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FLOW OF A NON-NEWTONIAN SECOND-ORDER FLUID BETWEEN TORSIONALLY OSCILLATING DISCS WITH UNIFORM SUCTION AND INJECTION

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The flow of an incompressible second-order fluid due to torsional oscillations of two infinite discs when the lower disc is subjected to uniform injection and the upper disc to an equal suction, has been considered by expanding the velocity components and the pressure in powers of the amplitude of oscillations. The effects of second-order forces on the steady part of radial, transverse and axial velocity at different phase difference (in case of transverse velocity) have been shown graphically. The results obtained are compared with those foa Newtonian fluid. The transverse shearing stress and moment on the discs have also been obtained.

1. INTRODUCTION

The torsional oscillations of a plane in Newtonian fluids have been discussed by Rosenblat[1]. He has also discussed the case when the Newtonian fluid is confined between two torsionally oscillating discs[2]. These problems have been extended by Srivastava[3-5] for Reiner-Rivlin and second-order fluids. Recently Sharma and Gupta[6] have considered a general case of flow between two torsionally oscillating discs. Kelly[7] has studied the problem considered by Rosenblat[1] for the case of large suction.

The present investigation deals with the study of flow of a second-order fluid between two infinite torsionally oscillating discs with uniform suction and injection. The flow functions and pressure are expanded in powers of the amplitude (assumed to be small) of the oscillations of the disc. The non-Newtonian effects are exhibited through two dimensionless parameters τ_1 ($= \mu_2 n/\mu_1$) and τ_2 ($= \mu_3 n/\mu_1$), where μ_1, μ_2, μ_3 are the coefficients of Newtonian viscosity, elsastico-viscosity and cross-viscosity; n being the uniform frequency of the oscillations. The results have their special importance as they are obtained in the most general form and are in good agreement with those obtained by Srivastava[5] and Sharma and Gupta[6] as special cases. The variation of steady parts of radial, axial and transverse velocity with suction parameter A and second-order parameter τ_2 (based on the relation $\tau_1 = \alpha\tau_2$ where $\alpha = -0.2$ as for 5.46% poly-iso-butylene type solution in cetane at 30°C (Markowitz[8])) is shown graphically

only in case of $s_1 = 1$ and $s_2 = -1$ i.e. when the discs perform torsional oscillations in opposite sense with equal angular velocity.

2. Formulation of the Problem

The constitutive equation of an incompressible second-order fluid as suggested by Coleman and Noll[9] can be written as :

$$\tau_{ij} = -p\delta_{ij} + 2\,\mu_1\, d_{ij} + 2\,\mu_2\, e_{ij} + 4\,\mu_3\, c_{ij} \qquad \ldots(1)$$

where

$$\left.\begin{aligned} d_{ij} &= \tfrac{1}{2}\,(v_{i,j} + v_{j,i}) \\ e_{ij} &= \tfrac{1}{2}\,(a_{i,j} + a_{j,i}) + v^m_{,i}\; v_{m,j} \\ \text{and} \\ c_{ij} &= d^m_i\; d_{mj}, \end{aligned}\right\} \qquad \ldots(2)$$

τ_{ij} is the stress tensor and p, the hydrostatic pressure; v_i and a_i are the velocity and the acceleration vectors.

The equation (1) together with the momentum equation for no extraneous force

$$\rho\left(\frac{\partial v_i}{\partial t} + v^m\, v_{i,m}\right) = \tau^m_{i,m} \qquad \ldots(3)$$

and the equation of continuity for incompressible fluid

$$v^i_{,i} = 0 \qquad \ldots(4)$$

where ρ is the density of the fluid and (,) represents covariant differentiation, form the set of governing equations.

In a three dimensional cylindrical set of co-ordinates (r, θ, z) the system consists of two discs of infinite radius at a distance 'd' apart (coinciding with the plane $z = 0$ and $z = d$) performing rotatory oscillations of the type $rs_1\ \Omega \cos(nt)$ and $rs_2\ \Omega \cos(nt)$ of small amplitude $\epsilon\ (= \Omega/n)$, about a perpendicular axis $r = 0$ with different angular velocities $s_1\,\Omega$ and $s_2\,\Omega$. The uniform injection and suction w_0 is applied on the lower ($z = 0$) and the upper ($z = d$) discs respectively and the space between the discs is occupied by a homogeneous, incompressible second order fluid. Assuming u, v and w as the radial, transverse and axial components of the velocity respectively, the relevant boundary conditions of the problem are

$$\left.\begin{aligned} z = 0;\quad u = 0,\quad v = r\,s_1\,\Omega\cos(nt),\quad w = w_0 \\ z = d;\quad u = 0,\quad v = r\,s_2\,\Omega\cos(nt),\quad w = w_0. \end{aligned}\right\} \qquad \ldots(5)$$

