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PROPAGATION OF A CRACK DUE TO MAGNETOELASTIC SHEAR WAVES IN A PERFECT ELASTIC CONDUCTOR

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In this paper the propagation of a crack due to magnetoelastic shear waves in an isotropic elastic perfect conductor is investigated. The stress intensity factor at the crack tip is determined for a constant loading which is assumed to propagate at a constant velocity. Following the method of Matczyński and Sokolowski, it has been shown that the stress intensity factor decreases as the length of the crack increases, the effect of the primary magnetic field being distinctly marked. The Wiener—Hopf technique has been used to solve this problem.

1. INTRODUCTION

The problem of the determination of the stress and strain fields in elastic solids containing cracks of finite dimensions has received considerable attention during recent years. A knowledge of the stress field in the vicinity of a crack is a prerequisite for the prediction of fracture and failure of the material. Elastic waves propagating in a solid are modified by the presence of cracks. The application of a sudden disturbance to the surface of an elastic body gives rise to elastic waves which encounter internal flaws such as cracks and a complicated pattern of diffracted waves is generated[1]. Achenbach[1] has investigated the conditions for crack propagation upon diffraction of an incident wave by a crack. Mal[2] has found the dynamic stress-intensity factor for a non-axisymetric loading of the penny-shaped crack. He has also discussed the interaction of elastic waves with a penny-shaped crack[3]. Sih and Loeber[4] have studied the torsional vibrations of an elastic solid containing a penny-shaped crack. They have also discussed the normal compression and radial shear wave scattering at a penny-shaped crack in an elastic solid[5]. The diffraction of an elastic wave by a spherical or circular cylinder inclusion has been investigated by several authors (see, in

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particular, Pao and Mow[6], Griffin and Miklowitz[7], Datta[8]). A recent review on scattering of elastic waves has been given by Datta[9]. Matczyński and Sokolowski[10] have discussed the propagation of cracks in an antiplane state of strains. Matczyński[11] has considered the motion of a crack in an antiplane state of strain of an elastic strip.

Recently, the propagation of magnetoelastic waves, with special attention to surface wave problems in linear magnetic bodies and for conductors, has been discussed by several authors. Without entering details of research work in this field we may mention the review papers of Maugin[12,13] and Viktorov[14], where many relevant references can be found. Among the most recent studies we mention that of Chattopadhyay and Maugin[15] concerning the diffraction of magnetoelastic shear waves by a rigid strip.

In the present paper, following the method presented by Matczyński and Sokolowski[10], we discuss the propagation of a crack due to magnetoelastic shear waves in a perfect electricity conductor (or a body which can be so assumed for all practical purposes). The stress intensity factor at the crack tips is determined for a constant loading which is assumed to propagate at a constant velocity. The mathematical problem is solved by using the Wiener-Hopf technique.

2. Formulation of the Problem

In a rectangular system $\{x_k;\ k = 1, 2, 3\}$ of coordinates let us consider the problem of an infinite strip of electrically conducting material of thickness $2h$ weakened in the middle point $x_2 = 0$ by a semi-infinite crack $x_1 < 0$. The upper and lower surfaces (i.e., $x_2 = \pm h$) are rigidly clamped. The surfaces of the crack are subjected to the action of forces $M_{23} = p(x_1)$. The crack and the load propagate at a constant speed s in the positive direction along the x_1-axis.

If u, v and w are the displacement components in the x_1, x_2 and x_3 directions, then for a propagation of a plane transverse wave in the positive x_1-direction polarized parallel to the plane we may take $u = v = 0$ and $w = w(x_1, x_2, t)$.

