + \frac{1}{e_t\ e_{k+1}^2}\right]$$

(equation continued on p. 754)

$$= \sum_{i_1,\ldots,i_{k+1}} \left[\prod_{t=1}^{k+1} \left(1 \Big/ e_t^{i_t} \right) \right]. \qquad \ldots(30)$$

In RHS of (30), $i_1 + \ldots + i_{k+1} = 2$.

Using (26) and (29) in (30), we get the third identity,

$$\left(\prod_{t=1}^{k} e_t \right) \sum_{t=1}^{k} \frac{(-1)^{k-1}}{e_t^3 \prod_{\substack{j=1 \\ j \neq t}}^{k} (e_t - e_j)} = \sum_{i_1,\ldots,i_k} \left[\prod_{t=1}^{k} \frac{1}{e_t^{i_t}} \right] \qquad \ldots(30a)$$

where

$$i_1 + \ldots + i_k = 2.$$

In general, we have, for a positive integer r,

$$\left(\prod_{t=1}^{k} e_t \right) \sum_{t=1}^{k} \frac{(-1)^{k-1}}{e_t^{r+1} \prod_{\substack{j=1 \\ j \neq t}}^{k} (e_t - e_j)} = \sum_{i_1,\ldots,i_k} \left(\prod_{t=1}^{k} \frac{1}{e_t^{i_t}} \right) \qquad \ldots(31)$$

where $i_1 + \ldots + i_k = r$ we can verify (31) easily for general r. Equations (26), (29), (30a), (31) can easily be verified with some real numbers $e_1, e_2, \ldots e_k$.

For example, if $k = 2$, $e_1 = 2$, $e_2 = 1$, then LHS of (31) is

$$2(-1)\left[\frac{1}{2^{r+1}} - 1 \right] = 2 - \frac{1}{2^r}. \qquad \ldots(32)$$

If $r = 2s - 1$, then RHS of (31) corresponding to tuples $(2s - 1, 0), (2s - 2, 1) \ldots, (s, s - 1)$, respectively, is

$$\left(\frac{1}{2^{2s-1}} + 1 \right) + \left(\frac{1}{2^{2s-2}} + \frac{1}{2} \right) + \ldots + \left(\frac{1}{2^s} + \frac{1}{2^{s-1}} \right)$$

$$= \frac{1}{2^s} \left(1 + \ldots + \frac{1}{2^{s-1}} \right) + \left(1 + \frac{1}{2} + \ldots + \frac{1}{2^{s-1}} \right)$$

$$= \frac{1}{2^{s-1}} \left(1 - \frac{1}{2^s} \right) + 2 \left(1 - \frac{1}{2^s} \right)$$

which is (32) for $r = 2s - 1$.

Similarly if $r = 2s$, then corresponding to the tuples, $(2s, 0)$, $(2s - 1, 1)$ ..., (s, s), we get RHS of (31) as

$$\left(\frac{1}{2^{2s}} + 1\right) + \left(\frac{1}{2^{2s-1}} + \frac{1}{2}\right) + \ldots + \left(\frac{1}{2^s}\right)$$

$$= \frac{1}{2^s}\left(1 + \ldots + \frac{1}{2^s}\right) + \left(1 + \frac{1}{2} + \ldots + \frac{1}{2^{s-1}}\right)$$

$$= \frac{1}{2^{s-1}}\left(1 + \frac{1}{2^{s-1}}\right) + 2\left(1 - \frac{1}{2^s}\right)$$

which is equal to (32) with $r = 2s$.

Comments : (1) Calculation of tail probabilities in (15) is slightly heavy, since two finite sums on i_t, s_t and one infinite sum on r_t have to be evaluated for k along with coefficients $A_s(r, \alpha)$. However on computers, it can easily be handled and further $A_s(r, \alpha)$ can easily be generated using the recurrence relation given in Lingappaiah[4]. (2) Once the tail probabilities are obtained, one can compare this probability for the case when $\theta_1 = \ldots = \theta_{k+1}$ to when at least one is not equal. The same thing can be applied to the case $\lambda_1 = \ldots = \lambda_{k+1}$. (3) Distribution of the random variable (sample size) is taken as Poisson given in (1) for simplicity. One could take, generalised Poisson or generalized negative binomial, and the like as given in Gupta and Gupta[3] or in Consul[2]. But analysis becomes more complex though the main development of this paper remains the same. (4) As the equality of parameters in $(k + 1)$ Poisson can be tested, similarly, if we take generalised Poisson, other parameters can also be dealt with. (5) In section 3, fixed sample size is dealt with and the same can easily be extended to variable sample size case too. However, the identities do not apply since they are based on fixed n_t's.

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