The velocity components for the axisymmetric flow compatible with the continuity criterion can be taken as[5]

$$\left.\begin{aligned} u &= r\,\Omega\,F'(\zeta,\tau) \\ v &= r\,\Omega\,G(\zeta,\tau) \\ w &= -\,2d\,\Omega\,F(\zeta,\tau) \end{aligned}\right\} \qquad \ldots(6)$$

where $F(\zeta, \tau)$ and $G(\zeta, \tau)$ are non-dimensional functions of the dimensionless variable $\zeta = z/d$, $\tau = nt$ and a prime denotes differentiation with respect to ζ.

Using the velocity field (6) the boundary conditions (5) are transformed to

$$\left.\begin{aligned} &\zeta = 0;\ F = -A,\ F' = 0,\ G = s_1 e^{it} \text{ (real part)} \\ &\zeta = 1;\ F = -A,\ F' = 0,\ G = s_2 e^{it} \text{ (real part)} \end{aligned}\right\} \qquad \ldots(7)$$

where $A\ (= W_0/2d\,\Omega)$ is the suction parameter. Making use of the expressions (6) the equations of motion in dimensionless form provide

$$\begin{aligned} \frac{1}{\xi}\frac{\partial P}{\partial \xi} &= F''' + \tau_1\left[\frac{\partial F'''}{\partial \tau} - 2\epsilon\,(FF^{iv} - F''^2)\right] + \tau_2\epsilon \\ &\quad [F''^2 - G'^2 - 2F'\,F''] \\ &\quad - R\left[\frac{\partial F'}{\partial \tau} + \epsilon\,(F'^2 - G^2 - 2FF'')\right] \qquad \ldots(8) \end{aligned}$$

$$\begin{aligned} 0 = G'' + \tau_1\left[\frac{\partial G''}{\partial \tau} + 2\epsilon\,(F''\,G' - FG'')\right] + 2\tau_2\ \epsilon\,(F''G' - F'G'') \\ - R\left[\frac{\partial G}{\partial \tau} + 2\epsilon\,(F'G - FG')\right] \qquad (9) \end{aligned}$$

$$\begin{aligned} -\frac{\partial P}{\partial \zeta} = 2F'' + 2\tau_1\left[\frac{\partial F''}{\partial \tau} - 2\epsilon\,(11\,F'\,F'' + FF''')\right] - 28\,\tau_2\,\epsilon F'F'' \\ - 2\xi^2\,\epsilon\,(2\tau_1 + \tau_2)\,(F''\,F''' + G'G'') \\ - 2R\left(\frac{\partial F}{\partial \tau} - 2\epsilon\,FF'\right) \qquad \ldots(10) \end{aligned}$$

where $R\left(= \dfrac{\rho n d^2}{\mu_1}\right)$ is Reynold's number; $P\ (= p/\mu_1\,\Omega)$ and $\xi\ (= r/d)$ are non-imenpsional pressure and radii. Equations (8) and (10) suggest the following form for the non-dimensional pressure

$$P = -\,P_1(\zeta, \tau) + \xi^2\,P_2(\zeta, \tau) \qquad \ldots(11)$$

Substituting expression (11) in equation (10) and equating the terms independent of ξ and coefficient of ξ^2 on both the sides we get equations in P_1 and P_2. Integration of the first equation provides us P_1 while integration of the second provides P_2 as

$$P_2(\zeta, \tau) = \epsilon(2\tau_1 + \tau_2)(F''^2 + G'^2) + \lambda(\tau) \qquad \ldots(12)$$

where $\lambda(\tau)$ is the constant of integration. Using (11) and (12), equation (8) becomes

$$R\left[\frac{\partial F'}{\partial \tau} + \epsilon(F'^2 - G^2 - 2FF'')\right] = F^{iv} + \tau_1\left[\frac{\partial F'''}{\partial \tau} - 2\epsilon(F''^2 + 2G'^2 + FF^{iv})\right] - \tau_2\epsilon[F''^2 + 3G'^2 + 2F'F'''] - 2\lambda. \qquad \ldots(13)$$

