Indian J. pure appl. Math., 16 (9) : 1106-1113, September 1986

PAIRWISE SET-CONNECTED MAPPINGS IN BITOPOLOGICAL SPACES

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(*Received* 25 *July* 1985)

In this paper, attempts have been made to generalize the concept of set-connected mappings in bitopological spaces. Such a mapping has been characterized and some of its properties have been studied. Finally, its relationships with pairwise continuous and pairwise weakly continuous mappings have been investigated.

1. INTRODUCTION

Kwak[5] introduced in a topological space a new type of mapping, called set-connected mapping which is weaker than a continuous map. Noiri[6] further investigated different properties of this mapping and showed that every weakly continuous surjection is set connected, but not conversely. In this note, we have studied the corresponding results in the bitopological setting. Since a bitopological space, as introduced by Kelly[4] as a natural structure, is a richer structure than that of a topological space, it is of much use and importance to study the generalizations of topological notions and implications in bitopological situation. Apart from this, it is interesting to notice the interactions of the two topologies in a bitopological space with reference to a study of a bitopological notion. These are the underlying reasons for extending and studying set connected maps in a bitopological space.

Section 2 opens up with the study of different properties of p-connected mapping (henceforth, the word "pairwise" shall be abbreviated as "$p-$"), specially in relation to its inherent applications to p-connected spaces, as introduced by Pervin[7]. Relations of this mapping with p-continuous and p-weakly continuous mappings have been discussed in section 3 and 4 respectively.

Throughout this note X and Y always represent bitopological spaces (X, Q_1, Q_2) and (Y, P_1, P_2) respectively and $f: X \to Y$ denotes a mapping from X into Y. If $A \subset X$, then the subspace A is the bitopological space $(A, (Q_1)_A, (Q_2)_A)$ induced by

(X, Q_1, Q_2). $Q_i-\text{cl } A$ and Q_i-int A respectively shall denote the closure and interior of A in (X, Q_i), for $i = 1, 2$. An ij-coset in any space (X, Q_1, Q_2) is a Q_i-closed and Q_j-open set in X and i, j can assume values 1 and 2 with $i \neq j$.

2. *P*-set Connected Mappings

Definition 2.1—A bitopological space X is said to be 12-connected (21-connected) between A and B (where A, B are non-empty subsets of X) iff there exists no 12-coset (21-coset) F such that $A \subset F \subset X - B$.

Clearly, X is 12-connected between A and B iff it is 21-connected between B and A.

Definition 2.2—X is said to be p-connected between A and B (where A, B are nonempty subsets of X) iff X is 12-connected as well as 21-connected between A and B.

Remark 2.3 : If X is ij-connected between A and B and $C \supset A$, $D \supset B$, then X is ij-connected between C and D.

Definition 2.4—A mapping $f: X \to Y$ is said to be p-set connected iff $f(X)$ is ij-connected between $f(A)$ and $f(B)$ in the subspace bitopology whenever X is ij-connected between A and B, where $i, j = 1, 2$ and $i \neq j$.

Remark 2.5 : It is clear that the definition 2.4 can equivalently be modified by writting 12 or 21 in place of ij.

Lemma 2.6—If a subspace M of a bitopological space X is ij-connected between A and B then so is the whole space (where, as usual, $i, j = 1, 2$ and $i \neq j$).

Proof : If there is any ij-coset F in the whole space X with $A \subset F \subset X-B$, then $F \cap M$ is ij-coset in M with $A \subset F \cap M \subset M - B$.

Lemma 2.7—A subspace M which is 12- (or 21-) coset in X, is ij-connected between A and B if X is ij-connected between two subsets A and B of M.

Proof : If F is any 12-coset in M, then F is also 12-coset in X, as M is 12-coset in X. This fact implies that M is 12-connected between A and B if X is so. Now if X is 21-connected between A and B then it is 12-connected between B and A; so M is 12-connected between B and A which implies that M is 21-connected between A and B. Similar argument is applicable when M is 21-coset in X. Hence the lemma is proved.

