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THE DISTRIBUTION OF THERMAL STRESS IN A BONDED CYLINDER WITH AN EXTERNAL CRACK

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Thermoelastic equilibrium of an infinite elastic isotropic cylinder which is embedded with isotropic infinite medium having an axisymmetric external crack is investigated. It is assumed that the thermal conditions on the upper surface of the crack are identical with those on the lower surface and the free boundary of the solids are kept at zero temperature. The plane of the crack is perpendicular to the axis of the cylinder. Mixed boundary conditions are applied with the help of suitable harmonic and biharmonic functions which satisfy the equation of equilibrium. By the theory of dual integral equations the problem is reduced to a set of Fredholm integral equation of the second kind. These equations are solved numerically. Expressions for various quantities of physical interest are derived.

INTRODUCTION

In recent years linear elastic solutions of crack problems have been given by many research workers because of their importance in the theory of fracture mechanics[1-3]. But there have been relatively few studies in axisymmetric problems of an emmbedded elastic zone. Earlier, we have investigated axisymmetric problem of external crack in a bonded cylinder and a problem of rigid disc situated symmetrically at the centre of an embedded elastic cylinder[4,5]. Here we have considered thermal stresses in an infinite isotropic solid bonding an infinite cylinder with an external crack covering outside of a circle $r \geqslant a, z = 0$. The temperature is prescribed on the faces on the faces of the external crack. The present problem is the thermal part of our problem considered earlier[4]. The axis of the cylinder is perpendicular to the plane of the crack.

It is assumed that all other faces of the solids are kept at zero temperature while the temperature is prescribed on the faces of the external crack. The external crack is situated in a plane normal to the axis of the cylinder occupying the region $z = 0, r \geqslant a$ where a is greater than the radius of the cylinder. There are many ways of solving the equations of thermoelastic equilibrium but we here adopt the method employed by Fox[6] and more recently by Shail[7].

In this method a harmonic function $\Omega\ (r, z)$ is used to represent the temperature T and the components of displacement Vector u, w by the relation.

$$u = 0, w = \frac{\Omega\ (r, z)}{\mu} \qquad \ldots(1.1)$$

where μ denotes shear modulus. This is a particular solution of thermoelastic equilibrium equation. By adding it to the solution for the isothermal problem given by Sneddon[8]

$$u = -\frac{1}{2\mu}\frac{\partial^2 \chi}{\partial R \partial z}, \quad w = \frac{1}{2\mu}\left[2\ (1 - \eta)\ \nabla^2 \chi - \frac{\partial^2 \chi}{\partial z^2}\right] \qquad \ldots(1.2)$$

where χ is a biharmonic function and η denotes Poisson's ratio. A solution of this problem is obtained and the geometry of the problem is given in Fig. 1.

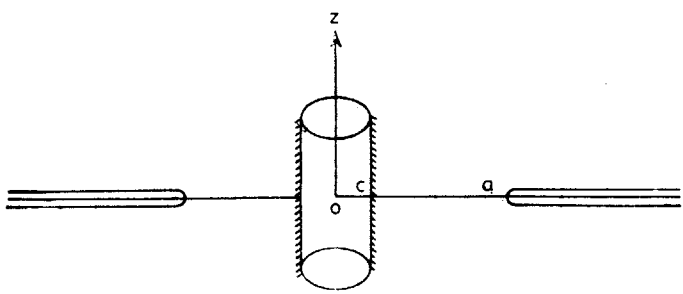

FIG. 1. Bonded cylinder with an external crack.

2. FORMULATION OF THE PROBLEM

In axisymmetric case the displacement vector u assumes the form $(u, 0, w)$ in cylindrical polar coordinate system (r, ϑ, z). The stress in the medium is caused by a prescribed temperature to the faces of flat external crack. The temperature distribution is however known. The boundary conditions for the present problem are as follows.

$$r = c, z > 0, t = 0$$

$$w^1\ (c, z) = w^2\ (c, z) \qquad \ldots(2.1)$$

$$u^1(c, z) = u^2(c, z) \qquad \ldots(2.2)$$

$$\sigma^1_{rr}(c, z) = \sigma^2_{rr}(c, z) \qquad \ldots(2.3)$$

$$\sigma^1_{rz}(c, z) = \sigma^2_{rz}(c, z) \qquad \ldots(2.4)$$

where c is the radius of the cylinder while on the plane of the crack $z = 0$,

$$\sigma^2_{zz} = -2\mu\, p(r) \qquad \ldots(2.5)$$

$$w^2 = 0 \quad c < r < a \qquad \ldots(2.6)$$

$$\sigma^2_{rz} = 0 \quad c < r < \infty \qquad \ldots(2.7)$$

where superscripts 1, 2, signify the two region L_1 ($0 \leqslant |r| \leqslant |c|$) and L_2 ($r \geqslant |c|, z \geqslant 0$). In case of isothermal condition we opt a set of biharmonic function as follows :