3. Solution of the Problem

For finding out the value of the functions F, G and λ we expand them in powers of ϵ, considered small. Substituting the series

$$F(\zeta, \tau) = -A + \sum_{n=0}^{\infty} \epsilon^n F_n(\zeta, \tau), \; G(\zeta, \tau) = \sum_{n=0}^{\infty} \epsilon^n G_n(\zeta, \tau)$$

and

$$\lambda(\tau) = \sum_{n=0}^{\infty} \epsilon^n \lambda_n(\tau) \qquad \ldots(14)$$

into eqns. (13) and (9) and equating terms independent of ϵ and coeffiicient of ϵ, we obtain

$$R\frac{\partial F_0'}{\partial \tau} = F_0^{iv} + \tau_1\frac{\partial F_0'''}{\partial \tau} - 2\lambda_0 \qquad \ldots(15)$$

$$R\left[\frac{\partial F_1'}{\partial \tau} + F_0'^2 - G_0^2 + 2F_0''(A - F_0)\right] = F_1^{iv} + \tau_1\left[\frac{\partial F_1'''}{\partial \tau} - 2\left(F_0''^2 + 2G_0'^2 + F_0^{iv}(F_0 - A)\right)\right] - \tau_2\left[F_0''^2 + 3G_0'^2 + 2F_0'F_0'''\right] - 2\lambda_1 \qquad \ldots(16)$$

$$R\frac{\partial G_0}{\partial \tau} = G_0'' + \tau_1\frac{\partial G_0''}{\partial \tau} \qquad \ldots(17)$$

$$R\left[\frac{\partial G_1}{\partial \tau} + 2\left\{F_0'G_0 + (A - F_0)G_0'\right\}\right] = G_1'' + \tau_1\left[\frac{\partial G_1''}{\partial \tau}\right.$$

(equation continued on p. 935)

$$+2\left\{F_0''\ G' + (A - F_0)\ G_0''\right\}\Big] + 2\tau_2\left(F_0''\ G_0' - F_0'\ G_0'\right) \quad \ldots(18)$$

Solving eqns. (15-18) under the boundary conditions

$$F_m(0) = F_m(1) = 0,\ F_m'(0) = F_m'(1) = 0$$

$$G_0(0) = s_1 e^{-\tau},\ G_0(1) = s_2 e^{\tau},\ G_{m+1}(0) = G_{m+1}(1) = 0$$

$$(\text{for } m = 0, 1, 2, \ldots) \quad \ldots(19)$$

we get

$$F_0(\zeta, \tau) = 0,\ \lambda_0(\tau) = 0$$

$$G_0(\zeta, \tau) = \frac{1}{\cosh 2\alpha - \cos 2\beta}[s_1\{\cos\tau\{\cosh\alpha(\zeta-2)\cos\beta\zeta - \cos\alpha\zeta\cos\beta(\zeta-2)\} + \sin\tau\{\sinh\alpha\zeta\sin\beta(\zeta-2) - \sinh\alpha(\zeta-2)\sin\beta\zeta\}\} + s_2\{\cos\tau\{\cosh\alpha(\zeta+1)\cos\beta(\zeta-1) - \cosh\alpha(\zeta-1)\cos\beta(\zeta+1)\} + \sin\tau\{\sinh\alpha(\zeta-1)\sin\beta(\zeta+1) - \sinh\alpha(\zeta+1)\sin\beta(\zeta-1)\}\}]$$

$$G_1(\zeta, \tau) = \frac{1}{\cosh 2\alpha - \cos 2\beta}[s_1\zeta\{(a\cos\tau - b\sin\tau)\{\cosh\alpha(\zeta-2)\cos\beta\zeta - \cosh\alpha\zeta\cos\beta(\zeta-2)\} + (a\sin\tau + b\cos\tau) \times \{\sinh\alpha\zeta\sin\beta(\zeta-2) - \sinh\alpha(\zeta-2)\sin\beta\zeta\}\} + s_2(\zeta-1)\{(a\cos\tau - b\sin\tau)\{\cosh\alpha(\zeta+1)(\cos\beta(\zeta-1) - \cosh\alpha(\zeta-1)\cos\beta(\zeta+1)\} + (a\sin\tau + b\cos\tau)\{\sinh\alpha(\zeta-1)\sin\beta(\zeta+1) - \sinh\alpha(\zeta+1)\sin\beta(\zeta-1)\}\}] \quad \ldots(20)$$

