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# SOME GENERALIZATIONS OF COUNTABLE COMPACTNESS

D. N. SARKHEL

*Department of Mathematics, University of Kalyani, Kalyani 7412335, West Bengal*

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A class of spaces which includes the class of countably compact spaces and $H$-closed spaces is introduced. It is shown that these spaces share several inreresting properties of countably compact spaces, including pesudocompactness.

## 1. INTRODUCTION

Fleischman[3] introduced the notion of star compactness, which for Hausdorff spaces coincides with countable compactness. In this paper, we introduce the more general notions of $\alpha$-starcompactness for $\alpha = 1, 2, 3, \ldots, \omega$. Fleischman observed that the product of a starcompact space with a compact space may fail to be starcompact. We show that such a product is necessarily 1-starcompact, and in general the product of an $\alpha$-starcompact space with a compact space is always $\alpha$-starcompact.

We also explore the relations between compactness and $\alpha$-starcompactness and show, among other things, that the following implications hold, and also provide examples to show that these cannot be reversed even for Hausdorff spaces.

Starcompact $\Rightarrow$ 1-starcompact, $H$-closed $\Rightarrow$ 1-starcompact, countably compact $\alpha$-starcompact $\Rightarrow$ $(\alpha + 1)$-starcompact $\Rightarrow$ $\omega$-starcompact $\Rightarrow$ pseudocompact, $(\alpha < \omega)$.

## 2. NOTATIONS AND DEFINITIONS

Throught this paper, $X$ and $Y$ will denote arbitrary nonvoid topological spaces, and $\alpha$ a nonzero ordinal not exceeding the smallest transfinite ordinal, $\omega$.

Let $\mathscr{V}$ be a family of subsets of $X$. For $A \subset X$, we denote by $\mathscr{V}(A)$ (or, $\mathscr{V}(a)$ if $A = \{a\}$) the star of $A$ relative to $\mathscr{V}$, that is, the union of the members of $\mathscr{V}$ intersecting $A$. Then for $\alpha < \omega$ we define the $\alpha$-star, $\mathscr{V}(A ; \alpha)$, of a relative to $\mathscr{V}$ as follows : $\mathscr{V}(A ; 1) = \mathscr{V}(A)$ and $\mathscr{V}(A; \alpha + 1) = \mathscr{V}(\mathscr{V} A; \alpha))$ for $\alpha = 1, 2, 3 \ldots$.

Clearly $\mathscr{V}(A;\ \alpha)$ is the set of points $x \in X$ for which there are sets (not necessarily all distinct) $V_i \in \mathscr{V}$, $i = 1, 2, \ldots, \alpha$, such that $x \in V_\alpha$ and $V_{i-1} \cap V_i \neq \theta$ for ail $i$, where $V_0 = A$; in this case we say that $(A, V_1, V_3, \ldots, V_\alpha)$ is a 'chain' joining $A$ and $x$.

We now introduce the following definition.

*Definition* 1—The space $X$ is $\omega$-starcompact if, for [every open cover $\mathscr{V}$ of $X$, there exist a finite family $\{B_1, B_2, \ldots, B_n\} \subset \mathscr{V}$ and an $\alpha < \omega$ such that $X = \bigcup_{i=1}^{n} \mathscr{V}(B_i;\ \alpha)$. If the same $\alpha < \omega$ holds for every open cover of $X$, then $X$ is called $\alpha$-starcompact.

For completeness, we state here some known definitions (see Dugundji[1] or Engelking[2] for the terms not defined here).

*Definition* 2 (Fleischman[3])—The space $X$ is starcompact if, for every open cover $\mathscr{V}$ of $X$, there is a finite set $\{b_1, b_2, \ldots, b_n\} \subset X$ such that $X = \bigcup_{i=1}^{n} \mathscr{V}(b_i)$.

*Definition* 3—The space $X$ is

(i) compact (countably compact) if every [every countable] open cover of $X$ has a finite subcover,

(ii) $H$-closed or absolutely closed if every open cover of $X$ has a finite subfamily whose union is dense in $X$,

(iii) pseudocompact if every real valued continuous function on $X$ is bounded,

(iv) hypocompact of strongly paracompact ([paracompact], (metacompact or weakly paracompact]) if every open cover of $X$ has an open star-finite ([locally finite], [point-finite]) refinement.

