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UNSTEADY FREE CONVECTION MHD BOUNDARY LAYER FLOW NEAR A THREE-DIMENSIONAL STAGNATION POINT

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The unsteady free convection boundary layer hydromagnetic flow near a stagnation point of a three-dimensional body with applied magnetic field and time-dependent wall temperature has been studied. Both semi-similar and self-similar cases have been considered. The equations governing the above flow have been solved numerically using an implicit finite-difference scheme due to Keller. The magnetic field is found to reduce both the heat transfer and skin friction. The effect of the variation of the wall temperature with time and of mass transfer is found to be more pronounced on the heat transfer than on the skin friction. In self-similar case, for decelerating flow, there is temperature overshoot in the presence of magnetic field, but in semi-similar case overshoot occurs even without magnetic field due to the unsteadiness.

1. INTRODUCTION

Transient free convection occurs in a fluid when the temperature changes cause density variations which give rise to buoyancy forces. Excellent reviews of free convection problems are given by Ede[1] and Gebhart[2]. This process of heat transfer has several engineering applications such as in nuclear reactor, power transformers etc. It is observed that in a nuclear reactor considerable reduction in heat transfer occurs by applying a magnetic field. Recently, Brar[3] considered the unsteady free convection laminar flow past an infinite plate in the presence of a uniform magnetic field. Subsequently, Soundalegekar *et al.*[4] studied the effect of mass transfer and convection currents in MHD Stokes flow problem for a vertical plate. However, the unsteady free convection flow over three-dimensional bodies in the presence of a magnetic field has not been studied so far.

The objective of this paper is to present the results of unsteady laminar incompressible free convection boundary layer flow of an electrically conducting fluid at the three-dimensional stagnation point in the presence of an applied magnetic field. The unsteadiness in the flow field has been caused by the wall temperature which varies with time. Both semi-similar and self-similar solutions have been obtained. The resulting equations have been solved numerically using an implicit finite-difference scheme[5-7].

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The results have been compared with the available results[6,9] and found to be in excellent agreement.

2. Governing Equations

2.1 *Semi-similar Equations*

Let us consider the unsteady laminar free convection incompressible flow of an electrically conducting fluid with an applied magnetic field B_0 in the z direction and also with mass transfer on the surface at a three-dimensional stagnation point with time-dependent wall temperature. The induced magnetic field is considered to be every small compared to the applied magnetic field and Hall effect is also neglected. The free stream temperature is assumed to be constant and also the dissipation terms and Joule heating are considered to be neglegible at the stagnation point. A locally orthogonal set of coordinates oxyz at the surface of a three-dimensional body is chosen in such a way that the origin coincides with the lowest stangnation point and x and y denote the (orthogonal) coordinates in the wall surface and z denotes the coordinate which is perpendicular to the wall. The appropriate semi-similar boundary-layer equations governing the above flow under Boussinesq's approximations in dimensionless form can be expressed as[8,9].

$$F'' + (f + cs)\,F' - F^2 - MF + \varphi\,G - F_{t^*} = 0 \qquad \ldots(1a)$$

$$S'' + (f + cs)\,S' - cS^2 - MS + \varphi\,G - S_{t^*} = 0 \qquad \ldots(1b)$$

$$Pr^{-1}\,G'' + (f + cs)\,G' - \varphi^{-1}\,\varphi_{t^*}\,G - G_{t^*} = 0. \qquad \ldots(1c)$$

The boundary conditions can be expressed as

$$\left.\begin{aligned} &F = S = 0,\ G = 1 \text{ at } \eta = 0 \\ &F = S = G = 0 \quad \text{as } \eta \to \infty \end{aligned}\right\}\ t^* > 0. \qquad \ldots(2)$$