The electromagnetic field is governed by Maxwell's equations

$$\underset{\sim}{\nabla} \cdot \mathbf{B} = 0, \quad \underset{\sim}{\nabla} \times \mathbf{H} = \frac{1}{c}\mathbf{J}$$

$$\underset{\sim}{\nabla} \cdot \mathbf{E} = 0, \quad \underset{\sim}{\nabla} \times \mathbf{E} = -\frac{1}{c}\frac{\partial \mathbf{B}}{\partial t} \qquad \ldots(2.1)$$

with electromagnetic state equation

$$\mathbf{B} = \mu_e \mathbf{H} \qquad \ldots(2.2)$$

and Ohm's law

$$\mathbf{J} = \sigma \left(E + \frac{1}{c} \frac{\partial \mathbf{u}}{\partial t} \times \mathbf{B} \right) \qquad \ldots(2.3)$$

where **E** is the electric field, **J** is the conduction current, **u** is the elastic displacement vector and **H** is the magnetic field which included both the primary field $\mathbf{H}_0$ and the induced field **h**. The light velocity in vacuum is noted c; μ_e and σ are the magnetic permeability and conduction coefficient, respectively, for a linear isotropic body. In writing down these equations we have used the Lorentz-Heaviside system of electromagnetic units, neglected the displacement current anda ssumed a relative electric dielectric constant equal to one. In the absence of body forces of non-electromagnetic origin, the equations governing the propagation of small elastic disturbances in which both mechanical and electromagnetic fields are taken into account, are

$$e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}) \qquad \ldots(2.4)$$

$$\mathbf{u} = (u_i) = (u, v, w) \qquad \ldots(2.5)$$

$$\tau_{ij} = \lambda e_{kk} \delta_{ij} + 2\mu e_{ij} \qquad \ldots(2.6)$$

and

$$\rho \frac{\partial \mathbf{u}^2}{\partial t^2} = \operatorname{div} \underset{\sim}{\tau} + \frac{1}{c} \mathbf{J} \times \mathbf{B}. \qquad \ldots(2.7)$$

That is, the infinite medium in which the rigid strip is placed is considered to be linear and isotropic in both mechanical and magnetic properties with Lamé coefficients λ and μ. Joule heating is assumed sufficiently small so that the energy equation does not come into play. In view of eqn. (2.1), eqns. (2.2) through (2.7) reduce to the following equations :

$$\frac{\partial H_1}{\partial x_1} + \frac{\partial H_2}{\partial x_2} = 0 \qquad \ldots(2.8)$$

$$\frac{\partial H_1}{\partial t} = \eta \nabla^2 H_1 , \frac{\partial H_2}{\partial t_2} = \eta \nabla^2 H_2 \qquad \ldots(2.9)_{1-2}$$

$$\frac{\partial H_3}{\partial t} = \eta \nabla^2 H_3 + \frac{\partial}{\partial x_1} \left(H_1 \frac{\partial w}{\partial t} \right) + \frac{\partial}{\partial x_2} \left(H_2 \frac{\partial w}{\partial t} \right) \qquad \ldots(2.9)_3$$

$$\mu_e (\mathbf{H} \underset{\sim}{.} \nabla) H_1 - \mu_e \frac{\partial}{\partial x_1} (\mathbf{H}^2/2) = 0 \qquad \ldots(2.10)_1$$

$$\mu_e (\mathbf{H} \underset{\sim}{.} \nabla) H_2 - \mu_e \frac{\partial}{\partial x_2} (\mathbf{H}^2/2) = 0 \qquad \ldots(2.10)_2$$

$$\mu_e (\mathbf{H} \cdot \underset{\sim}{\nabla}) \mathbf{H}_3 + \mu \nabla^2 w = \rho \frac{\partial^2 w}{\partial t^2} \qquad \ldots(2.10)_3$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x_1^2} + \frac{\partial^2}{\partial x_2^2}, \eta = c^2/\mu_e \sigma .$$