## 3. Properties of $\alpha$-Starcompact Spaces

It is clear from the definitions that starcompact $\Rightarrow$ 1-starcompact $\Rightarrow$ $\alpha$-starcompact $\Rightarrow$ $(\alpha+1)$-starcompact $\Rightarrow$ $\omega$-starcompact, $(\alpha < \omega)$. Also, countable compactness implies starcompactness (Fleischman[3], Theorem 1, p. 2). These in conjunction with Theorems 1, 2 below prove the desired implications.

*Theorem* 1—An $H$-closed space is starcompact.

Proof : Obvious.

*Theorem* 2—An $\omega$-starcompact space is pseudocompact.

PROOF : Given a real valued continuous function $f$ on $X$, let $\mathscr{V}$ denote the family of nonvoid open sets $V \subset X$ such that

$$|f(x) - f(y)| < 1 \text{ for all } x, y \in V. \qquad \ldots(1)$$

Clearly $\mathscr{V}$ covers $X$. For any $V_0 \in \mathscr{V}$ and any $\alpha < \omega$, consider any $x \in \mathscr{V}(V_0; \alpha)$. There is a chain $(V_0, V_1, \ldots, V_\alpha)$ of members of $\mathscr{V}$ joining $V_0$ and $x$. Select $a_0 \in V_0$ and $a_i \in V_{i-1} \cap V_i$, $i = 1, 2, \ldots, \alpha$. Then $a_{i-1}, a_i \in V_{i-1}$ for $i = 1, 2, \ldots, \alpha + 1$, where $a_{\alpha+1} = x$. Since $|f(a_0) - f(x)| \leqslant \sum_{i-1}^{\alpha+1} |f(a_{i-1}) - f(a_i)|$, it follows from (1) that $f$ is bounded on $\mathscr{V}(V_0; \alpha)$. This evidentlyimplies that $f$ is bounded on $X$, if $X$ is $\omega$-starcompact.

Fleischman[3] proved that, for metacompact spaces, starcompactness is equivalent to compactness. We omit the somewhat similar proofs of the following analogous results. A stronger result is known[6] in completely regular spaces.

*Theorem* 3—The following conditions are equivalent :

(i) $X$ is compact;

(ii) $X$ is paracompact and $H$-closed;

(iii) $X$ is paracompact and $\alpha$-starcompact for some $\alpha < \omega$;

(iv) $X$ is hypocompact and $\omega$-starcompact.

*Theorem* 4—If $X$ is $\alpha$-starcompact [starcompact] and $f: X \to Y$ is continuous, then $f(X)$ is $\alpha$-starcompact [starcompact].

PROOF : We give the proof for $\omega$-starcompactness, from which the proofs in other cases will be clear.

Let $\mathscr{V}$ be any open cover of the subspace $f(X)$ of $Y$. Then the family $\mathscr{U} = \{f^{-1}(V) \mid V \in \mathscr{V}\}$ is an open cover of $X$. Since $X$ is $\omega$-starcompact, there is an $\alpha < \omega$ such that $x$ can be covered by finitely many $\alpha$-stars $\mathscr{U}(f^{-1}(V); \alpha) = V^*$ (say), $V \in \mathscr{V}$. It suffices therefore to show that $f(V^*) \subset (V; \alpha)$ for every $V \in \mathscr{V}$.

To this end, let $x \in V^*$. Then there are sets $V_1, V_2, \ldots V_\alpha \in \mathscr{V}$ such that $x \in f^{-1}(V_\alpha) \cap f^{-1}(V_{i-1}) \cap f^{-1}(V_i) \neq \phi$ for $i = 1, 2, \ldots, \alpha$, where $V_0 = V$. Then, $f(x) \in v_\alpha$ and $V_{i-1} \cap V_i \neq \phi$ for $i = 1, 2, \ldots, \alpha$. Hence $f(x) \in \mathscr{V}(V; \alpha)$, which in fact completes the proof.

*Corollary* 5—If $X = \Pi_t X_t$ and is $\alpha$-starcompact (starcompact) then each factor $X_t$ is $\alpha$-starcompact (starcompact).

*Theorem* 6—If $X$ is $\alpha$-starcompact (starcompact) and $y$ is compact, then $X \times Y$ is $\alpha$-starcompact (1-starcompact).

PROOF : We give the proof for the case $\alpha = \omega$. from which the proof for the case $\alpha < \omega$ will be clear. When $x$ is starcompact, then it is 1-starcompact and, hence, the corresponding result follows from the case $\alpha = 1$.