The initial conditions (i. e., at $t^* = 0$) can be written as

$$F'' + (f + cs)\,F' - F^2 - MF + G = 0 \qquad \ldots(3a)$$

$$S'' + (f + cs)\,S' - cS^2 - MS + G = 0 \qquad \ldots(3b)$$

$$Pr^{-1}\,G'' + (f + cs)\,G' = 0 \qquad \ldots(3c)$$

where

$$\left.\begin{aligned} &\eta = Gr^{1/4}\,z/L,\ Gr = \beta g\,(T_{w0} - T_\infty)\ L^3\,\nu^{-2} \\ &t^* = \nu\,L^{-2}\,Gr^{1/2}t,\ u = \nu\,L^{-2}\,x\,Gr^{1/2}\,F(\eta, t^*) \end{aligned}\right\} \qquad \ldots(4a)$$

$$\left.\begin{aligned} &v = \nu\,cL^{-2}\,yGr^{1/2}\,S(\eta, t^*),\ w = -\nu\,L^{-1}\,Gr^{1/4}\,(f + cs) \\ &(T - T_\infty)/(T_w - T_\infty) = G(\eta, t^*),\ (T_w - T_\infty)/(T_{w0} - T_\infty) = \varphi(t^*) \end{aligned}\right\} \qquad \ldots(4b)$$

$$M = \sigma B_0^2 \; L^2 (Gr)^{-1/2}/\mu, \; f' = F, \; s' = S, f = \int_0^\eta F \, d\eta + f_w$$

$$s = \int_0^\eta S \, d\eta, \; s_w = 0, \; f_w = - (w)_w \; L \, Gr^{-1/4} \, \nu^{-1}. \qquad \text{...(4c)}$$

Here x, y and z are principal, transverse and normal directions, respectively; u, v and w are the velocity components in the x, y and z directions, respectively; η is the dimensionless independent variable; t and t^* are the dimensional and dimensionless times, respectively; f and s are the dimensionless stream functions; F and S are the dimensionless velocity components in the x and y directions, respectively; g is the acceleration due to gravity; T and G are the dimensional and dimensionless temperatures, respectively ; Gr is the Grashof number; Pr is the Prandtl number; M is the magnetic parameter; σ is the electrical conductivity; B_0 is the applied magnetic field; L is the characteristic length; T_w and T_{w0} are the temperatures at any time t^* and at $t^*=0$, respectively; μ is the coefficient of viscosity; c is the parameter characterizing the nature of the three-dimensional stagnation-point flow ($c = 0$ and 1 for two-dimensional and axisymmetric stagnation-point flows, respectively); T_∞ is the free stream temperature; ν is the kinematic viscosity; prime denotes derivatives with respect to η and the subscript t^* denotes the partial differentiation with respect to t^*. f_w is the surface mass transfer parameter ($f_w \gtrless 0$ according to whether there is suction or injection) and f_w is a constant if the normal velocity at the wall $(w)_w$ is taken as a constant as L, Gr and ν are all constants. Also $\varphi(t^*)$ is an arbitrary function of t^* indicating the unsteadiness in the wall temparature and has a continuous first derivative for $t^* \geqslant 0$. It has been assumed that the flow is steady and changes to unsteady for $t^* > 0$. The steady-state equations (3) in the absence of the magnetic field (i. e. $M = 0$) are essentially the same as those of Poots[8] except for some minor differences (which are due to slightly different transformations). However, they are identical to those of Poots[8] for $c = 0$ and 1. Also, they are identical with those of Banks[9] if the terms cs is replaced by $\tilde{s}$. It may be remarked that the unsteady three dimensional stagnation point boundary layer equations with four independent variables (x, y, z, t) can be reduced to semi-similar equations with two independent variables (η, t^*) as given in (1) by using the transformations (4).

The skin-friction and heat-transfer coefficients can be written as

$$C_f = \mu \, (\partial u/\partial z)_w/[\rho \, \nu^2 \, x/L^3] \; = Gr^{3/4} \; F'_w$$

$$C_f = \mu \, (\partial v/\partial z)_w/[\rho \, \nu^2 \; y/L^3] \; = Gr^{3/4} \; c \, S'_w \qquad \text{...(5)}$$

$$Nu = L \, (\partial T/\partial z)_w/(T_{w0} - T_\infty) = Gr^{1/4} \, \varphi(t^*) \, G'_w$$

where C_f and $\bar{C}_f$ are the skin-friction coefficients in the x and y directions, respectively F'_w and S'_w are the skin-friction parameters in the x and y directions, respectively; Nu and G'_w are, respectively, the heat-transfer coefficient (Nusselt number) and heat-transfer parameter and ρ is the density.