Theorem 2.8—A mapping $f: X \to Y$ is p-set connected iff $f^{-1}(F)$ is 12-coset in X for any 12-coset F in $f(X)$.

Proof : Let f be p-set connected and F be any 12-coset in $f(X)$. If $f^{-1}(F)$ is not 12-coset in X then X is 12-connected between $f^{-1}(F)$ and $X - f^{-1}(F)$ and therefore,

as f is p-set connected, $f(X)$ is 12-connected between $f(f^{-1}(F))$ and $f(X - f^{-1}(F))$. But $f(f^{-1}(F)) = F \cap f(X) = F$ and $f(X - f^{-1}(F)) = f(X) - F$ imply that F is not 12-coset in $f(X) - a$ contradiction. Hence $f^{-1}(F)$ is 12-coset in X.

Conversely, let the condition hold for f and let X be 12-connected between A and B. If $f(X)$ is not 12-connected between $f(A)$ and $f(B)$, then there exists a 12-coset F in $f(X)$ such that $f(A) \subset F \subset f(X) - f(B)$. But we find that $A \subset f^{-1}f(A) \subset f^{-1}(F), f^{-1}(F) \subset X - B$ and $f^{-1}(F)$ is 21-coset in X. This contradicts that X is 12-connected between $f(A)$ and $f(B)$ i.e., f is p-set connected.

Remark 2.9 : (a) Similarly as in the above theorem, we can show that f is p-set connected iff $f^{-1}(F)$ is 21-coset in X for any 21-coset F in $f(X)$. (b) If f is p-set connected, then $f^{-1}(F)$ is 12-coset (21-coset) in X for any 12-coset (21-coset) F in Y.

Definition 2.10[7]—A bitopological space (X, Q_1, Q_2) is called pairwise connected (p-connected, according to our abbreviation) iff X cannot be expressed as the union of two non-empty disjoint sets A and B such that $(A \cap Q_1 - \text{cl } B) \cup (Q_2 - \text{cl } A \cap B) = \phi$.

It is shown[7] that $(X, Q_1 Q_2)$ is pairwise connected iff X cannot be expressed as the union of two non-empty disjoint sets A and B such that A is Q_1-open and B is Q_2-open.

Now Lemmas 2.11 and 2.12, stated below, follow immediately.

Lemma 2.11—Every mapping $f: X \to Y$ such that $f(X)$ is p-connected, is p-set connected.

Lemma 2.12—Let $f: X \to Y$ be a p-set connected mapping. If X is p-connected, then $f(X)$ is p-connected.

Lemma 2.13—Let $f: X \to Y$ be p-set connected and A ($\subset X$) be such that $f(A)$ is 12-coset (or 21-coset) in $f(X)$. Then the restriction $f|A : A \to Y$ is p-set connected.

PROOF : Let A be 12-connected between C and D. By Lemma 2.6, X is 12-connected between C and D, so $f(X)$ is 12-connected between $f(C)$ and $f(D)$. As $f(A)$ is 12-coset (or 21-coset) in $f(X)$, by Lemma 2.7, $f(A)$ is 12-connected between $f(C)$ and $f(D)$. Hence by Remark 2.5, the lemma is proved.

Theorem 2.14—Let $f: X \to Y$ be p-set connected, b_i-open (i.e., $f: (X, Q_i) \to (Y, P_i)$ is open for $i = 1, 2$), surjection and $f^{-1}(y)$ be p-connected for each y of Y. Then for any 12- (or 21-) coset F in Y, F is p-connected iff $f^{-1}(F)$ is p-connected.