Now, let $\mathscr{U}$ be any opencover of $X \times Y$. For every $(x, y) \in X \times Y$, we select a member $U(x, y)$ of $\mathscr{U}$ and open sets $X(x, y)$ containing $x$ in $X$ and $Y(x, y)$ containing $y$ in $Y$, such that $X(x, y) \times Y(x, y) \subset U(x, y)$. For each $x_t \in X$, the sets $Y(x_t, y)$ for all $y \in Y$ from an open cover of the compact space $Y$, so, there are finitely many points $x_{tj} \in Y$, $j = 1, 2, \ldots, n(t)$, such that

$$Y = \bigcup_{j=1}^{n(t)} Y(x_t, x_{tj}). \qquad \ldots(1)$$

Let now $V(x_t)$ denote the open set $\bigcap_{j=1}^{n(t)} X(x_t, x_{tj})$ containing $x_t$ in $X$. Then $\mathscr{V} = \{V(x_t) \mid x_t \in X\}$ is an open cover of the $\omega$-starcompact space $X$. So, there are finitely many points $a_1, a_2, \ldots, a_m \in X$ and an $\alpha < \omega$, such that $X = \bigcup_{i=1}^{m} \mathscr{V}(V(a_i); \alpha)$.

Now, given $(x, y) \in X \times Y$, we have $x \in \mathscr{V}(V(a_i); \alpha)$ for some $i \leqslant m$. So, there are points $x_t \in X$, $t = 1, 2, \ldots, \alpha$, such that $x \in V(x_\alpha)$ and $V(x_{t-1}) \cap V(x_t) \neq \phi$ for $t = 1, 2, \ldots, \alpha$, where $x_0 = a_i$. Also, by (1), $y \in Y(x_t, x_{tj(t)})$ for some $j(t) \in \{1, 2, \ldots, n(t)\}$, $t = 0, 1, 2, \ldots, \sigma$. Noting that

$$V(x_t) \times Y(x_t, x_{tj(t)}) \subset X(x_t, x_{tj(t)}) \times Y(x_t, x_{tj(t)})$$

$$\subset U(x_t, x_{tj(t)}) = U_t \text{ (say)}$$

for $t = 0, 1, 2, \ldots, \alpha$, we have $(x, y) \in U_\alpha$ and $(b_t, y) \in U_{t-1} \cap U_t$ for $t = 1, 2, \ldots, \alpha$, where we select $b_t \in V(x_{t-1}) \cap V(x_t)$. Therefore, $(x, y) \in \mathscr{U}(U_0; \alpha)$ where $U_0 = U(x_0, x_{0j(0)}) = U(a_i, a_{ij(i)})$.

Thus $X \times Y$ is covered by the $\alpha$-stars of the sets $U(a_i, a_{ij}) \in \mathscr{U}$ relative to $\mathscr{U}$, $j = 1, 2, \ldots, n(i)$ and $i = 1, 2, \ldots, m$. Hence $X \times Y$ is $\omega$-starcompact.

## 4. SOME COUNTER EXAMPLES

*Example* 1—For any $\alpha < \omega$, we show an $\alpha$-starcompact Hausdorff space that is not starcompact (and, hence, not countably compact), nor even $(\alpha - 1)$-starcompact when $\alpha \geqslant 2$. Take a compact interval $I = [c, d]$ on the real line, and express it as the union of pairwise disjoint sets $A_1, A_2, \ldots, A_{2\alpha}$, each dense in $I$ with

$c, d \in A_{2\alpha}$. Let $E_n = A_n$ for $n = 1,3,5, \ldots, 2\alpha - 1$, and $E_n = A_{n-1} \cup A_n \cup A_{n+1}$ for $n = 2, 4, 6, \ldots, 2\alpha$, where $A_{2\alpha+1} = A_{2\alpha}$. Note that $E_i \cap E_j$ is dense in $I$ if $|i - j| = 0$ or 1, or if $i$ and $j$ are consecutive even integers; also, for every $x \in I$ there is a unique index $n(x) \leqslant 2\alpha$ such that $x \in A_{n(x)}$.

Now, let $X$ denote the set $I$ endowed with the topology $\mathcal{T}_\alpha(X)$ consisting of all subsets $G \subset I$ such that, for every $x \in G$, there is an open interval $I(x)$ satisfying $x \in I(x) \cap E_{n(x)} \subset G$. Then $X$ is clearly a Hausdorff space.