$$\phi = -2\mu_1 \int_0^\infty \lambda^{-2} [\{A(\lambda) + 4(1 - \eta_1) B(\lambda)\} I_0(\lambda r) - \lambda r\, B(\lambda)\, I_1(\lambda r)] \sin \lambda z\, d\lambda, \quad -\infty < z < \infty, \quad 0 \leqslant r \leqslant c \qquad \ldots(2.8)$$

$$\psi = -2\mu_2 \int_0^\infty \lambda^{-2} [\{C(\lambda) - 4(1 - \eta_2) D(\lambda)\} K_0(\lambda r) - \lambda r\, D(\lambda)\, K_1(\lambda r)] \sin \lambda z\, d\lambda - 2\mu_2 \int_0^\infty \lambda^{-3} F(\lambda)(2\eta_2 + \lambda z)\, e^{-\lambda z} J_0(\lambda r)\, d\lambda; \quad c \leqslant r \leqslant \infty, \quad -\infty < z < \infty. \qquad \ldots(2.9)$$

In consideration (1.1) for the particular solution of $\bar{u}$ we take;

$$\Omega(R, z) = 2\mu_2 \int_0^\infty t(\lambda)\, e^{-\lambda z}\, \frac{J_0(\lambda r)\, y_1(\lambda) - y_0(\lambda r)\, J_1(\lambda)}{J_1^2(\lambda) + y_1^2(\lambda)}\, d\lambda \qquad \ldots(2.10)$$

$t(\lambda)$ is related to the temperature $T(r, 0)$ on the plane of the crack by the relation

$$t(\lambda) = -\alpha(1+\eta_2)\mu_2 \int_c^\infty r\,T(r,0)\,[J_0(\lambda r)\,y_1(\lambda) - y_0(\lambda r)\,J_1(\lambda)]\,d\lambda \qquad \ldots(2.11)$$

where α is the coefficient of thermal expansion.

For the isothermal complementary solution we take $\psi(r, z)$ to be the biharmonic function expressed by the relation

$$\psi(r,z) = -2\mu^2(1-2\eta^2)\int_0^\infty \lambda^{-2}(C(\lambda)+\lambda z\,D(\lambda))\,e^{-\lambda z}\,\frac{J_0(\lambda r)\,y_1(\lambda) - y_0(\lambda r)\,J_1(\lambda)}{J_1^2(\lambda)+y_1^2(\lambda)}\,d\lambda. \qquad \ldots(2.12)$$

3. Solution to the Equilibrium Equation

With the help of (2.10), (2.11), (2.12) and (1.1) we have

$$u = \int_0^\infty [C(\lambda)+\lambda z\,D(\lambda) - D(\lambda)]\,\frac{J_1(\lambda r)\,y_1(\lambda) - y_1(\lambda r)\,J_1(\lambda)}{J_1^2(\lambda)+y_1^2(\lambda)}\,e^{-\lambda z}\,d\lambda \qquad \ldots(3.1)$$

$$w = \int_0^\infty [2(1-2\eta_2)\,D(\lambda) + C(\lambda) + \lambda z\,D(\lambda) + 2t(\lambda)]\,[J_0(\lambda r)\,y_1(\lambda) - y_0(\lambda r)\,J_1(\lambda)]\,(J_1^2(\lambda)+y_1^2(\lambda))^{-1}\,e^{\lambda z}\,d\lambda \qquad \ldots(3.2)$$

$$\sigma_{rr} = -2\mu\int_0^\infty \lambda\,[\{(1+2\eta_2)\,D(\lambda) - \lambda z\,D(\lambda) - A(\lambda) - t(\lambda)\}\,\{J_0(\lambda r)\,y_1(\lambda) - y_0(\lambda r)\,J_1(\lambda)\} + (\lambda r)^{-1}\{C(\lambda)+\lambda z\,D(\lambda) - D(\lambda)\}\{J_1(\lambda r)\,y_1)\,\lambda - y_1(\lambda r)\,J_1(\lambda)\}]\,(J_1^2(\lambda)+y_1^2(\lambda))^{-1}\,e^{-\lambda z}\,d\lambda \qquad \ldots(3.3)$$

$$\sigma_{\theta\theta} = -2\mu\int_0^\infty \lambda\,e^{\lambda z}\{2\eta_2\,D(\lambda) - t(\lambda)\}\,\frac{\{J_0(\lambda r)\,y_1(\lambda) - J_1(\lambda r)\,y_0(\lambda r)\}}{J_1^2(\lambda)+y_1^2(\lambda)}\,d\lambda$$