2.2. *Self-similar Equations*

It has been found that if the wall temperature varies inversely as a quadratic function of time t^* and the normal velocity at the wall as a squareroot of a linear function of time then the unsteady three-dimensional stagnation-point boundary layer equations with four independent variables admit similarity solutions (i. e., the governing equations are reduced to ordinary differential equations with η as the independent variable under suitable transformations (8)]. The self-similar equations are given by

$$F''' + (f + cs)\, F'' - F^2 - MF + G - \alpha\,(F + \eta F'/2) = 0 \qquad \ldots(6a)$$

$$S''' + (f + cs)\, S'' - cS^2 - MS + G - \alpha\,(S + \eta\, S\,/2) = 0 \qquad \ldots(6b)$$

$$Pr^{-1}\, G'' + (f + cs)\, G' - \alpha\,(2G + \eta G') = 0. \qquad \ldots(6c)$$

The boundary conditions are

$$\left.\begin{aligned} &F = S = 0, \quad G = 1 \text{ at } \eta = 0 \\ &F = S = G = 0 \text{ as } \eta \to \infty \end{aligned}\right\} \qquad \ldots(7)$$

where

$$\eta = Gr^{1/4}\, L^{-1}\, z\, (1 - \alpha t^*)^{-1/2}$$

$$u = \nu\, L^{-2}\, x\, Gr^{1/2}\, (1 - \alpha t^*)^{-1}\, F \qquad \ldots(8a)$$

$$v = \nu\, cL^{-2}\, y\, Gr^{1/2}\, (1 - \alpha t^*)^{-1}\, S$$

$$\left.\begin{aligned} &w = -\nu\, L^{-1}\, Gr^{1/4}\, (1 - \lambda t^*)^{-1/2}\, (f + cs) \\ &(T - T_\infty)/(T_w - T_\infty) = G,\ (T_w - T_\infty)/(T_{w0} - T_\infty) = (1 - \alpha t^*)^{-2} \end{aligned}\right\} \qquad \ldots(8b)$$

$$f_w = -(w)_w\, L\, \nu^{-1}\, (Gr)^{-1/4}\, (1 - \alpha t^*)^{1/2}. \qquad \ldots(8c)$$

It is evident from (8c) that if the normal velocity at the wall $(w)_w$ varies as $(1 - \alpha t^*)^{-1/2}$, then the mass transfer parameter f_w will be a constant as L, ν and Gr are constants.

The skin-friction and heat-transfer coefficients can be expressed as

$$C_f = \mu\, (\partial u/\partial z)_w/[\rho\, \nu^2\, x/L^3] = \bar{G}r^{3/4}\, F'_w$$

$$\bar{C}_f = \mu\, (\partial v/\partial z)_w/[\rho\, \nu^2 y/L^3] = \bar{G}r^{3/4}\, c\, S'_w$$

$$Nu = L\,(\partial T/\partial z)_w/(T_w - T_\infty) = \bar{G}r^{1/2}\,G_w'$$

$$\bar{G}r = \beta g\,(T_w - T_\infty)\,L^3\,\nu^{-2}.$$

3. Results and Discussion

3.1. *Semi-similar results*

The set of partial differential equations (1) under boundary conditions (2) and initial conditions (3) has been solved numerically using a comparatively new and efficient implicit finite-difference scheme developed by Keller[5]. In recent years, this scheme has been widely used for the solution of variety of boundary layer problems[6,7]. We have considered the effect of step size $\Delta\eta$ and Δt^* and the edge of the boundary layer represented by η_∞ on the solution with a view to optimize them. Finally, the computations were carried out with $\Delta\eta = 0.01$, $\Delta t^* = 0.05$, and $4 \leqslant \eta_\infty \leqslant 8$ and the results presented here are independent of the step size and η_∞ at least up to 4th decimal place. The variation of wall temperature with time has been taken to be of the form $\varphi\,(t^*) = 1 - \epsilon t^{*2}$. Since most three-dimensional bodies of practical interest lie between cylinder ($c = 0$) and sphere ($c = 1$), we have confined our studies to the range $0 \leqslant c \leqslant 1$.