Let $\mathcal{V}$ be any open cover of $X$. For each $x \in I$ we selecr an open set $V(x) \in \mathcal{V}$ and an open interval $I(x)$ such that

$$x \in I(x) \cap E_{n(x)} \subset v(x) \text{ for all } x \in I. \qquad \ldots(1)$$

Clearly $I \subset \cup \{I(a) \mid a \in A\}$ for some finite subset $A \subset I$. Then, given $x \in X (= I)$, we have $x \in I(a)$ for some $a \in A$. We shall show that $x \in \mathcal{V}(V(a); \alpha)$, providing thereby that $X$ is $\alpha$-starcompact.

*Case I*: $|n(x) - n(a)| = 0$ or 1, or, $n(x)$ and $n(a)$ are consecutive even integers. Here, $E_{n(x)} \cap E_{n(a)}$ is dense in $I$. Since $x \in I(a) \cap I(x)$, it follows that $(E_{n(x)} \cap E_{n(a)}) \cap (I(a) \cap I(x)) \neq \theta$, whence by (1) $V(a) \cap V(x) \neq \theta$. So $x \in \mathcal{V}(V(a); \alpha)$.

*Case II*: not *Case I*. Here, there is at least one even integer between $n(a)$ and $n(x)$. Assume $n(a) < n(x)$; otherwise, just interchange $a$ and $x$ in what follows. Let $s$ be the first even integer greater than $n(a)$, and $t$ be the last even integer less than $n(x)$. Setting $x_{s-2} = a$ and $x_{t+2} = x$, we select successively the points

$$x_{i+2} \in A_{i+2} \cap I(x_i) \cap I(x_{i+2}), \quad i = s - 2, s, s + 2, \ldots, t - 2.$$

Then $x_{i+2} \in I(x_i) \cap I(x_{i+2})$. for $i = s - 2, s, s + 2, \ldots, t - 2$ and $x_i \in I(x_i) \cap I(x_{i+2})$. So, by denseness character of $E_i \cap E_j$ in $I$, for $i = s - 2, s, s + 2, \ldots, t$ we have

$$(E_{n(x_i)} \cap E_{n(x_{i+2})}) \cap (I(x_i) \cap I(x_{i+2})) \neq \theta$$

whence by (1) $V(x_i) \cap V(x_{i+2}) \neq \phi$. Also, $a \in V(a) = V(x_{s-2})$ and $x \in V(x) = V(x_{t+2})$. Since the number of the indices $s, s + 2, s + 4, \ldots, t + 2$ cannot exceed $(t + 2)/2 \leqslant (2\alpha)/2 = \alpha$, we conclude that $x \in \mathcal{V}(V(a); \alpha)$.

Next, taking $\alpha = 1$, we show that $X$ is not starcompact. Fix $a \in A_1$ and a strictly increasing sequence $\{a_n\} \subset A_1$ converging to $a$. Let $I_1 = (a, d]$, $I_2 = [c, a_1)$ and $I_n = (a_{n-2}, a_{n-1})$ for $n \geqslant 3$. Then $\mathcal{U} = \{A_1, I_1, I_2, I_3, \ldots\}$ is an open cover of $X$. Given any finite subset $B \subset X$, clearly there is an index $m$ such that $I_m \cap B = \phi$. Fix $x \in I_m \cap A_2$. Then $I_m$ is the only member of $\mathcal{U}$ containing $x$. Since $I_m \cap B = \phi$, we cannot have $x \in \mathcal{U}(B)$. Hence $X$ is not starcompact.

Finally, let $\alpha > 2$. Fix a strictly increasing sequence $\{c_n\} \subset A_{2\alpha}$ converging to $d$, with $c_1 = c$. Select $a_n \in I_n \cap A_1$ where $I_n = (c_n, c_{n+1})$, and set $J_n = (c_n, a_n) \cup (a_n, c_{n+1})$. Then the family $\mathscr{U}$ of the set $E_{2\alpha}$ together with the sets $I_n \cap A_1$ and $J_n \cap E_i$ $(n = 1, 2, \ldots; i = 1, 2, \ldots, 2\alpha - 1)$ is an open cover of $x$. Given anyfini te family $\mathscr{B} \subset \mathscr{U}$, clearly there is an index $m$ such that none of the sets $I_m \cap A_1$, $J_m \cap E_2$, $J_m \cap E_3$, ..., $J_m \cap E_{2\alpha-1}$ belongs to $\mathscr{B}$. We shall show that $a_m$ is not $\mathscr{U}(B; \alpha - 1)$ for any $B \in \mathscr{B}$, proving thereby that $x$ is not $(\alpha - 1)$ − starcompact.