where,

$$\alpha = \left(\frac{R}{2}\right)^{1/2}\left[\frac{\sqrt{1+\tau_1^2}+\tau_1}{1+\tau_1^2}\right]^{1/2}$$

$$\beta = \left(\frac{R}{2}\right)^{1/2}\left[\frac{\sqrt{1+\tau_1^2}-\tau_1}{1+\tau_1^2}\right]^{1/2}$$

$$a = \left(2\alpha\beta\tau_1^2 + (\alpha^2 - \beta^2)\tau_1 - 1\right)/\left(1+\tau_1^2\right)$$

and

$$b = ((\beta^2 - \alpha^2)\ \tau_1^2 + (2\alpha\beta + 1)\tau_1)/\left(1 + \tau_1^2\right). \qquad ...(21)$$

The function $F_1(\zeta, \tau)$ and the constant $\lambda_1(\tau)$ are of the following form,

$$F_1(\zeta, \tau) = f(\zeta) + h(\zeta)\, e^{2i\tau}$$

$$\lambda_1(\tau) = c + d\, e^{2i\tau}. \qquad ...(22)$$

This form is suggested by G_0^2 and $G_0'^2$ which are composed of a steady term and a term of frequency $2n$. Complex notation has been adopted here with the convention that only the real parts of the complex quantities have physical meaning. Substituting the expressions for F_0, F_1, G_0, λ_1 into (16) and equating the coefficient of $e^{2i\tau}$ and terms independent of it, we get two equations giving $f(\zeta)$ and constant 'c' obtained as

$$f(\zeta) = KK_2\,\alpha^3 [2\beta\zeta\left\{\zeta^2\left(s_1^2 + s_2^2\right) - \zeta\left(2s_1^2 + s_2^2\right) + s_1^2\right\}\cos 2\beta$$

$$- \left\{2\zeta^3\left(s_1^2 + s_2^2\right) - 3\zeta^2\left(s_1^2 + s_2^2\right) + s_1^2\right\}\sin 2\beta$$

$$+ 2\beta\zeta\left\{\zeta^2\left(s_1^2 + s_2^2\right) - \zeta\left(s_1^2 + 2s_2^2\right) + s_2^2\right\}$$

$$- s_1^2 \sin 2\beta(\zeta - 1) - s_2^2 \sin 2\beta\zeta + 2s_1 s_2 \{2\zeta^2(2\zeta - 3)(\sin\beta$$

$$- \beta\cos\beta) - 2\beta\zeta\cos\beta + \sin\beta(2\zeta - 1) + \sin\beta\}\cosh\alpha]$$

$$+ K K_1 \beta^3\left[\left\{2\zeta^3\left(s_1^2 + s_2^2\right) - 3\zeta^2\left(s_1^2 + s_2^2\right) + s_1^2\right\}\sinh 2\alpha\right.$$

$$- 2\alpha\zeta\left\{\zeta^2\left(s^2 + s_2^2\right) - \zeta\left(2s_1^2 + s_2^2\right) + s_1^2\right\}\cosh 2\alpha - 2\alpha\zeta\left\{\zeta^2\right.$$

$$\left(s_1^2 + s_2^2\right) - \zeta\left(s_1^2 + 2s_2^2\right) + s_2^2\left.\right\} + s_1^2 \sinh 2\alpha(\zeta - 1)$$

$$+ s_2^2 \sinh 2\alpha\zeta + 2s_1 s_2 \{2\zeta^2(2\zeta - 3)(\alpha\cosh\alpha - \sinh\alpha) + 2\alpha\zeta\cosh$$

$$- \alpha - \sinh\alpha(2\zeta - 1) - \sinh\alpha\}\cos\beta] \qquad ...(23)$$

$$c = 6\ K K_2\,\alpha^3\left[\beta\left(s_1^2 + s_2^2\right)\cos 2\beta - \left(s_1^2 + s_2^2\right)\sin 2\beta\right.$$

$$\left. + \beta\left(s_1^2 + s_2^2\right) + 4 s_1 s_2 (\sin\beta - \beta\cos\beta)\cosh\alpha\right]$$