PROOF : We prove the theorem for the case when F is 12-coset in Y. Similar will be the proof when F is 21-coset. Let $f^{-1}(F)$ be not p-connected for some 12-coset F in Y. Then there exists a 12-coset K with $\phi \neq K \neq f^{-1}(F)$, in the subspace bitopological space $(f^{-1}(F), Q_1|f^{-1}(F), Q_2|f^{-1}(F))$. We show that $f(K)$ is 12-coset in F with $\phi \neq f(K) \neq F$. As $f^{-1}(y)$ is p-connected, either $f^{-1}(y) \subset K$, or $f^{-1}(y) \subset f^{-1}(F) - K$, for all $y \in F$ and so $f(K) \neq F (\neq \phi)$ and $f(K) \cap f(f^{-1}(F) - K)$

$= \phi$. As f is surjective, $f(K) \cup f(f^{-1}(F) - K) = F$ and so $f(f^{-1}(F) - K) = F - f(K)$. Now f being *bi*-open $f|f^{-1}(F)$ is *bi*-open onto F and hence $f(K)$ is 12-coset in F. This implies that F is not p-connected. So F is p-connected implies $f^{-1}(F)$ is p-connected.

Next, since $f(f^{-1}(F)) = F$ which is 12-coset in Y, by Lemma 2.13, the restriction $f|f^{-1}(F) : f^{-1}(F) \to Y$ is p-set connected. Now by Lemma 2.12 it follows that $f|f^{-1}(F)(f^{-1}(F)) = F$ is p-connected if $f^{-1}(F)$ is p-connected. Hence the theorem.

Theorem 2.15—Let $f : X \to Y$ be a p-set connected mapping. Then for each $p \in f(X)$, $f(Q_i - \text{cl } f^{-1}(p)) \subset C_p$, where C_p is any ij-coset in $f(X)$ (or in Y) containing the point p (where, as usual, $i, j = 1, 2$ and $i \neq j$).

PROOF : As C_p is ij-coset in $f(X)$ (or in Y) and f is p-set connected $f^{-1}(C_p)$ is ij-coset in X (by Theorem 2.8 and Remark 2.9 (b)). Also $f^{-1}(p) \subset f^{-1}(C_p)$, therefore $Q_i - \text{cl } (f^{-1}(p)) \subset f^{-1}(C_p)$. Hence $f(Q_i - \text{cl } (f^{-1}(p)) \subset ff^{-1}(C_p) \subset C_p$ and the theorem is proved.

Theorem 2.16—Let (X, Q_1, Q_2) be a bitopological space. If a sequence $\{p_n\}$ converges to p in (X, Q_i) and also if p has a p-connected Q_i-open neighbourhood, then there exists some n_0 such that X is p-connected between p and p_n, for all $n \geq n_0$ $(i = 1, 2)$.

PROOF : If p and p_n are contained in a p-connected set then X is p-connected between p and p_n.

Definition 2.17[1]—Let (X, Q_1, Q_2) be a bitopological space. Q_1 is said to be locally connected with respect to Q_2 if for each point x in X and each Q_1-open neighbourhood U of x there is a pairwise connected Q_1-open set G such that $x \in G \subset U$. (X, Q_1, Q_2) is pairwise locally connected if Q_1 is locally connected w. r. t. Q_2 and Q_2 is locally connected w.r.t. Q_1.

Corollary 2.18—If a sequence $\{x_n\}$ converges to x in (X, Q_i) and Q_i is locally connected w.r.t. Q_j then there exists some n_0 such that X is p-connected between p and p_n, for all $n \geq n_0$, where $i = 1$ and $j = 2$, or $i = 2$ and $j = 1$.

Definition 2.19[3]—In a bitopological space (X, Q_1, Q_2), Q_i is said to be extremally disconnected w.r.t. Q_j iff Q_j-closure of any Q_i-open set is Q_i-open $(i, j = 1, 2$ and $i \neq j)$. The space is p-extremally disconnected iff Q_1 is extremally disconnected w.r.t. Q_2 and Q_2 is extremally disconnected q.r.t. Q_1.