(equation continued on p. 860)

$$+ \frac{2\mu}{r}\int_0^\infty \lambda\, e^{-\lambda z}\{C(\lambda)+\lambda z\, D(\lambda)-D(\lambda)\}\,\frac{\{J_1(\lambda r)\, y_1(\lambda)-y_1(\lambda r)\, J_1(\lambda)\}}{J_1^2(\lambda)+y_1^2(\lambda)}d\lambda \quad ..(3.4)$$

$$\sigma_{zz} = -2\mu\int_0^\infty \lambda\, e^{-\lambda z}\{(1-2\eta_2)\, D(\lambda)+C(\lambda)+\lambda z\, D(\lambda)+t(\lambda)\}$$

$$\{J_0(\lambda r)+y_1(\lambda)-y_0(\lambda r)\times J_1(\lambda)\}\,(J_1^2(\lambda)+y_1^2(\lambda))^{-1}\,d\lambda \quad ...(3.5)$$

$$\sigma_{rz} = 2\mu\int_0^\infty \lambda\,\{C(\lambda)-2\eta_2\, D(\lambda)+\lambda z\, D(\lambda)+t(\lambda)\}\,\{J_1(\lambda r)\, y_1(\lambda)$$

$$-y_1(\lambda r)\, J_1(\lambda)\}\,(J_1^2(\lambda)+y_1^2(\lambda)^{-1}\, e^{-\lambda z}\,d\lambda \quad ...(3.6)$$

where J_l and Y_l are modified Bessel's function. It is clear from (3.2) and (3.6) that (2.6) and (2.7) will be satisfied if we choose

$$C(\lambda) = 2\eta_2\, D(\lambda) - t(\lambda). \quad ...(3.7)$$

The mixed boundary conditions (2.5) and (2.7) lead to dual integral equation

$$\int_0^\infty \lambda\,\{(1-2\eta_2)\, D(\lambda)+A(\lambda)+t(\lambda)\}\,\frac{J_0(\lambda r)\, y_1(\lambda)-y_0(\lambda r)\, J_1(\lambda)}{J_1^2(\lambda)+y_1^2(\lambda)}\,d\lambda$$

$$= p(r),\ a < r < \infty \quad ...(3.8)$$

$$\int_0^\infty \{2(1-2\eta_2)\, D(\lambda)+C(\lambda)+2t(\lambda)\}\,\frac{J_1(\lambda r)\, y_1(\lambda)-y_0(\lambda r)\, J_1(\lambda)}{J_1^2(\lambda)+y_1^2(\lambda)}\,d\lambda$$

$$= 0,\ c \leqslant r \leqslant a \quad ...(3.9)$$

We will deal this pair of dual integral equation through the method adopted by Shrivastava and Narain[9]. It is found that

$$D(\lambda) = \int_a^\infty g(u)\,\{y_1(\lambda)\cos\lambda u - J_1(\lambda)\sin\lambda u\}\,du - \frac{t(\lambda)}{2(1-\eta_2)}. \quad ...(3.10)$$

Where the function $g(u)$ is determined from the Fredholm integral equation

$$g(t) - \int_a^\infty g(u)\, K(t,u)\,du = \frac{2}{\pi}\int_t^\infty \frac{r\, p(r)\, dr}{\sqrt{r^2-t^2}}$$

(equation continued on p. 861)

$$+ \frac{2}{\pi} \int_0^\infty \frac{t(\lambda)}{2(1-\eta^2)} \frac{J_1(\lambda)\cos\lambda t + y_1(\lambda)\sin\lambda t}{J_1^2(\lambda) + y_1^2(\lambda)} \frac{d\lambda}{\lambda} \quad \ldots(3.11)$$

where the Kernel can be determined with the help of (2.9) and (2.1) to (2.4) as given in our earlier paper[4]. The ultimate form of the Kernel can be obtained by substituting $sc = \xi$ and expanding the Kernel in a polynomial as[4]

$$K(u,t) = \int_0^\infty \frac{L_1}{c}\Big[T_0 + (T_1 - T_0(t+u)\frac{\xi}{c}$$

$$+ \left(T_0\frac{(t+u)^2}{2} - T_1(t+u) + T_2\right)\frac{\xi^2}{c^2}$$

$$+ \left(-\frac{T_0(t+u)^3}{6} + \frac{T_1}{2}(t+u)^2 - T_2(t+u) + T_3\right)\frac{\xi^3}{c^3}$$