(equation continued on p. 937)

$$+ 6KK_1\ \beta^3\Big[\Big(s_1^2 + s_2^2\Big)\sinh 2\alpha - \alpha\Big(s_1^2 + s_2^2\Big)\cosh 2\alpha$$

$$- \alpha\Big(s_1^2 + s_2^2\Big) + 4s_1 s_2\,(\alpha\cosh\alpha - \sinh\alpha)\cos\beta\Big] \quad ...(24)$$

where,

$$K = \frac{R\left(1+\tau_1^2\right)^{-1/2}\alpha^{-3}\beta^{-3}}{16\,(\cosh 2\alpha - \cos 2\beta)}$$

$$K_1 = 4\tau_1 + 3\tau_2 - \sqrt{1+\tau_1^2} \text{ and } K_2 = 4\,\tau_1 + 3\,\tau_2 + \sqrt{1+\tau_1^2}.$$

...(25)

The expression for $h(\zeta)$ is completed and it is cumbersome to separate its real and imaginary parts. The non-dimensional transverse component of the velocity and dimensionless steady parts of radial and axial velocity are given by

$$V = \frac{v}{r\Omega} = \frac{1}{\cosh 2\alpha - \cos 2\beta}[s_1\{\{\cos\tau - A\epsilon\zeta\,(a\cos\tau - b\sin\tau)\}$$

$$\times\{\cosh\alpha\,(\zeta-2)\cos\beta\zeta - \cosh\alpha\zeta\cos\beta\,(\zeta-2)\}$$

$$+\{\sin\tau - A\epsilon\tau\,(a\sin\tau + b\cos\tau)\}\{\sinh\alpha\zeta\sin\beta\,(\zeta-2)$$

$$-\sinh\alpha\,(\zeta-2)\sin\beta\zeta\}\} + s_2\{\{\cos\tau - A\epsilon\,(\zeta-1)\,(a\cos\tau$$

$$- b\sin\tau)\}\{\cosh\alpha\,(\zeta+1)\cos\beta\,(\zeta-1) - \cosh\alpha\,(\zeta-1)$$

$$\times\cos\beta\,(\zeta+1)\} + \{\sin\tau - A\epsilon\,(\zeta-1)\,(a\sin\tau + b\cos)$$

$$\tau)\}\{\sinh\alpha\,(\zeta-1)\sin\beta\,(\zeta+1) - \sinh\alpha\,(\zeta+1)$$

$$\sin\beta\,(\zeta-1)\}\}] \quad ...(26)$$

$$U_s = \frac{u_s}{r\Omega} = \epsilon KK_2\alpha^3[2\beta\Big(3\zeta^2\Big(s_1^2 + s_2^2\Big) - 2\zeta\Big(2s_1^2 + s_2^2\Big) + s_1^2\Big\}\cos 2\beta$$

$$- 6\zeta\Big(s_1^2 + s_2^2\Big)(\zeta-1)\sin 2\beta + 2\beta\Big\{3\zeta^2\Big(s_1^2 + s_2^2\Big)$$

$$- 2\zeta\Big(s_1^2 + 2s_2^2\Big) + s_2^2\Big)\Big\} - 2\beta\Big\{s_1^2\cos 2\beta\,(\zeta-1) + s_2^2\cos 2\beta\zeta\Big\}$$

$$+ 4s_1 s_2\{6\zeta\,(\zeta-1)\,(\sin\beta - \beta\cos\beta) - \beta\,(\cos\beta - \cos\beta$$

$$(2\zeta-1))\}\cosh\alpha] + \epsilon KK_1\beta^3\Big[6\zeta\Big(s_1^2 + s_2^2\Big)(\zeta-1)\sinh 2\alpha$$

$$- 2\alpha\Big\{3\zeta^2\Big(s_1^2 + s_2^2\Big) - 2\zeta\Big(2s_1^2 + s_2^2\Big) + s_1^2\Big\}\cosh 2\alpha$$

$$- 2\alpha \left\{ 3\zeta^2 \left(s_1^2 + s_2^2 \right) - 2\zeta \left(s_1^2 + 2s_2^2 \right) + s_2^2 \right\}$$

$$+ 2\alpha \left(s_1^2 \cosh 2\alpha (\zeta - 1) + s_2^2 \cosh 2\alpha\zeta \right) + 4s_1 s_2 \{6\zeta (\zeta - 1)$$

$$\times (\alpha \cosh \alpha - \sinh \alpha) + \alpha (\cosh \alpha - \cosh \alpha (2\zeta - 1))\} \cos \beta] \quad ...(27)$$