Definition 2.20[4]—A bitopological space (X, Q_1, Q_2) is said to be pairwise Hausdorff if given distinct points x, y of X, there is a Q_1-open set U and a Q_2-open set V, such that $x \in U, y \in V, U \cap V = \phi$.

Theorem 2.21—Let $f : X \to Y$ be p-set connected and Y be p-Hausdorff and P_i is extremally disconnected w.r.t. P_j in Y. Then the graph $G(f)$ of f is closed in $(X \times Y, Q_j \times P_i)$ $(i, j = 1, 2$ and $i \neq j)$.

PROOF : Let $(x, y) \notin G(f)$ so that $y \neq f(x)$. Then y has a P_i-open neighbourhood whose P_j-closure, say V, does not contain $f(x)$ (since Y is p-Hausdorff). As P_i is extremally disconnected w.r.t. P_j, V is ji-coset in Y and so $f^{-1}(V)$ is ji-coset in X [by Remark 2.9 (b)] and $x \notin f^{-1}(V)$. Let $U = X - f^{-1}(V)$. Then U is Q_j-open in X containing x and $f(U) \cap V = \phi$. Thus $U \times V$ is an open neighbourhood of (x, y) in $(X \times Y, Q_j \times P_i)$ with $(U \times V) \cap G(f) = \phi$.
Hence $G(f)$ is closed in $(X \times Y, Q_j \times P_i)$.

Remark 2.22 : In the above theorem, one might expect that $G(f)$ is closed in $(X \times Y, Q_i \times P_i)$ (for $i = 1, 2$). That this is false is shown by the next example.

Example 2.23—Let $X = Y = R$ = real line. Let $Q_1 = P_1$ = cofinite topology on R and $Q_2 = P_2$ = discrete topology. Let $f : X \to Y$ be the identity map. Then f is clearly p-set connected, $(Y, P_1 P_2)$ is p-Hausdorff and P_1 is extremally disconnected w.r.t. P_2. But $G(f)$ is not closed in $(X \times Y, Q_1 \times P_1)$. In fact, $G(f)$ is dense in $(X \times Y, Q_1 \times P_1)$.

Theorem 2.24—Let $f : X \to Y$ be a mapping and $g : X \to (X \times Y, Q_1 \times P_1, Q_2 \times P_2)$ be given by $g(x) = (x, f(x))$, for $x \in X$. Then f is p-set connected if g is so.

PROOF : Let F be 12-coset in $f(X) \subset Y$. Then $X \times F$ is 12-coset in the subspace $X \times f(X) \subset X \times Y$ (we note that the topologies on $X \times f(X)$, namely $Q_i \times P_i / f(X)$ and that inherited as a subspace from $Q_i \times P_i$ are identical, for $i = 1, 2$). As $g(X) \subset X \times f(X)$, $(X \times F) \cap g(X)$ is 12-coset in $g(X)$ and so $g^{-1}[(X \times F) \cap g(X)]$ is 12-coset in X by Theorem 2.8. But $g^{-1}[(X \times F) \cap g(X)] = g^{-1}(X \times F) = f^{-1}(F)$. Hence f is p-set connected.

Remark 2.25 : Converse of the above theorem is false as is seen from example below.

Example 2.26—Let $X = Y = R$, the real line; $Q_1 = P_1$ = the left hand topology and $Q_2 = P_2$ = the right hand topology. Let $f : X \to Y$ be defined as $f(x) = x$ if $x \neq 0$ and $f(0) = -1$. Here f is p-set connected as Y has no nontrivial 12- or 21-coset. Now $[0, \infty) \times [0, \infty)$ is closed in $Q_1 \times P_1$, $(0,\infty) \times (0,\infty)$ is open in $Q_2 \times P_2$ and $([0,\infty)) \times [0,\infty)) \cap g(X) = ((0,\infty) \times (0,\infty)) \cap g(X) = K$ (say). Thus K is 12-coset in the subspace $g(X)$ of $X \times Y$. But X has no non-trivial 12-coset whereas $g^{-1}(K) = (0, \infty)$. Hence g is not p-set connected.