$$+ \left(\frac{T_0(t+u)^4}{24} - \frac{T_1}{6}(t+u)^3 + \frac{T_2}{2}(t+u)^2\right.$$

$$\left. - T_3(t+u) + (T_4\right)\frac{\xi^4}{c^4}$$

$$+ \left(-\frac{T_0(t+u)^5}{120} + \frac{T_1}{24}(t+u)^4 - \frac{T_2(t+u)^3}{6}\right.$$

$$\left. + \frac{T_3(t+u)^2}{2} - T_4(t+u) + T_5\right)\frac{\xi^5}{c^5}$$

$$+ \ldots\ldots\ldots\Big]\, d\xi \quad \ldots(3.12)$$

where

$$T_0 = (3-2\eta_2)M_1 I_0 + (1-2\eta_2)M_3 I_1 + \mu c^{-1}(1-2\eta_2)M_5 I_1$$

$$- M_7 I_1 \mu \quad \ldots(3.13)$$

$$T_1 = M_2 I_0 (3-2\eta_2)t + CM_1 I_1 + (M_4(1-2\eta_2)I_1 t - M_3 CI_0$$

$$+ \mu c^{-1} M_6 (1-2\eta_2) I_1 t - (\mu c M_7 I_0 + \mu M_8 (I_1 t)$$

$$- \mu M_9 M_1 I_1 \quad \ldots(3.14)$$

$$T_2 = (c\,t\,M_2 I_1 - M_1 I_0) + (M_3 I_1 - M_4 ct I_0) + \mu c^{-1} M_5 I_1 (1+c^2)$$

$$+ \mu (M_7 I_1 - M_8 ct I_0) - \mu (M_9 M_{11} I_1 + M_9 M_{10} I_0 c) \quad \ldots(3.15)$$

$$T_3 = -M_2 I_0 t + M_4 I_1 t + \mu (M_6 (1 + c^2) c^{-1} I_1 t - M_5 I_0) + \mu M_8 I_1 t + \mu (M_9 M_{10} I_1 - M_9 M_{11} ct I_0) \quad ...(3.16)$$

$$T_4 = -\mu t (M_6 I_0 - M_9 M_{11} I_1) \quad ...(3.17)$$

$$M_1 = -\left[(\alpha_3 + \alpha_2 (1 - 2\eta_2)) I_1 - (\alpha_{13} I_0 + \alpha_8 I_1)(L_2 - L_4 (1 - 2\eta_2) L_3^{-1}\right] \quad ...(3.18)$$

$$M_2 = -\alpha_2 I_1 + (\alpha_{13} I_0 + \alpha_8 I_1) L_4 L_3^{-1} \quad ...(3.19)$$

$$M_3 = -(\alpha_3 + \alpha_2 (1 - 2\eta_2)) I_0 - (\alpha_{14} I_0 + \alpha_9 I_1) (L_2 - L_4 (1 - 2\eta_2)) L_3^{-1} \quad ...(3.20)$$

$$M_4 = -\alpha_2 I_0 + (\alpha_{14} I_0 + \alpha_9 I_1) L_4 L_3^{-1} \quad ...(3.21)$$

$$M_5 = -(\alpha_6 + \alpha_5 (1 - 2\eta_2)) I_1 - (\alpha_{15} I_0 + \alpha_{10} I_1) (L_2 - (1 - 2\eta_2) L_4) L_3^{-1} \quad ...(3.22)$$

$$M_6 = \alpha_5 I_1 + (\alpha_{15} I_0 + \alpha_{10} I_1) L_4 L_3^{-1} \quad ...(3.23)$$

$$M_7 = -(I_1 - \lambda I_0) (\alpha_6 + \alpha_5 (1 - 2\eta_2)) \quad ...(3.24)$$

$$M_8 = -(I_1 - \lambda I_0) \alpha_5 \quad ...(3.25)$$

$$M_9 = -(\alpha_{16} I_0 + \alpha_{11} I_1) \quad ...(3.26)$$

$$M_{10} = (L_2 - L_4 (1 - 2\eta_2)) L_3^{-1} \quad ...(3.27)$$

$$M_{11} = -L_4 L_3^{-1}. \quad ...(3.28)$$

Supposing E_0, E_1, E_2, E_3, E_4 for the consecutive terms of (3.12). The ingetrals are approximated numerically by using Simpson's one third rule after puting

$$\xi = \frac{x - 1}{x + 1} \text{ and } \eta_1 = \eta_2 = \frac{1}{3} \text{ and } \mu = 0.6.$$