Inside a perfect conductor $\eta = 0$ so that, with eqns. $(2.9)_{1-2}$ we see that x_1 and x_2-components of the magnetic field must be independent of time, and this implies that there will be no perturbations in the corresponding components of the magnetic field. Assuming the primary magnetic field to be uniform throughout the space, we can therefore write

$$H_1 = H_{01}, \; H_2 = H_{02}, \; H_3 = H_{03} + h_3 \qquad \ldots(2.11)$$

where h_3 is the only surviving component of the induced magnetic field. Equation (2.8) is satisfied and eqns. $(2.10)_{1-2}$, after use of eqn. (2.11), give

$$H_{03} \frac{\partial h_3}{\partial x_1} = H_{03} \frac{\partial h_3}{\partial x_2} = 0. \qquad \ldots(2.12)$$

From the above equations we infer that waves can be propagated only if the magnetic field is normal to their polarization direction. Now, if θ is the angle at which a wave crosses the primary manetic field, eqn. $(2.9)_3$ reduces to

$$\frac{\partial h_3}{\partial t} = \frac{\partial}{\partial t} \left(H_0 \cos \theta \frac{\partial w}{\partial t_1} + H_0 \sin \theta \frac{\partial w}{\partial x_2} \right) \qquad \ldots(2.13)$$

where H_0 is the magnitude of the primary magnetic field. Integrating in time the above equation, we get

$$h_0 = H_0 \left(\cos \theta \frac{\partial w}{\partial x_1} + \sin \theta \frac{\partial w}{\partial x_2} \right) \qquad \ldots(214)$$

if $h_3 = 0$ initially.

Using the above result, eqn. $(2.10)_3$ takes on the form

$$\rho \frac{\partial^2 w}{\partial t^2} = \mu \nabla^2 w + \rho c_A^2 \frac{\partial^2 w}{\partial x_2^2} \qquad \ldots(2.15)$$

where we have defined the squared Alfvén speed by

$$c_A^2 = \mu_e H_0^2 / \rho. \qquad \ldots(2.16a)$$

Now, the surface tractions at a mechanically free boundary of unit normal $\underset{\sim}{\nu}$ are

$$[[M_{ij}\,\nu_j]] = [[m_{ij} + \tau_{ij}]]\,\nu_j = 0 \qquad \ldots(2.16b)$$

where m_{ij} is the Maxwellian stress tensor in the present theory and the symbolism [[..]] indicates the jump of its enclosure at the surface. In general, we have Dunkin and Eringen[16]

$$m_{ij} = \mu_e \left(H_i\,H_j - \frac{1}{2}\,\mathbf{H}^2\,\delta_{ij} \right). \qquad \ldots(2.17)$$

After linearization, this yields

$$m_{ij} = \mu_e\,[H_{0i}\,h_j + H_{0j}\,h_i - (\mathbf{H}_0\,\mathbf{h})\,\delta_{ij}]. \qquad \ldots(2.18)$$

With the help of eqn. (2.14), the above equation becomes

$$m_{23} = \mu_e\,H_0^2\,\sin\theta\,\left(\cos\theta\,\frac{\partial w}{\partial x_1} + \sin\theta\,\frac{\partial w}{\partial x_2}\right). \qquad \ldots(2.19)$$

Now let us replace the system (x_1, x_2, x_3) by the convective system (x, y, z) according to the Galilean transformation law

$$x_1 = x + st\,,\; x_2 = y\,,\; x_3 = z \qquad \ldots(2.20)$$

where s is the velocity of the motion of the system (x, y, z).

For the sake of simplicity we shall assume that the primary magnetic field is set normal to the direction of propagation. Therefore, $\theta = \pi/2$. Now, substituting $\theta = \pi/2$ and the transformation (2.20) into eqns. (2.15) and (2.16) we arrive at

$$\beta^2\,\frac{\partial^2 w}{\partial x^2} + (1 + \epsilon_H)\,\frac{\partial^2 w}{\partial y^2} = 0 \qquad \ldots(2.21)$$