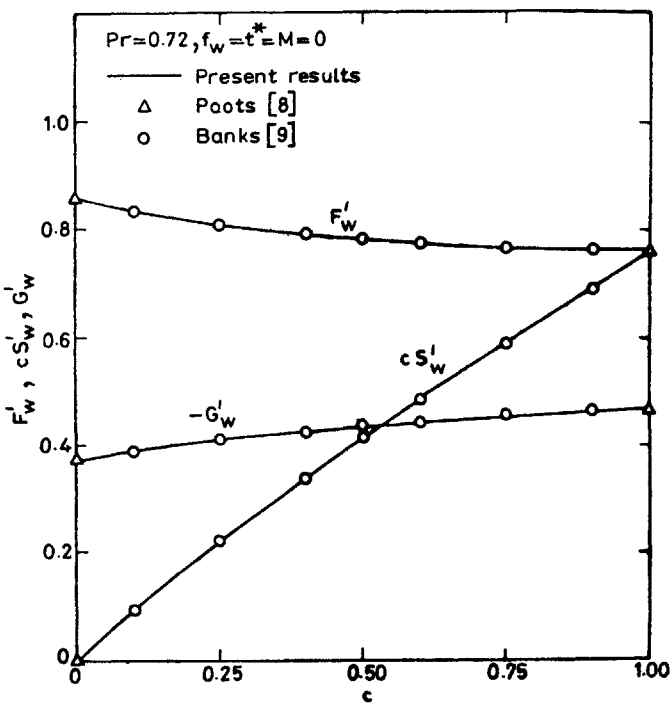

Fig. 1. Comparison of skin-friction and heat-transfer results for steady-state case.

With a view to assess the accuracy of the present method, the skin friction and heat transfer results for the steady-state case ($t^* = 0$) have been compared with those of Poots[8] and Banks[9] and they are found to be in excellent agreement (Fig. 1).

The effect of magnetic parameter M, time t^*, and mass transfer parameter f_w on the skin friction and heat transfer parameters $\left(F'_w , S'_w , -G'_w \right)$ has been shown in Figs. 2 a, b. It is observed that both skin friction and heat transfer reduce as the magnetic parameter M or time t^* or injection rate ($f_w < 0$) increases. The effect of magnetic parameter M is found to be more pronounced on F'_w, S'_w and G'_w for small or

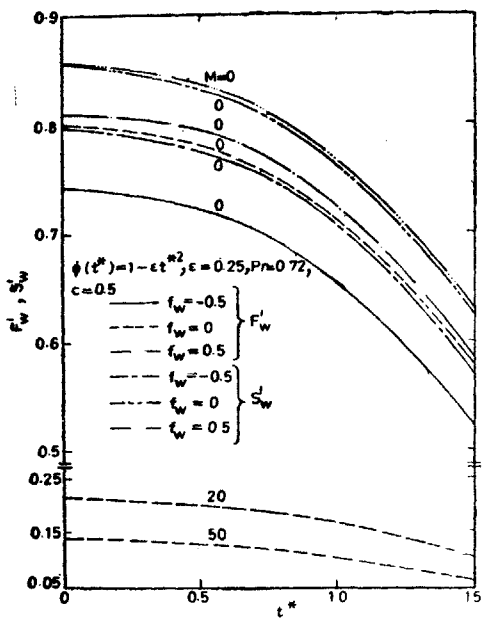

FIG. 2 (a) Effect of magnetic parameter M and mass transfer parameter f_w on skin-friction parameters (semi-similar flows)