Suppose $a_m \in \mathscr{U}(B; \alpha - 1)$ for some $B \in \mathscr{B}$. Then there are sets $B_1, B_2, B_p \in \mathscr{U}$, $p < \alpha$, with $a_m \in B_1$ and $B_p = B$, such that $B_i \cap B_{i+1} \neq \theta$ for $1 \leqslant i < p$. Deleting, if necessary, certain members of the chain which may be contained in some others, we may and do suppose that $B_i \not\subset B_j$ for $i \neq j$. Since $2\alpha \geqslant 4$, clearly the only member of $\mathscr{U}$ containing $a_m$ is $I_m \cap A_1$. So $B_1 = I_m \cap A_1 \notin \mathscr{D}$, and hence $B_1 \neq B_p$, so that $p \geqslant 2$. Then the only member of $\mathscr{U}$ which is not contained in $B_1$ but intersects $B_1$ is $J_m \cap E_2$. So $B_2 = J_m \cap D_2 \notin \mathscr{B}$, and hence $B_2 \neq B_p$, so that $\alpha \geqslant p \geqslant 3$. By definitions of $E_2$ and $E_3$, $J_m \cap E_3 \subset J_m \cap E_3 \subset J_m \cap E_2 = B_2$ and, hence, $J_m \cap E_3$ is not in the chain. Therefore, since $2^{\aleph} > 6$, the only member of $\mathscr{U}$ which is not contained in $B_1$ or $B_2$ but intersects $B_2$ is $J_m \cap E_4$. So $B_3 = J_m \cap E_4 \notin \mathscr{B}$, and hence $B_3 \neq B_p$, so that $\alpha \geqslant p > 4$. Proceeding thus, we get $B_p = J_m \cap E_{2p-2} \notin B$. But $B_p = B \in \mathscr{B}$. This contradiction proves the result.

*Example* 2—We show an $\omega$-starcompact Hausdorff space that is not $\alpha$-starcompact for any $\alpha < \omega$. Let $X$ be the union of the pairwise disjoint sets $\{0\}$, $X_1$, $X_2$, $X_3$ ..., where $X_k = [1/(2k + 1), 1/2k]$. Let each $X_k$ be endowed with the topology $\mathscr{T}_k(X_k)$ as defined in Example 1, and let $X$ be endowed with the topology consisting of all subsets $G \subset X$ such that $G \cap X_k$ is open in $X_k$ for each $k$ and, if $0 \in G$ then $X_k \subset G$ for all but finitely many $k$. Then $X$ is Hausdorff. Since, by Example 1, $X_{k+1}$ is not $k$-starcompact, $X$ cannot be $k$-starcompact. For, if $X$ were $k$-starcompact then, being an open-and-closed subspace of $X$, obviously $X_{k+1}$ would also be.

On the other hand, given any open cover $\mathscr{V}$ of $X$, there is a $V_0 \in \mathscr{V}$ with $0 \in V_0$. Also, there is an index $m$ such that $X_k \subset V_0$ for all $k > m$. But $\mathscr{V}_k = \{V \cap X_k \mid V \in \mathscr{V}\}$ is an open cover of $X_k$. Since $X_k$ is $k$-starcompact, there are finitely many members $V_{kj}$ $(j = 1, 2, \ldots, j(k))$ of $\mathscr{V}$ such that

$$X_k = \bigcup_{j=1}^{j(k)} \mathscr{V}_k(V_{kj} \cap X_k; k),$$

Since obviously $\mathscr{V}_k(V_{kj} \cap X_k; k) \subset \mathscr{V}(V_{kj}; m)$ for all $k \leqslant m$ and all $j \leqslant j(k)$ and since $\bigcup_{k>m} X_k \subset V_0 \subset \mathscr{V}(V_0; m)$, it follows that

$$X = \mathscr{V}(V_0; m) \cup \bigcup_{k=1}^{m} \bigcup_{j=1}^{j(k)} \mathscr{V}(V_{kj}; m).$$

Hence $X$ is $\omega$-starcompact, as we intended to show.