and

$$M_{yz} = \rho c_T^{-2}\,\frac{\partial w}{\partial y} \qquad \ldots(2.22)$$

where we have set

$$\beta^2 = 1 - \left(s^2/c_T^2\right),\; \epsilon_H = c_A^2/c_T^2$$

$$c_T^2 = \mu/\delta,\; c_T^{-2} = c_T^2\,(1 + \epsilon_H).$$

Equations (2.21) will be solved under the following boundary conditions :

$$w(x, y) = 0 \text{ for } |x| < \infty, y = h \qquad \ldots(2.23)_1$$

$$w(x, y) = 0 \text{ for } x > 0, y = 0 \qquad \ldots(2.23)_2$$

$$M_{yz}(x, y) = p(x) \text{ for } x < 0, y = 0. \qquad \ldots(2.23)_3$$

3. Solution of the Problem

We shall use the two-sided Fourier integral transform defined by

$$F(\alpha, y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} f(x, y)\, e^{i\alpha x}\, dx, \; f(x, y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty+i\gamma}^{+\infty+i\gamma} F(\alpha, y)\, e^{-i\alpha x}\, d\alpha \qquad \ldots(3.1)$$

where $i\gamma$ is the imaginary number lying in the common strip of regularity of all transforms appearing in the solution and α is the transform parameter which is complex. We shall also use the following representation by Titchmark as[7]

$$F(\alpha, y) = F^-(\alpha, y) + F^+(\alpha, y) \qquad \ldots(3.2)$$

where

$$F^-(\alpha, y) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} f(x, y)\, e^{i\alpha x}\, dx \qquad \ldots(3.3)$$

$$F^+(\alpha, y) = \frac{1}{\sqrt{2\pi}} \int_{0}^{\infty} f(x, y)\, e^{i\alpha x}\, dx$$

are regular functions in the respective half planes Im $\alpha < \gamma_2$ and Im $\alpha < \gamma_1$. Applying the Fourier integral transform (3.3) to eqns. (2.21) and (2.22), we obtain

$$\frac{d^2 w(\alpha, y)}{dy^2} - \alpha^2 \beta_1^2 \, w(\alpha, y) = 0 \qquad \ldots(3.4)$$

and

$$M_{yz}(\alpha, y) = \rho c_T^{-2} \frac{dw(\alpha, y)}{dy} \qquad \ldots(3.5)$$

where

$$\beta_1^2 = \beta^2/(1 + \epsilon_H).$$

The solution of eqn. (3.4) is

$$W(\alpha, y) = A(\alpha) \sinh(\alpha\beta_1 y) + B(\alpha) \cosh(\alpha\beta_1 y) \quad \ldots(3.6)$$

where $A(\alpha)$ and $B(\alpha)$ are unknown functions. Substituting from eqn. (3.6) into eqn. (3.5), we have

$$M_{yz}(\alpha, y) = \rho c_T^{-2} \alpha\beta_1 [A(\alpha) \cosh(\alpha\beta_1 y) + B(\alpha) \sinh(\alpha\beta_1 y)]. \quad \ldots(3.7)$$

Now, applying the Fourier integral transform (3.1) to the three boundary conditions (2.23) and using eqns. (3.6) and (3.7) we have the following Wiener-Hopf equation [compare Matcyński and Sokolowski[10], p. 971]

$$W^-(\alpha, 0) = -\frac{1}{\rho c_T^{-2} \beta_1} \frac{\tanh \alpha\beta_1 h}{\alpha} \left[M_{yz}^+(\alpha) + P(\alpha) \right] \quad \ldots(3.8)$$

where

$$p(\alpha) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{0} p(x) e^{i\alpha x} dx. \quad \ldots(3.9)$$