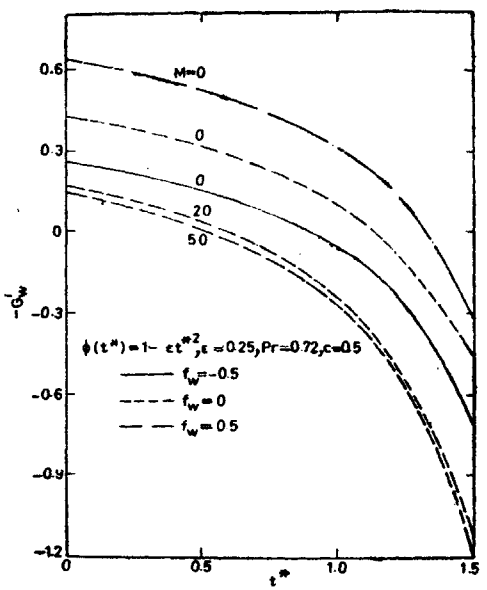

FIG. 2 (b) Effect of magnetic parameter M and mass transfer prrameter f_w on heat transfer parameter (semi-similar flow).

moderate values of M ($M < 20$) then for large value of M ($M \geqslant 20$). This implies that the skin friction and heat transfer reach nearly an asymptotic state when M becomes large. Furthermore, for $M \geqslant 20$, S'_w is nearly same as F'_w and therefore, it has not been shown in Fig. 2a. The effect of t^* is found to be more pronounced on the heat transfer as compared to the skin friction. The effect of suction is just opposite to that of injection. The reason for the above behaviour of skin friction and heat transfer with M, t^* and f_w ($f_w < 0$) is due to the increase in the boundary layer thickness as these parameters increase whch results in the reduction of the heat transfer and

skin friction. Also, we find that the heat-transfer parameter $-G'_w \gtrless 0$ as $t^* \lesssim t_1^*$ (for example, $t_1^* \simeq 0.6$ for $M = 20$ and $f_w = 0$) which implies that the heat is transferred from the wall to the fluid when $t^* < t_1^*$ and there is no heat transfer when $t^* = t_1^*$ (Fig. 2b). Such a behaviour of heat transfer can be attributed to the reduction of wall temperature with time, the wall temperature at $t^* = 0$ being maintained at a higher temperature than the free stream temperature. When the wall temperature is considered to be increasing with time $\varphi(t^*) = 1 + \epsilon t^{*2}$ with initial wall temperature higher than the free stream temperature (i. e., $T_{w0} > T_\infty$), there is a transfer of heat from the wall to the fluid for all t^* (not shown in the figure).

The effect of c, the parameter characterizing the nature of stagnation-point flow, on the skin friction and heat transfer $\left(F'_w \;\; S'_w \;, -G'_w\right)$ is shown in Fig. 3. It is observed that in contrast to the forced flow[10], the skin friction parameters F'_w and S'_w increase as c decreases from 1 to 0 for all values of t^*, but the heat transfer parameter $-G'_w$ decreases.

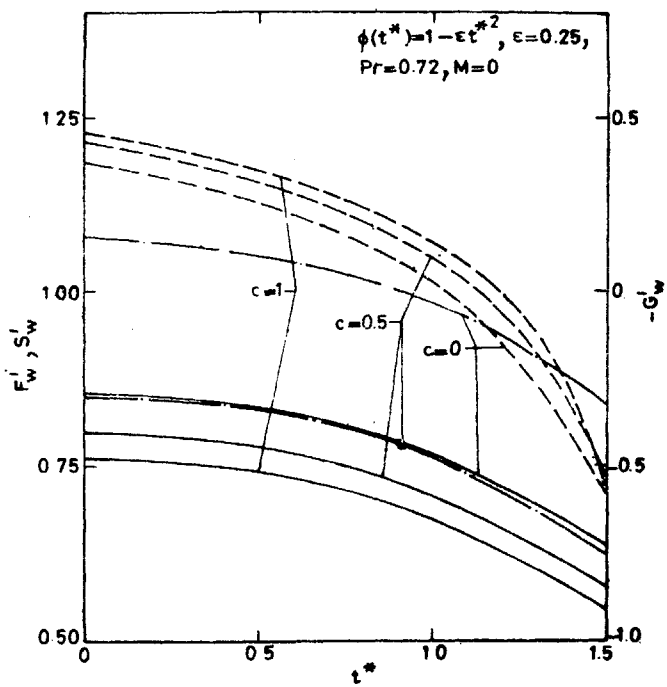

FIG. 3. Effect of c on skin-friction and heat-transfer parameter (semi-similar flow).