3. P-SET CONNECTED MAPPINGS AND P-CONTINUOUS MAPPINGS

Definition 3.1[7]—A mapping $f : X \to Y$ will be said to be p-continuous iff the induced maps $f : (X, Q_1) \to (Y P_1)$ and $f : (X, Q_2) \to (Y, P_2)$ are continuous.

By virtue of Theorem 2.8 it follows that :

Theorem 3.2—Every p-continuous mapping is p-set connected. The following example shows that the converse is false.

Example 3.3—Let $X = Y = R$, the real line, and $P_1 = Q_1 =$ the usual topology on R and $P_2 = Q_2 =$ the countable complement topology on R. Let $f : X \to Y$ be defined by $f(x) = x$, $x \neq 0$ and $f(0) = 1$. Then f is p-set connected, since there is no proper 12- or 21-coset in $f(X) = Y - \{0\}$. But $f : (X, Q_1) \to (Y, P_1)$ is not continuous at $x = 0$ and hence $f : X \to Y$ is not p-continuous.

Now, we proceed to find the conditions under which a p-set connected mapping is p-continuous.

Theorem 3.4—Let $f : X \to Y$ be p-set connected. If Y be p-Hausdorff and P_1 (or P_2) is extremally disconnected w.r.t. P_2 (or P_1), then $f|C : C \to Y$ is constant for every p-connected subset C of X.

PROOF : Let $x, y \in C$ and $x \neq y$. If possible, let $f(x) \neq f(y)$ in Y. Then there exists 21-coset V in Y such that $f(x) \in V$ and $f(y) \notin V$, since Y is p-Hausdorff and P_1 is extremaly disconnected w.r.t. P_2. Now, $f^{-1}(V)$ is 21-coset in X as f is p-set connected and therefore, $f^{-1}(V) \cap C$ is a non-empty proper 21-coset in the subspace C. This condradicts that C is p-connected. Hence $f(x) = f(y)$, for all $x, y \in C$, i.e., $f|C : C \to Y$ is constant.

Corollary 3.5—Let $f : X \to Y$ be p-set connected, Y be p-Hausdorff and P_1 (or P_2) is extremally disconnected w.r.t. P_2 (or P_1) and for each $x \in X$ it has a Q_i-open p-connected neighbourhood. Then $f^{-1}(y)$ is Q_i-open in X for each $y \in Y$, for $i = 1, 2$.

PROOF : Follows from the fact that for each $x \in f^{-1}(y)$ there is a Q_i-open p-connected set U with $x \in U \subset f^1(y)$.

Remark 3.6 : In the above corollary, if for each $x \in X$, x has a Q_i-open p-connected neighbourhood and a Q_2-open p-connected neighbourhood, then f is p-continuous. Thus

Corollary 3.7—If $f : X \to Y$ be p-set connected, X be p-locally connected and Y be p-extremally disconnected and p-Hausdorff, then f is p-continuous.

Definition 3.8—A space (X, Q_1, Q_2) is p-c-compact iff every cover of any Q_i-closed set in X by Q_j-open sets of X has finite subfamily whose union is Q_i-dense in the Q_i-closed set, for $i, j = 1, 2$ and $i \neq j$.

Theorem 3.9—Let Y be p-extremally disconnected, p-c-compact and p-Hausdorff. Then $f : X \to Y$ is p-continuous if it is p-set connected.