$$W_s = \frac{w_s}{d\Omega} = 2A - 2\epsilon KK_2\alpha^3 \Big[2\beta\zeta \left\{ \zeta^2 \left(s_1^2 + s_2^2 \right) - \zeta \left(2s_1^2 + s_2^2 \right) + s_1^2 \right\} \cos 2\beta - \left\{ 2\zeta^3 \left(s_1^2 + s_2^2 \right) - 3\zeta^2 \left(s_1^2 + s_2^2 \right) + s_1^2 \right\} \sin 2\beta + 2\beta\zeta \left\{ \zeta^2 \left(s_1^2 + s_2^2 \right) - \zeta \left(s_1^2 + 2s_2^2 \right) + s_2^2 \right\} - s_1^2 \sin 2\beta (\zeta - 1) - s_2^2 \sin 2\beta\zeta + 2s_1 s_2 \{2\zeta^2 (2\zeta - 3) (\sin \beta - \beta \cos \beta) - 2\beta\zeta \cos \beta + \sin \beta (2\zeta - 1) + \sin \beta\} \cosh \alpha] - 2\epsilon KK_1 \beta^3 \Big[\left\{ 2\zeta^2 \left(s_1^2 + s_2^2 \right) - 3\zeta^2 \left(s_1^2 + s_2^2 \right) + s_1^2 \right\} \sinh 2\alpha - 2\alpha\zeta \left\{ \zeta^2 \left(s_1^2 + s_2^2 \right) - \zeta \left(2s_1^2 + s_2^2 \right) + s_1^2 \right\} \cosh 2\alpha - 2\alpha\zeta \left\{ \zeta^2 \left(s_1^2 + s_2^2 \right) - \zeta \left(s_1^2 + 2s_2^2 \right) + s_2^2 \right\} + s_1^2 \sinh 2\alpha (\zeta - 1) + s_2^2 \sinh 2\alpha\zeta + 2s_1 s_2 \{2\zeta^2 (2\zeta - 3) (\alpha \cosh \alpha - \sinh \alpha) + 2\alpha\zeta \cosh \alpha - \sinh \alpha (2\zeta - 1) - \sinh \alpha\} \cos \beta]. \quad ...(28)$$

For $s_1 = 1$, $s_2 = 0$, $A = 0$ and $A = 0$ the results are in good argeement with those obtained by Srivastava[5] and Sharma and Gupta[6] respectively.

4. Numerical Computation, Result and Discussions

The numerical calculations have been carried out for the case $s_1 = 1$ and $s_2 = -1$ taking Reynolds number $R = 0.5$ and amplitude of oscillation $\epsilon = 0.2$. The variation of steady part of radial velocity U_s for different values of second-order parameter τ_2 is shown through Fig. 1. It is found that U_s increases near both the discs with an increase in τ_2 and attains its maxima near both the discs and minima exactly in the middle of the gap-length. The curves are symmetrical about the line $\zeta = 0.5$. The steady part for a radial velocity in case of Newtonian fluid ($\tau_2 = 0$) is less than that for a second-order fluid near both the discs while it is higher in the middle of the gap-length.

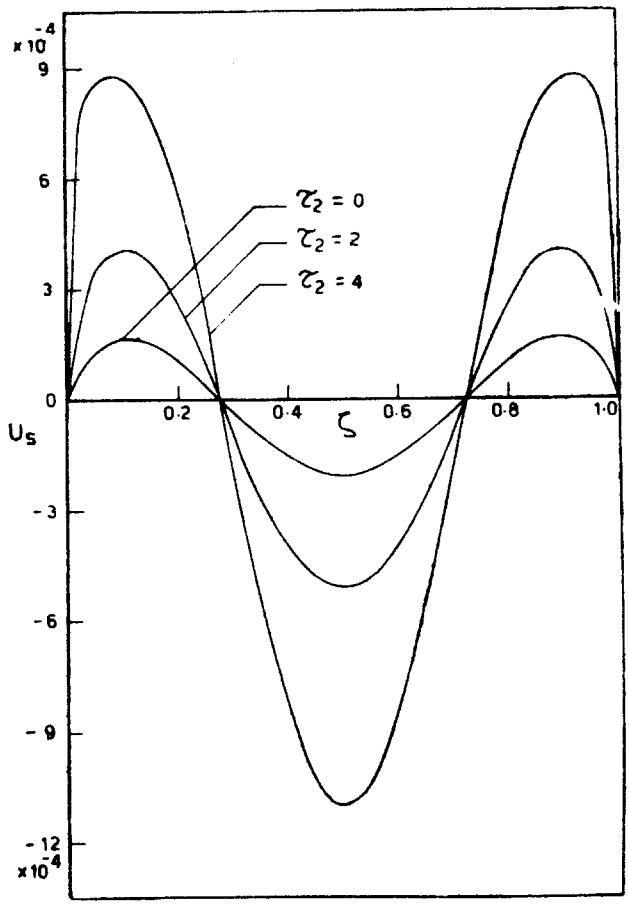

FIG. 1. 1-Variation of steady radial velocity Us with τ_2.