The velocity profile in th x direction F and the temperature profile G are shown in Fig. 4. Since the behaviour of the velocity profile in the y direction S is qualitatively

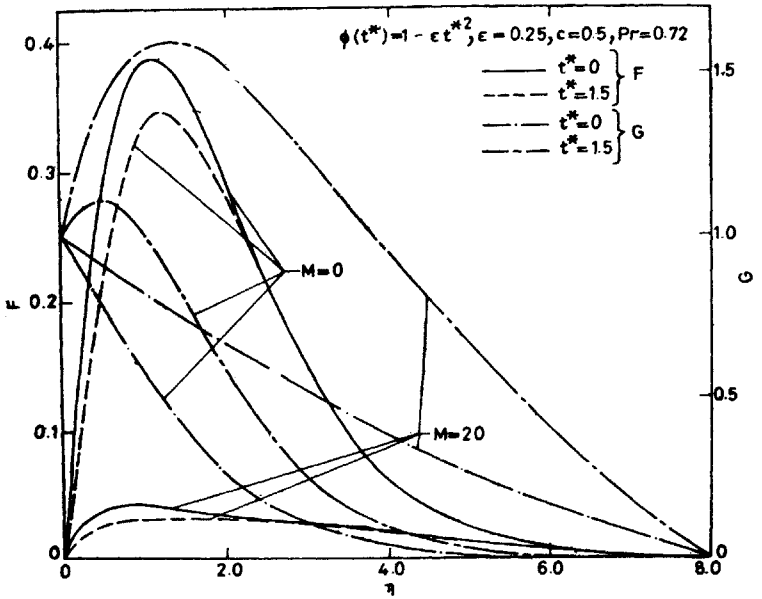

FIG. 4. Velocity profiles in the x direction and temperature profiles (semi-similar flow).

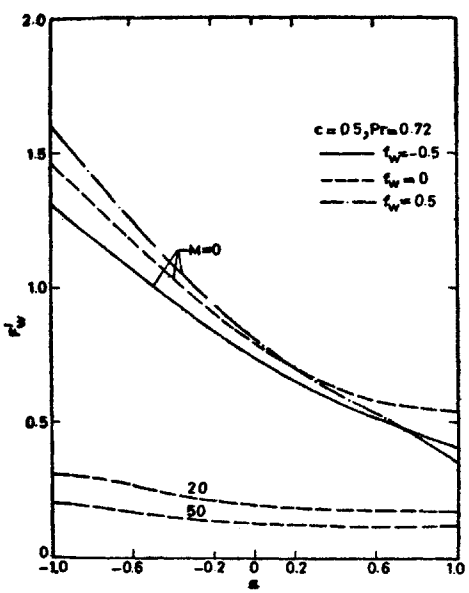

FIG. 5(a) Effect of magnetic parameter M and mass transfer parameter f_w on skin-friction parameter in the x direction (self-similar flow).

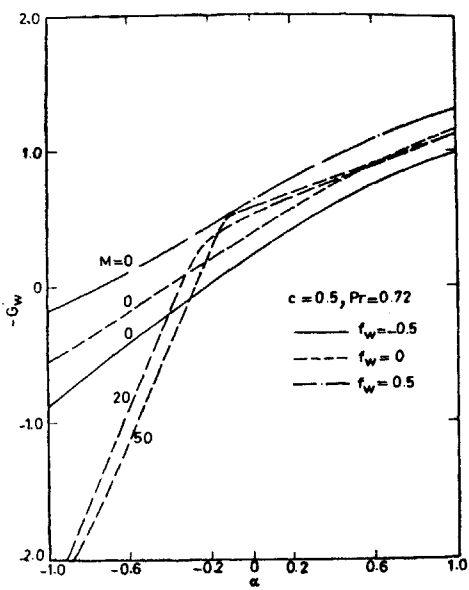

FIG. 5(b) Effect of magnetic parameter M and mass transfer parameter (self-similar flow).

same as that of F, it is not shown in the figure. It is observed that the maximum value of F decreases as M or t^* increases. Also, the temperature profile G for $t^* = 1.5$ exceeds the free stream value near the wall and the magnitude of this "overshoot" increcases with magnetic parameter M. This implies that the heat is being transferred from the fluid to the wall so that the wall is heated instead of being cooled. Although the wall is maintained at a higher temperature than the free stream temperature at $t^* = 0$. This is due to the reduction of the wall temperature with time and direction of the heat transfer is reversed when the wall temperature becomes less than the free stream temperature.