The values of the integrals become

$$E_0 = 7.325652 - 0.165040\, c^{-1} \quad ...(3.29)$$

$$E_1 = 12.417221 + 9.582731\, c^{-1}t + 7.004371\, c^{-1}u - 0.232337\, c^{-2}t - 0.162391\, c^{-2}u \quad \ldots(3.30)$$

$$E_2 = 6.006873c^{-2}t^{-2} + 3.448638\, c^{-2}u^{-2} + 9.455512\, c^{-2}tu - 0.1482997\, c^{-3}\, t^2 - 0.080881\, c^{-3}\, u^2 - 0.229287\, c^{-3}\, tu + 17.525904\, c^{-1}\, t + 12.536939\, c^{-1}\, u - 10.259631\, c^{-2} - 0.4853259\, c^{-3}\, (1 + c^2) + 2.944246 \quad \ldots(3.31)$$

$$E_3 = 2.419360\, c^{-3}\, t^3 + 1.143596\, c^{-3}u^{-3} + 5.982316\, c^{-3}\, t^2u + 4.706553\, c^{-3}\, tu^2 = 0.060304\, c^{-4}\, t^3 = 0.0269171\, c^{-4}u^3 - 0.147519\, c^{-4}\, t^2u - 0.1141352\, c^{-4}\, tu^2 + 11.289841x^{-2}\, t^2 + 6.288422\, c^{-2}u^2 + 17.578264\, c^{-2}tu - 14.234655\, c^{-3}t - 10.260513\, c^{-3}u - 0.6848125\, c^{-4}t\,(1+c^2) - 0.4845096\, c^{-4}u\,(1+c^2) + 3.731800\, c^{-1}t + 2.741926\, c^{-1}u - 1.195441\, c^{-2} + 1.0869638\, c^{-3} \quad \ldots(3.32)$$

$$E_4 = 0.710285\, c^{-4}\, t^4 + 0.285403\, c^{-4}u^4 + 2.416259\, c^{-4}t^3u + 1.566494\, c^{-4}\, tu^3 + 2.987065\, c^{-4}t^2u^2 - 0.017812\, c^{-5}t^4 - 0.0067254\, c^{-5}u^4 - 0.0601614\, c^{-5}t^3u - 0.037988\, c^{-5}tu^3 - 0.073612\, c^{-5}t^2u^2 + 4.6011433\, c^{-3}t^3 + 2.098357\, c^{-3}u^3 + 11.300644\, c^{-3}\, t^2\, u + 8.797860\, c^{-3}\, tu^2 - 9.140708\, c^{-4}t^2 - 5.130403\, c^{-4}u^2 - 14.271111\, c^{-4}tu + 0.4714649\, c^{-5}\, (1 + c^2)\, t^2 - 0.242057\, c^{-5}\, (1 + c)\, u^2 - 0.713583\, c^{-5}\, (1 + c^2)\, tu + 1.584231\, c^{-2}t^2 + 1.337242\, c^{-2}u^2 + 2.921473\, c^{-2}tu + 1.781194\, c^{-4}t + 1.151515\, c^{-4}u - 1.955627\, c^{-3}t - 1.627825c^{-3}u. \quad \ldots(3.33)$$

Adjusting the values of these integrals in powers of c, the Kernel becomes

$$K(u, t) = 22.687119c^{-1} + (-0.650366 + 30.840435t)\, c^{-2} + 22.2832236\, uc^{-2} + (-11.455072 - 0.917149\, t$$

$$- 0.647104\, u + 18.880945\, t^2 + 11.074303\, u^2$$

$$+ 29.955249\, tu)\, c^{-3} + (0.6016379 - 16.190282\, t$$

$$- 11.888338\, u - 0.619864\, t^2 - 0.322945\, u^2$$

$$- 0.942871\, tu + 7.020503\, t^3 + 3.241954 u^3$$

$$+ 17.282860\, t^2 u + 13.504412\, tu^2)\, c^{-4} + \ldots\ldots \quad \ldots(3.34)$$