We have to solve the above eqn. (3.8) by the method of factorization following Sokolowski[18] and Matczyński[11]. The region of existenee of eqn. (3.8) is

$$-\frac{\pi}{2\beta_1 h} < -\epsilon < \operatorname{Im} \alpha < 0.$$

The kernel of eqn. (3.8) is

$$\frac{\tanh \alpha\beta_1 h}{\alpha} = H(\alpha)$$

which can be written in terms of Γ-functions as

$$H(\alpha) = \frac{\beta_1 h}{\pi} \frac{\Gamma\left(\frac{1}{2} - \frac{i\alpha\beta_1 h}{\pi}\right) \Gamma\left(\frac{1}{2} + \frac{i\alpha\beta_1 h}{\pi}\right)}{\Gamma\left(1 - \frac{i\alpha\beta_1 h}{\pi}\right) \Gamma\left(1 + \frac{i\alpha\beta_1 h}{\pi}\right)}$$

$$= \frac{\beta_1 h}{\pi} H^-(\alpha) H^+(\alpha) \quad \ldots(3.10)_1$$

where

$$H^{-}(\alpha) = \frac{\Gamma\left(\frac{1}{2} - \frac{i\alpha\beta_1 h}{\pi}\right)}{\Gamma\left(1 - \frac{i\alpha\beta_1 h}{\pi}\right)}, \quad H^{+}(-\alpha) = H^{-}(\alpha). \qquad \ldots(3.10)_2$$

The functions

$$\left\{\Gamma\left(1 + \frac{i\alpha\beta_1 h}{\pi}\right), \Gamma\left(\frac{1}{2} + \frac{i\alpha\beta_1 h}{\pi}\right)\right\}$$

and

$$\left\{\Gamma\left(1 - \frac{i\alpha\beta_1 h}{\pi}\right), \Gamma\left(\frac{1}{2} - \frac{i\alpha\beta_1 h}{\pi}\right)\right\}$$

are regular and non zero in the half planes $\mathrm{Im}\ \alpha < \pi/2\beta_1 h$ and $\mathrm{Im}\ \alpha > -\pi/\beta_1 h$. Using eqn. $(3.10)_1$ the Winer-Hopf equation (3.8) takes on the form

$$-\frac{\pi\rho\, c_T^{-2}}{h}\frac{\omega^{-}(\alpha)}{H^{-}(\alpha)} = H^{+}(\alpha)\, M_{yz}^{+} + E(\alpha) \qquad \ldots(3.11)$$

where

$$E(\alpha) = H^{+}(\alpha)\, p(\alpha) \qquad \ldots(3.12)$$

has to be decomposed into the expressions regular in the corresponding half planes[11] $\mathrm{Im}\ \alpha > -\epsilon$, $\mathrm{Im}\ \alpha < 0$, i.e.,

$$E(\alpha) = E^{+}(\alpha) - E^{-}(\alpha) \qquad \ldots(3.13)$$

where

$$E^{+}(\alpha) = \frac{1}{2i\pi}\int_{-\infty - i\varepsilon_1}^{\infty - i\varepsilon_1} \frac{E(\rho)}{\rho - \alpha}\, d\rho \qquad \ldots(3.14)_1$$

and

$$E^{-}(\alpha) = \frac{1}{2i\pi}\int_{-\infty + i\varepsilon_2}^{\infty + i\varepsilon_2} \frac{E(\rho)}{\rho - \alpha}\, d\rho \qquad \ldots(3.14)_2$$

in which $-\delta_1 < \text{Im } \alpha < \delta_2$ and $-i\delta_1$, $i\delta_2$ lie within the required strip of regularity and $E^+(\alpha)$, $E^-(\alpha)$ are regular in the respective half planes $\text{Im } \alpha > -\pi/2\beta_1 h$ and $\text{Im } \alpha < 0$. After use of the relation (3.13), eqn. (3.14) takes the form

$$-\frac{\pi\rho c_T^{-2}}{h}\frac{\omega^-(\alpha)}{H^-(\alpha)} + E^-(\alpha) = H^+(\alpha)\, M_{yz}^+(\alpha) + E^+(\alpha). \qquad \ldots(3.15)$$