3.2. *Self-similar Results*

The governing equations 6 (a) — (c) under boundary conditions (7) have been solved numerically as in the case of semi-similar case and the results are presented in Figs. 5 — 7. Figures 5 a-b present the effect of magnetic parameter M and mass transfer f_w on the skin-friction parameter in the x direction F'_w and the heat-transfer

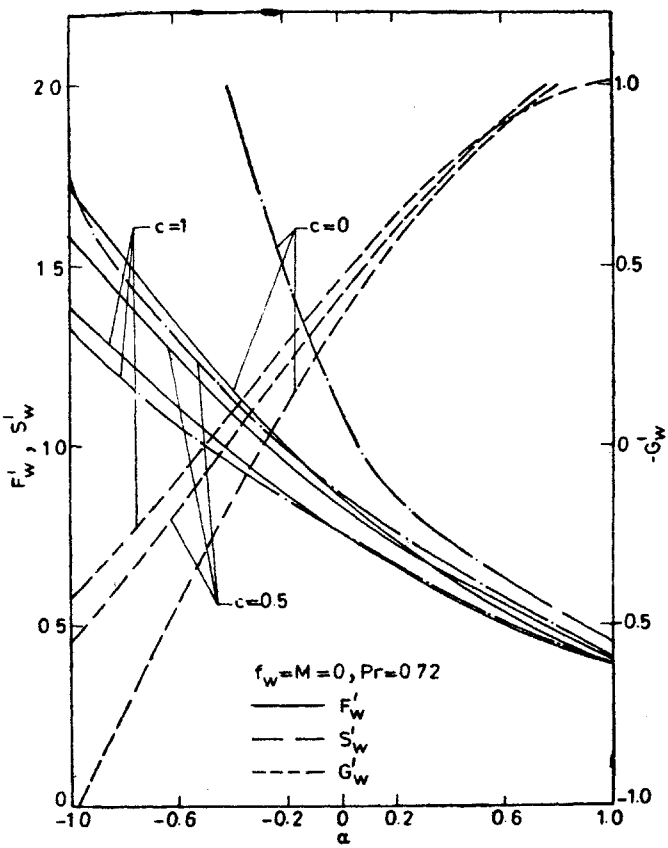

FIG. 6 Effect of c on skin-friction and heat-transfer parameters (self-similar flow).

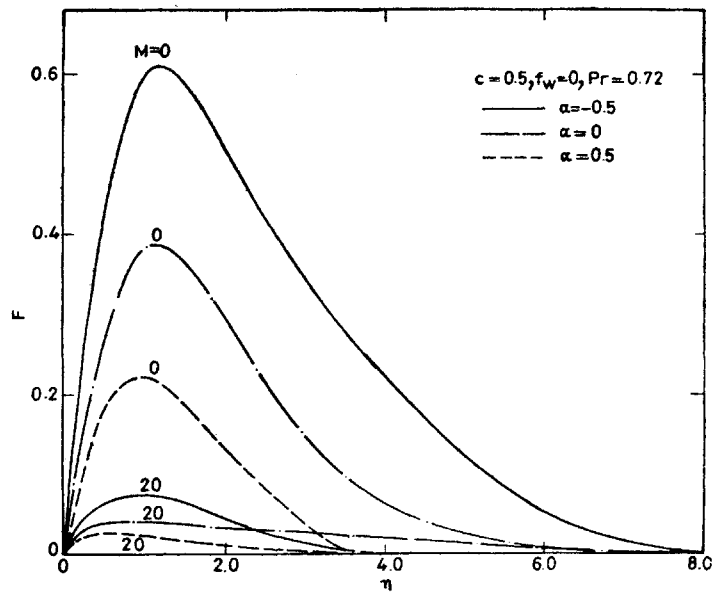

FIG. 7(a) Velocity profiles in the x direction (self-similar flow).