It is obvious that as c tends to infinity $K(t, u)$ tends to zero. If c is sufficiently large we assume $\beta = ut/c \geqslant a$ through which the series occurring in (3.34) is uniformly and absolutely convergent with the result the Kernel of (3.11) is not only bounded it is also $L_2(a, \infty)$ Srivastava and Mahajan[10]. Hence we get

$$K(u, t) = 22.687119\, u^{-1} t^{-1} \beta + (0.650566 + 30.840435\, t$$

$$+ 22.283236\, u)\, u^{-2} t^{-2} \beta^2 + (-11.455072 - 0.917149\, t$$

$$- 0.647100\, u + 18.880945 t^2 + 11.074303\, u^2$$

$$+ 29.955249\, tu)\, u^{-3} t^{-3} \beta^3 + (0.601638 + 16.190282\, t$$

$$- 11.888338\, u - 0.619865\, t^2 - 0.322945\, u^2 - 0.942971\, tu$$

$$+ 7.020503\, ut^3 + 3.241954\, u^3 + 17.282960\, t^2 u$$

$$+ 13.504412\, u^2)\, u^{-4}\ t^{-4}\, \beta^4 + \ldots \quad \ldots(3.35)$$

In particular if the temperature is not uniform i.e. if we set $p(r) = 0$, the free term in (3.11) presents an interesting excercise in closed form evaluation of definition integrals. Let

$$\phi(r, t) = \int_0^\infty \frac{J_1(\lambda r)\, Y_1(\lambda) - Y_0(\lambda r)\, J_1(\lambda)}{\lambda (J_1^2(\lambda) + Y_1^2(\lambda))} \{J_1(\lambda) \cos \lambda t + Y_1(\lambda) \sin \lambda t\}\, d\lambda$$

$$\ldots(3.36)$$

and $P(r)$ denote the free temperature in (3.11) them from (2.11) and (3.11) we have ;

$$P(r) = - \alpha \frac{(1 + \eta)\, \mu}{\pi (1 - \eta)} \int_0^\infty r\, \phi(r, t)\, T(r, 0)\, dr \quad \ldots(3.37)$$

$$\frac{\partial \phi}{dt} = 0,\ c < r < t$$

$$= (r^2 - t^2)^{-1/2},\ t < r < \infty. \quad \ldots(3.38)$$

Moreover

$$\phi(r,t) = \phi(r,\infty) - \int_0^\infty \frac{\partial \phi}{\partial \tau}\, d\tau \qquad \ldots(3.39)$$

(r,∞) can be shown to be $\frac{\pi}{2}$ through an application of Biemann Lebeseque theorem :

$$\phi(r,t) = \frac{\pi}{2}\, c \leqslant r \leqslant t \qquad \ldots(3.40)$$

$$= \sin^{-1}\frac{t}{r}\, r \geqslant t.$$

To illustrate the applicability of our method we now consider a special temperature distribution given by

$$T(r,0) = -\frac{K}{r^3},\ r \geqslant 1$$

and taking $p(r) = 0$, the equation (3.11) reduces to

$$g(t) - \int_a^\infty g(u)\, K(t,u)\, du = -\frac{K\alpha}{\pi}\frac{1+\eta_2}{1-\eta_2}\left[\frac{\pi}{2c} - \frac{1}{t}\right],\ t \geqslant a$$

$$g(t) - \int_a^\infty g(u)\, K(t,u)\, du = -\frac{2K\alpha}{\alpha}\left[\frac{\pi}{2c} - \frac{1}{t}\right],\ t \geqslant a. \qquad \ldots(3.41)$$

A quantity of physical interest that can be obtained with a use of our technique is the stress intensity factor. From (3.5) and (3.7) we see that

$$\frac{\sigma_{zz}}{2\mu_2}\Big|_{z=0} = \int_a^\infty \frac{\lambda D(\lambda) J_0(\lambda r) Y_1(\lambda) - Y_0(\lambda r) J_1(\lambda)}{J_1^2(\lambda) + Y_1^2(\lambda)}\, \alpha\lambda. \qquad \ldots(3.42)$$

This can be reduced in an alternative form as :

$$\frac{\sigma_{zz}}{2\mu_2}\Big|_{z=0} = \int_a^\infty g(u) \int_0^\infty \lambda \{Y_1(\lambda)\cos\lambda u - J_1(\lambda)\sin\lambda u\}$$

(equation continued on p. 866)

$$\left\{\frac{J_0(\lambda r)\,Y_1(\lambda) - (Y_0(\lambda r)\,J_1(\lambda)}{J_1^2(\lambda) + Y_1^2(\lambda)}\right\} d\lambda$$

$$+ \frac{\alpha(1+\eta_2)}{4(1-\eta_2)}\,T(r,0). \qquad \ldots(3.43)$$

The evaluation of the contour integral

$$\int_{\Gamma} Z\,J_1(z)\,\frac{J_2(rz) + i\,Y_0(rz)}{J_1(z) + i\,Y_1(z)}\,e^{iuz}\,dz \qquad \ldots(3.44)$$