Figures 2 and 3 represent the behaviour of transverse velocity respectively with τ_2, τ and A, τ. The transverse velocity V at $\tau = 0$, $\pi/3$ and $2\pi/3$ decreases with an increase in τ_2 (Fig. 2) while it increases with an increase in A at $\tau = 0$, $\pi/3$ (Fig. 3) and decreases at $\tau = 2\pi/3$. The transverse velocity in case of Newtonian fluid is higher (Fig.2) than that for a second order fluid for all τ throughout the gap-length.

The variation of steady part of axial velocity for different values of τ_2 and A has been represented through Figs. 4 and 5. It is seen that the steady part of axial velocity decreases with an increase in τ_2 and A in the first half of the gaplength while it increases in the second half. The steady part of axial velocity is minimum in the middle of the first half while maximum in the middle of the second half of the gaplength. The curves showing the variation of W_s are symmetric about the line $W_s = 2$ in Fig. 4 while in Fig. 5 about the line $W_s = 0,2$ and 4 for $A = 0,1$ and 2 respectively. It can

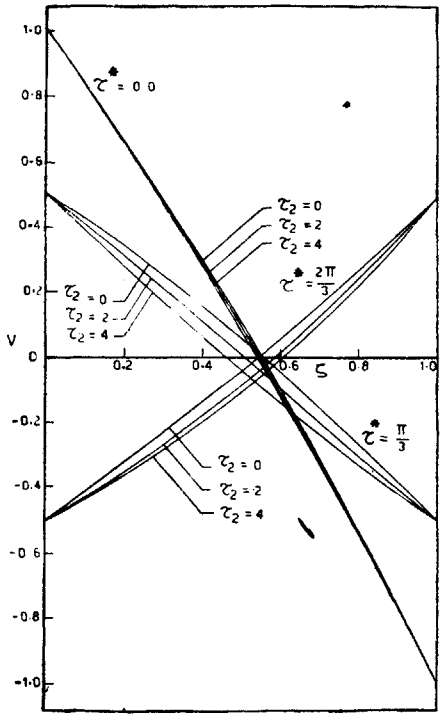

FIG. 2. Variation of transverse velocity V with τ, and τ.

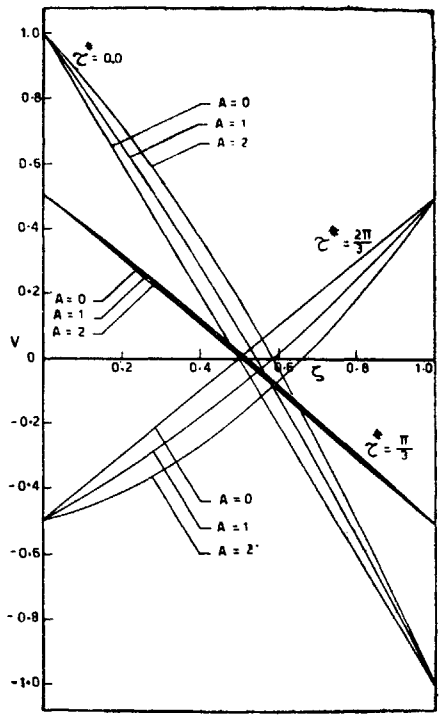

FIG. 3. Variation of transverse velocity V with A and τ.

be observed from Fig. 4 that the steady part of axial velocity in case of Newtonian fluid is higher than that for a second-order fluid in the first half while lower in the second half of the gaplength.