Since in eqn. (3.15) both sides represent regular and non-zero functions in the respective half planes $\text{Im } \alpha > -\pi/2\beta_1 h$ and $\text{Im } \alpha < 0$, by the Liouville theorem we have

$$W^-(\alpha) = \frac{h}{\pi\rho c_T^{-2}} H^-(\alpha) E^-(\alpha) \qquad \ldots(3.16)$$

regular for $\text{Im } \alpha < 0$, and

$$M_{yz}^+(\alpha) = -E^+(\alpha)/H^+(\alpha) \qquad \ldots(3.17)$$

regular for $\text{Im } \alpha > -\pi/2\beta_1 h$. The functions $E^+(\alpha)$ and $E^-(\alpha)$ are assumed to be analytic functions for $-(\pi/2\beta_1 h) < \text{Im } \alpha < 0$, so that they may be represented in the form

$$E^{\pm}(\alpha) = -\frac{1}{\alpha}\left[I - \frac{1}{2i\pi}\int_{-\infty-i\delta}^{\infty-i\delta}\frac{\rho E(\rho)}{\rho-\alpha}\,d\rho\right] \qquad \ldots(3.18)$$

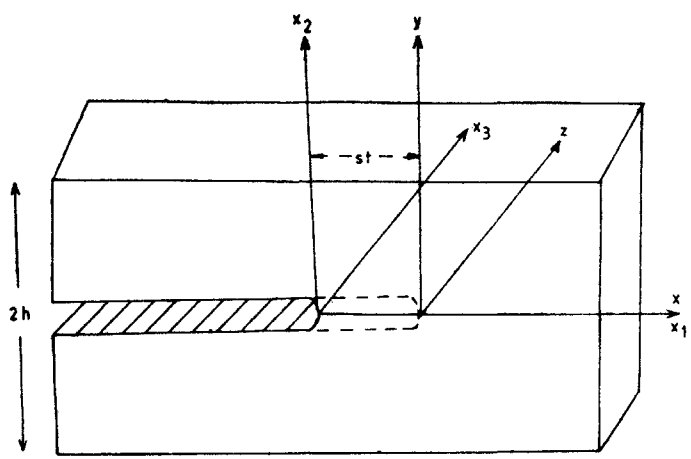

FIG. 1.

where

$$I = \frac{1}{2i\pi} \int_{-\infty - i\delta}^{+\infty - i\delta} E(\rho)\, d\rho \ , \ \delta_1 < \delta < \delta_2. \qquad \ldots(3.19)$$

Using

$$\frac{\Gamma(\alpha + K)}{\Gamma(\alpha)} \simeq \alpha^k, \ | \alpha | \to \infty \qquad \ldots(3.20)$$

and the properties of $E^+(\alpha)$, functions $W^-(\alpha)$ and $M^+_{yz}(\alpha)$ defined by eqns. (3.16) and (3.17) become

$$W^-(\alpha) = - \frac{I}{\rho\, c_T^{-2}\, \alpha} \sqrt{\frac{h}{i\pi\beta_1\alpha}} \qquad \ldots(3.21)$$