around the boundary of the first quadrant in the complex plane leads us to a useful identity

$$\int_0^{\infty} \lambda\,\{Y_1(\lambda)\cos\lambda u - J_1(\lambda)\cos\lambda u\}\,\{J_0(\lambda r)\,Y_1(\lambda) - Y_0(\lambda r)\,J_1(\lambda)\}$$

$$\left(J_1^2(\lambda) + Y_1^2(\lambda)\right)^{-1} \alpha\lambda$$

$$= \frac{1}{r}\frac{\partial}{\partial r}\,\frac{u\,H(u-v)}{\sqrt{u^2 - r^2}} + \int_0^{\infty} y\,L_1\Big[\Big\{K_0(yr)\,\Big(\alpha_3 I_1(yr) + (\alpha_{13} I_0(yr)$$

$$+ \alpha_8 I_1(yr) \times L_2 L_3^{-1}\Big) + (2\eta_2 K_0(yr) - yr\,K_1(yr))$$

$$\Big(\alpha_2 I_1(yr) - (\alpha_{13} I_0(yr) + \alpha_8 I_1(yr))\,L_4 L_3^{-1}\Big)\Big\}$$

$$\times \{(3 - y^2 - 2\eta_2)\,I_0(yr) + yr\,I_1(yr)\}$$

$$+ \Big\{K_0(yr)\,\Big(\alpha_3 I_0(yr) + (\alpha_{14} I_0(yr) + \alpha_9 I_1(yr))\,L_2\,L_3^{-1}\Big)$$

$$+ (2\eta_2 K_0(yr) - yr\,K_1(yr))\Big(\alpha_2 I_0(yr) - (\alpha_{14} I_0(yr)$$

$$+ \alpha_9 I_1(yr))\,L_4 L_3^{-1}\Big)\Big\} \times \{-yr\,I_0(yr) + (1-2\eta_2+y^2)\,I_1(yr)\}$$

$$+ \Big\{K_0(yr)\,\Big(-\alpha_6 I_1(yr) + (\alpha_{15} I_0(yr) + \alpha_{10} I_1(yr))\,L_2 L_3^{-1}\Big\}$$

(*equation continued on p.* 867)

$$+ \ (2\eta_2 K_0(yr) - yr K_1(yr))(-\alpha_5 I_1(yr)$$

$$- \ (\alpha_{15} I_0(yr) + \alpha_{10} I_1(yr)) \ L_4 L_3^{-1}\Big\}$$

$$\left\{\frac{\mu}{r}((1 - 2\eta_2 + y^2(1 + r^2)) I_1(yr) - y^3 r I_0(yr)\right\}$$

$$+ \ \Big\{K_0(yr)(\alpha_6(I_1(yr) - yr I_0(yr)) y^{-1} + (\alpha_{16} I_0(yr)$$

$$+ \ \alpha_{11} I_1(yr)) \ L_2 L_3^{-1}\Big) + (2\eta_2 K_0(yr) - yr K_1 yr)) (y^{-1} \alpha_5 (I_1(yr)$$

$$- \ yr(I_0(yr)) - (\alpha_{16} I_0(yr) + \alpha_{11} I_1(yr)) L_4 L_3^{-1}\Big\}$$

$$\{\mu(y(y^2 - 1) I_1(yr) - y^2 r I_0(yr)\}\Big] e^{-uy} dy, \ r \geqslant 1, u \geqslant 1$$

...(3.45)

where for $r = 0, 1$ Jr, Yr, Ir and Kr are Bessels functions and modified Bessel's functions of first and second kind. The values of α_i and L_i have the same notations as in Singh and Dubey[4]. There values are with regard to the variable y in place of s and the values of c is generalized to r.

The result of the second integrand of R.H.S. in (3.45) is directly borrowed from our earlier paper[4] through which equation (3.43) becomes :

$$\frac{\sigma_{zz}}{2\mu_2}\Big|_{z=0} = \frac{\alpha(1 + \eta_2)}{4(1 - \eta_2)} T(r, 0) + \frac{1}{r}\frac{\partial}{\partial r}\int_{\max(a,r)}^{\infty} \frac{u g(u)}{\sqrt{u^2 - r^2}} du$$

$$+ \int_0^{\infty} g(u)[-22.283215 \beta^2 + (0.647101 - 22.1486074) \beta^3$$

$$+ (11.444867 + 0.6460133 u - 11.051865 u^2) \beta^4$$

$$+ (- 0.600799 + 11.589288 u + 0.382804 u^2 - 3.24071 u^3) \beta^5$$

$$+ (- 0.6225883 u + 5.1304525 u^2 + 0.0269019 u^3$$

$$- 0.2852379 u^4) \beta^6 + ...] du. \qquad ...(3.46)$$

The stress intensity factor obtained in this way is equal to $\frac{\mu_2 \sqrt{2g(a)}}{\sqrt{a}}$.