The transverse shearing stress on the lower and upper disc respectively is obtained as

$$(\tau_{\theta z})_{z=0} = \frac{\mu_1 \xi \Omega}{\cosh 2\alpha - \cos 2\beta}[s_1 \{(\alpha \sinh 2\alpha + \beta \sin 2\beta)(\tau_1 \sin \tau - \cos \tau) + A\epsilon (\cosh 2\alpha - \cos 2\beta)\{(a\tau_1 + b)\sin \tau + (b\tau_1 - a)\cos \tau)\} - (\sin \tau + \tau_1 \cos \tau)(\alpha \sin 2\beta - \beta \sinh 2\alpha)\} + s_2 \{2\{\cos \tau - \tau_1 \sin \tau + A\epsilon \{(a - b\tau_1)\cos \tau - (b + a\tau_1)\sin \tau\}\}(\beta \cosh \alpha \sin \beta + \alpha \sinh \alpha \cos \beta) + 2\{\sin \tau + \tau_1 \cos \tau + A\epsilon \{(a - b\tau_1)\sin \tau + (b + a\tau_1)\cos \tau)\}\}(\alpha \cosh \alpha \sin \beta - \beta \sinh \alpha \cos \beta)\}] \quad \ldots(29)$$

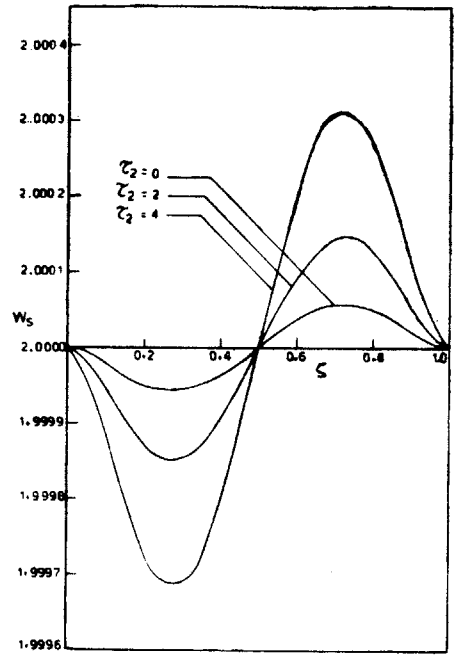

FIG. 4. Variation of steady axial velocity W_s with τ_2.

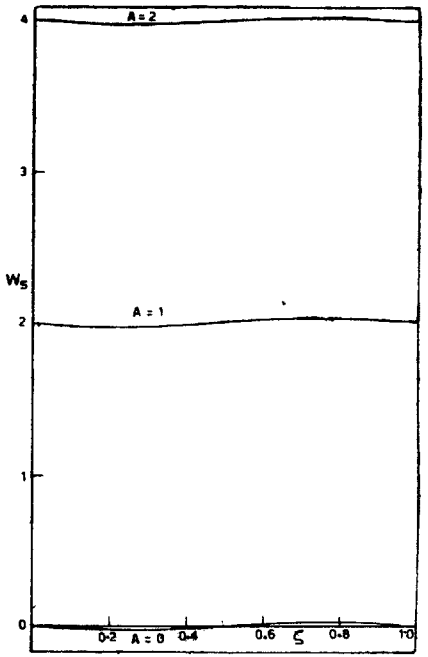

FIG. 5. Variation of steady axial velocity W_s with A.

and

$$(\tau_{\theta z})_{z=d} = \frac{\mu_1 \xi \Omega}{\cosh 2\alpha - \cos 2\beta} [s_1 \{2 \{(\tau_1 \sin \tau - \cos \tau) + A\epsilon \{(a - b\tau_1) \cos \tau - (b + a\tau_1) \sin \tau\}\} (\alpha \sinh \alpha \cos \beta + \beta \cosh \alpha \sin \beta) - 2 \{\sin \tau + \tau_1 \cos \tau - A\epsilon \{(a - b\tau_1) \times \sin \tau + (a\tau_1 + b) \cos \tau\}\} (\alpha \cosh \alpha \sin \beta - \beta \sinh \alpha \cos \beta)\} + s_2 \{(\cos \tau - \tau_1 \sin \tau) (\alpha \sinh 2\alpha + \beta \sin 2\beta) + A\epsilon \{(a\tau_1 + b) \sin \tau + (b\tau_1 - a) \cos \tau\} (\cosh 2\alpha - \cos 2\beta) + (\sin \tau + \tau_1 \cos \tau) (\alpha \sin 2\beta - \beta \sinh 2\alpha)\}]. \quad ...(30)$$

The moment on the upper or lower disc can be obtain by the expression

$$M = -\int_0^{\xi} 2\pi \xi^2 (\tau_{\theta z})_{\zeta=1 \text{ or } 0} \, d\xi. \quad ...(31)$$

It is obtained from the expressions that at $A = 0$, the transverse shearing stress and the moment on the disc $z = 0$ are equal to those for the disc $z = d$ in case of $s_1 = 1$, $s_2 = -1$ for all values of R, τ and τ_2.

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