*Example* 3—We show an 1-starcompact Hausdorff space that is not $H$-closed. Let $\Omega$ denote the first uncountable ordinal. The ordinal space $[0, \Omega)$ with the order topology is in fact countably compact and Hausdorff (Dugundji[1]. Ex. 1. p 228). But, considering the open cover $\{[0, \beta] \mid \beta < \Omega\}$ we see at once that it is not $H$-closed.

*Example* 4—We show a pseudocompact Hausdorff space that is not $\omega$-starcompact. Thus, our notion of $\alpha$-starcompactness is distinct from the notion of *-compactness introduced by Koszela *et al.*[4], which is equivalent to pseudocompactness as defined here.

Modifying Example 1, let the compact interval $I = [c, d]$ be expressed as the union of a sequence of pairwise disjoint sets $A_1, A_2, A_3, \ldots$, each dense in $I$ with $c, d \in A_1$. Let $E_n = A_n$ if $n$ is odd and $E_n = A_{n-1} \cup A_n \cup A_{n+1}$ if $n$ is even. Let $X$ denote the set $I$ endowed with the topology consisting of all subsets $G \subset I$ such that, for every $x \in G$, there is an open interval $I(x)$ satisfying $x \in I(x) \cap E_{n(x)} \subset G$, where $n(x)$ denotes the unique index for which $x \in A_{n(x)}$. Then $X$ is clearly Hausdorff.

Let $f$ be a real valued continuous function on $X$. For every $x \in X$ there is an open interval $I(x)$ containing $x$, such that $|f(x_1) - f(x_2)| < 2^{-n(x)}$ for all $x_1, x_2, \in V(x) = I(x) \cap E_{n(x)}$. Then there is a finite subset $A \subset I$ such that $I \subset \cup \{I(a) \mid a \in A\}$. Consider any $a \in A$ and any $x \in X \cap I(a)$. If $V(x) \cap V(a) \neq \phi$, taking $y \in V(x) \cap V(a)$ we have $|f(a) - f(x)| \leqslant |f(a) - f(y)| + |f(y) - f(x)| < 2^{-n(a)} + 2^{-n(x)} \leqslant 1$. Otherwise, as shown in Case II of Example 1 there are finitely many points $x_1, x_2, \ldots, x_{k+1}$ $(k = k(a, x))$, with $n(x) \neq n(x_j)$ for $i \neq j$, such that $a \in V(x_1)$, $x \in V(x_{k+1})$ and $V(x_i) \cap V(x_{i+1}) \neq \phi$ for $i = 1, 2, \ldots, k$. Taking $y_1 = a$, $y_{k+2} = x$ and $y_{i+1} \in V(x_i) \cap V(x_{i+1})$, we have

$$|f(a) - f(x)| \leqslant \sum_{i=1}^{k+1} |f(y_i) - f(y_{i+1})| < \sum_{i=1}^{k+1} 2^{-n(x_i)} < 1.$$

Consequently $f$ is bounded on $X \cap I(a)$, for every $a \in A$. Hence $f$ is bounded on $X$, proving that $X$ is pseudocompact.

On the other hand, consider the open cover $\mathscr{V} = \{E_1, E_2, E_3, \ldots\}$ of $X$. Choose any index $m$, and set $B = \bigcup_{i=1}^{2m} E_i$. For any $n > m$, the shortest chain of members of $\mathscr{V}$ joining $B$ and $E_{2n}$ is $(B, E_{2m+2}, E_{2m+4}, \ldots E_{2n})$. Since $n$ can be chosen arbitrarily large, it follows that $X \neq \mathscr{V}(B; k)$, for all $k$. This evidently shows that $X$ is not $\omega$-starcompact.

*Example* 5—We show that the product of two $\alpha$-starcompact spaces need not be $\alpha$-starcompact. Indeed, Dugundji[1] (p. 245) has presented an example (essentially

due to Novák[5]) of countably compact completely regular Hausdorff space $K$ such that $K \times K$ is not even pseudocompact (see also, Engelking[2], p. 149).

*Example* 6—We show that a closed subspace of an $\alpha$-starcompact space need not be of the same type. Indeed, in the $\alpha$-starcompact space $X$ of Example 1, $A_2$ is a closed subspace, which in fact coincides with the non-compact metrizable subspace $A_2$ of the real line, and hence it is not even pseudocompact (cf. Dugundji[1], 3.9, p. 232).

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