To solve (3.41) we put

$$g(t) = \frac{-K\alpha(1+\eta_2)G(t)}{\pi(1-\eta_2)}$$

and enumerated the numerical equation through successive integration by supposing.

$$G(t) = G_0(t) + G_1(t)\beta + G_2(t)\beta^2 = G_3(t)\beta^3 + \ldots\ldots \quad \ldots(3.47)$$

By iteration

$$G_0(t) = -\frac{1}{t}$$

$$G_1(t) = -22.687119\, a^{-1} t^{-1} + \frac{\pi}{2} u^{-1} t^{-1}$$

$$G_2(t) = -487.94159\, a^{-2} t^{-1} + 0.325183\, a^{-2} t^{-2} + 22.283236\, a^{-1} t^{-2}.$$

Hence

$$G(t) = -t^{-1}\,1 + (22.687119\, a^{-1} - 1.5714285 u^{-1})\beta + (487.94159\, a^{-2} - 0.325183\, t^{-1} a^{-2} - 22.28326\, a^{-1})\beta^2 + \ldots\ldots$$

i.e. we see that as $\beta \to 0$ $G(t) \alpha \frac{1}{t}$

with a substitution of $\eta_2 = \frac{1}{3}$. $\alpha = 1$ and

$$K' = \frac{0.15714285}{K}$$

we obtain the following results :

S. No.	a	K'	K
1.	2.0	0.3535534	0.1590909
2.	4.0	0.1250000	0.0397727
3.	6.0	0.0680413	0.0176767
4.	8.0	0.0441941	0.0099431
5.	10.0	0.0316227	0.0063636

where K is the variation of stress intensity factor in isothermal case as given in our earlier paper[4]. The variation of stress intensity factor with a is shown in Fig. 2.

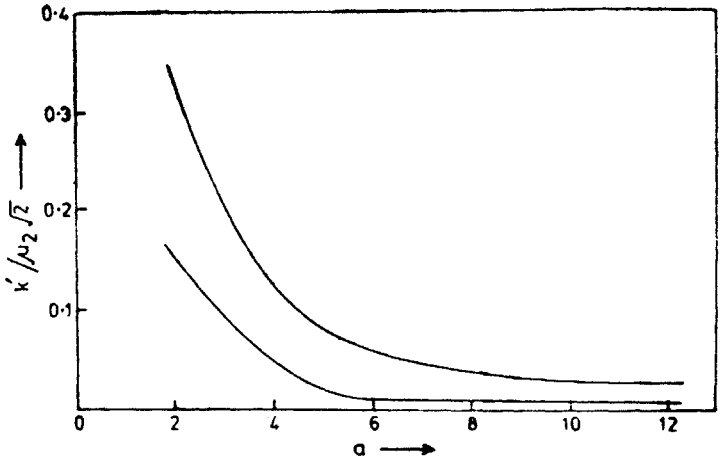

FIG. 2. Variation of stress intensity factor with a.

CONCLUSION

It is easy to conclude now that the fall of stress intensity factor at prescribed temperature is more rapid in comparison to the fall of stress intensity factor at normal temperature which can obviously be guessed through the Fig 2. It is interesting that the procedure adopted here is beneficial to evaluate normal component of displacement of crack too.

REFERENCES

1. F. Erdogan, *J. appl. Mech.* 32 (1965), 829.
2. A. I. Florence, and I. N. Goodier, *Int. J. engng Sci.* 1 (1963), 533-40.
3. K. Anzai, Y. Shindo, and A. Atsumi, *Int. J. engng Sci.* 15 (1981), 275-82.
4. K. Singh, and G. C. Dubey, *Indian J. pure appl. Math.* 16 (1985), 673-92.
5. K. Singh, and G. C. Dubey, *Indian J. pure appl. Math.* 16 (1985), 396-410.
6. N. Fox, *Proc. Lond. Math. Soc.* 11 (1961), 276.
7. R. Shail, *Int. J. engng Sci.* 3 (1965), 597.
8. J. T. Welch, and I. N. Sneddon, *Int. J. engng Sci.* 1 (1963), 411.
9. R. P. Srivastava, and Prem Narain, *Int. J. engng Sci.* 4 (1966), 689.
10. K. N. Srivastava, and R.L. Mahajan, *ZAMM* 15 (1971), 433-44.