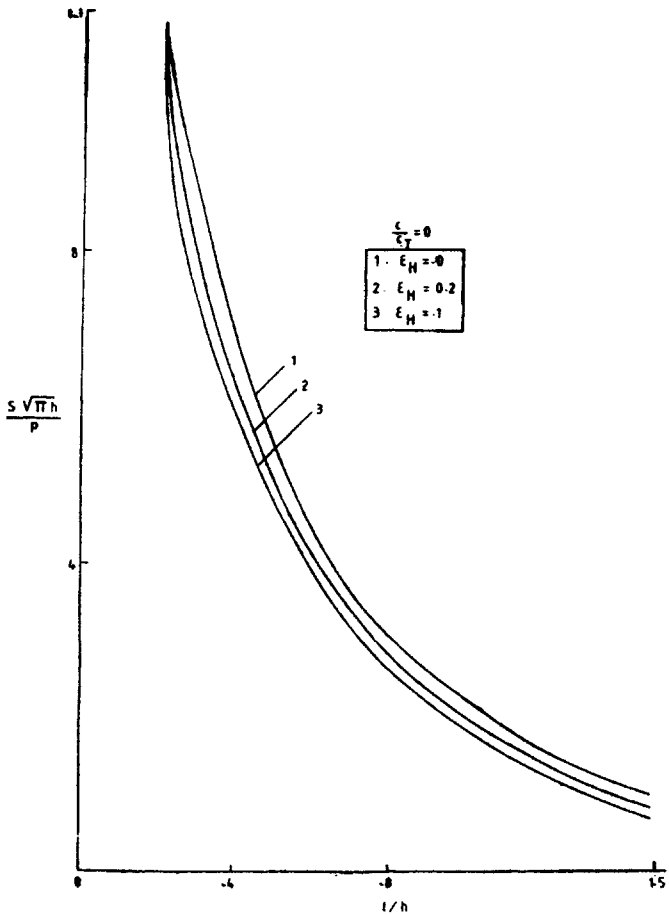

FIG. 2.

and

$$M_{yz}^{+}(\alpha) = I\sqrt{\frac{-i\beta_1 h}{\pi\alpha}}. \qquad \ldots(3.22)$$

The above two equations (3.21) and (3.22), by means of Abel's theorem, become

$$\omega(x) = \frac{2S}{\rho\beta_1 c_T^2}\sqrt{-x} \quad \text{for } x \to -\epsilon,\ \epsilon \to 0 \qquad \ldots(3.23)$$

and

$$M_{yz}(x) = \frac{S}{\sqrt{x}} \qquad \text{for } x \to +\epsilon,\ \epsilon \to 0 \qquad \ldots(3.24)$$

where S is called the stress intensity factor defined by

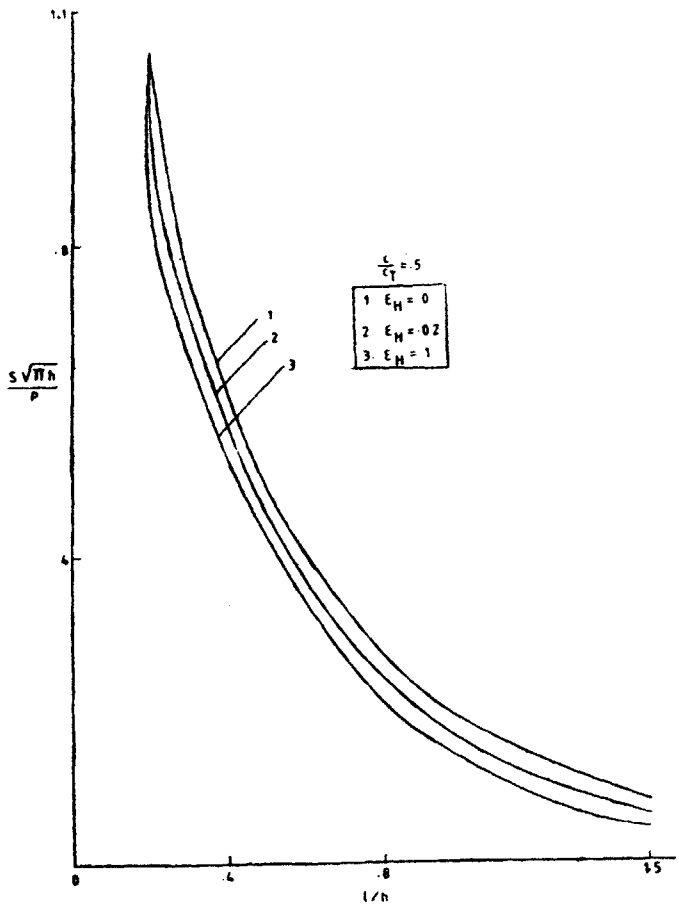

FIG. 3.

$$S = -i\, I\, (2\beta_1 h/\pi)^{1/2}. \qquad \ldots(3.25)$$

This quantity is the most interesting one from the point of view of crack stability.