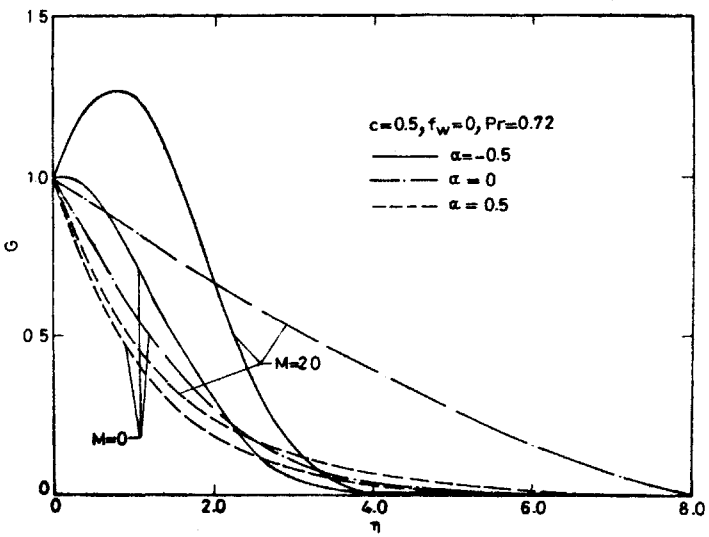

FIG. 7 (b) Temperature profiles (self-similar flow).

parameter $-G'_w$. Since the behaviour of the skin-friction parameter in the y direction S'_w is qualitatively same as that of F'_w it is not shown here to reduce the number of figures. It is found that F'_w or S'_w decreases as M increases whether the flow is accelerating

($\alpha > 0$) or decelerating ($\alpha < 0$). The effect of α on F'_w or S'_w is less pronounced for moderate and large values of M ($M \geqslant 20$). The effect of mass transfer on skin friction and heat transfer is more pronounced for decelerating flows ($\alpha < 0$) than for accelerating flows ($\alpha > 0$). In contrast to the skin friction case, the heat transfer is strongly affected by α for all values of M. It is observed that the heat transfer parameter $-G'_w \gtrless 0$ according to whether $\alpha \lesseqgtr \alpha_1$ ($\alpha_1 \simeq -0.35$ for $M = 20$, and $f_w = 0$). This indicates that the heat transfer takes place from the wall to the fluid for $\alpha > \alpha_1$ and from the fluid to the wall for $\alpha < \alpha_1$ and there is no transfer of heat for $\alpha = \alpha_1$.

The effect of c on the skin friction and heat transfer $\left(F'_w, S'_w, -G'_w\right)$ is presented in Fig. 6. It is observed that F'_w and S'_w increase as c decreases for all values of α, but $-G'_w$ decreases for $\alpha \leqslant 0.25$.

The velocity profile in the x direction F is shown in Fig. 7a and the temperature profile G in 7b. Since the behaviour of S is qualitatively same as that F, it is not shown here. We find that the maximum value of F is more for decelerating flow ($\alpha < 0$) than for acceleating flow ($\alpha > 0$) and the magnetic parameter M reduces it considerably. It is also observed that for decelerating flow ($\alpha = -0.5$) and for magnetic parameter $M = 20$, the temperature G near the wall exceeds the free stream temperature. This implies that the heat is being transferred from the fluid to the wall and therefore the wall will be heated instead of being cooled, although the wall is maintained initially at a higher temperature than the free stream temperature. Similar behaviour has been observed for semi-similar cases discussed earlier.

4. Conclusions

The heat transfer and skin friction are found to decrease as the magnetic parameter or time or injection rate increases. However, the effect of large value of the magnetic parameter is rather weak. As the wall temperature varies with time, the direction of heat transfer changes. The nature of stagnation point flows also exerts a strong influence on the skin friction and heat transfer. The unsteadiness in the flow field and the magnetic field cause the fluid to attain higher temperature than the free stream, although initially the wall is maintained at a higher temperature than the free stream temperature.

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