4. Numerical Calculation and Discussion

In this section we consider that the edges $y = 0$ of the crack are loaded at $x = -l$ by a force of intensity P which is constant. Let

$$p(x) = p\,\delta(x + l). \qquad \ldots(4.1)$$

Substituting from this into eqn. (3.9) we have

$$p(\alpha) = \delta\, \frac{P}{\sqrt{2\pi}}\, e^{-i\alpha l}. \qquad \ldots(4.2)$$

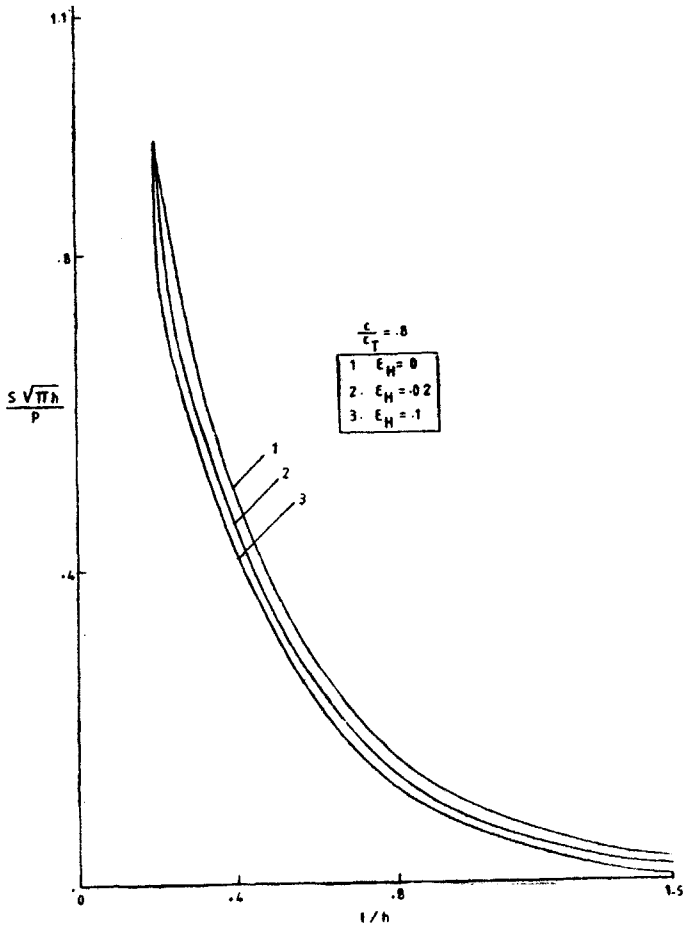

Fig. 4.

Using now eqns. (3.10), (3.12), (3.19) and (4.1), we obtain

$$S = -\frac{P}{\sqrt{\beta_1 h}}\, e^{(-\pi l/2\beta_1 h)}\, \frac{1}{2i\pi} \int_{\epsilon-i\infty}^{\epsilon+i\infty} \frac{\Gamma(p)}{\Gamma\left(p+\frac{1}{2}\right)}\, e^{(\pi i p/\beta_1 h)}\, dp \qquad \ldots(4.3)$$

where $0 < \epsilon < \frac{1}{2}$. Integrating we finally obtain

$$S = -\frac{P}{\sqrt{\pi\beta_1 h\left[\exp\left(\frac{\pi l}{h\beta_1}\right)-1\right]}}. \qquad \ldots(4.4)$$

We have calculated $S\sqrt{\pi h}/P$, i.e., the stress intensity factor, against (l/h) for different values of c/c_T (0, 5, .8) and ϵ_H (0, .02, .1) and drawn the corresponding curves (Fig. 2-4). From these graphs it is clear that the stress intensity factor decreases as (l/h) increases and the same nature follows for $\epsilon_H \neq 0$.

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