PROOF : Let f be not p-continuous. Then there exists a P_i-closed set F in Y such that $f^{-1}(F)$ is not Q_i-closed in X (for $i = 1$ or 2). Let $x \in Q_i - \mathrm{cl}(f^{-1}(F)) - f^{-1}(F)$. Then X is ij-connected between $f^{-1}(F)$ and x. So $f(X)$ is ij-connected between $f(f^{-1}(F))$ and $f(x)$. By Lemma 2.6 and Remark 2.3, we obtain that Y is ij-connected between $F (\supset ff^{-1}(F))$ and $f(x)$. Now as Y is p-Hausdorff, for each

$y \in F$ there exists a P_j-open neighbourhood V_y of y in Y such such that $f(x) \notin P_i-\text{cl}\ (V_y)$. Then the family $\{V_y : y \in F\}$ is a cover of F by P_j-open sets in Y. By p-c-compactness, there exists finite set of points $y_1, y_2, \ldots, y_n$ in F such that $F \subset P_i-\text{cl}\ (\bigcup_{r=1}^{n} V_{y_r}) = V$ (say). Then V is ij-coset in Y, since Y is p-extremally disconnected and $P_i-\text{cl}\ (\bigcup_{r=1}^{n} V_{y_r}) = \bigcup_{r=1}^{n} P_i-\text{cl}\ (V_{y_r})$. Also, $f(x) \notin V$, since $f(x) \notin P_i-\text{cl}\ (V_y)$, for any $y \in F$. This contradicts that Y is ij-connected between F and $f(x)$. Hence f is p-continuous.

4. P-set Connected Mapping and P-weakly Continuous

Definition 4.1[2]—A mapping $f : (X, Q_1, Q_2) \to (Y, P_1\ P_2)$ is said to be $Q_1\ P_1$-weakly continuous w.r.t. P_2 if for each point x of X and each P_1-neighbourhood V of $f(x)$ there exists a Q_1-neighbourhood U of x such that $f(U) \subset P_2-\text{cl}\ (V)$.

The mapping f will be called p-weakly continuous if it is $Q_1\ P_1$ weakly continuous w.r.t. P_2 as well as $Q_2\ P_2$ weakly continuous w.r.t. P_1

Lemma 4.2[2]—A mapping $f : X \to Y$ is $Q_1\ P_1$-weakly continuous w.r.t. P_2 iff for each P_1-open set V of Y, $f^{-1}(V) \subset Q_1-\text{int}\ (f^{-1}(P_2-\text{cl}\ V))$.

Lemma 4.3[2]—If a mapping $f : X \to Y$ is $Q_1\ P_1$ weakly continuous w.r.t. P_2 then $Q_1-\text{cl}\ (f^{-1}(V)) \subset f^{-1}(P_1-\text{cl}\ V)$, for any P_2-open set V.

Theorem 4.4—If a surjection $f : (X, Q_1, Q_2) \to (Y, P_1, P_2)$ is p-weakly continuous, then f is p-set connected.

Proof: Let V be 12-coset in $Y\ (= f(X))$. Since V is P_2-open, by Lemma 4.2 $f^{-1}(V) \subset Q_2-\text{int}\ (f^{-1}(P_1-\text{cl}\ V)) = Q_2-\text{int}\ (f^{-1}(V))$, since V is P_1-closed. This implies $f^{-1}(V) = Q_2-\text{int}\ (f^{-1}(V))$ and hence $f^{-1}(V)$ is Q_2-open in X. Again by Lemma 4.3, $Q_1-\text{cl}\ (f^{-1}(V)) \subset f^{-1}(P_1-\text{cl}\ V) = f^{-1}(V)$ so that $Q_1-\text{cl}\ (f^{-1}(V)) = f^{-1}(V)$ and hence $f^{-1}(V)$ is Q_1-closed in X. Thus $f^{-1}(V)$ is 12-coset in X. Then by Theorem 2.8, f is p-set connected.

Remark 4.5 : The following example shows that in the above theorem the condition "p-weakly continuous" can not be relaxed by $Q_1\ P_1$-weakly continuous w.r.t. P_2 or $Q_2\ P_2$-weakly continuous w.r.t. P_1.

Example 4.6—Let $X = Y = R$, the real line, Q_1, Q_2 be respectively the discrete and indiscrete topology on R; P_1 the lower limit topology and P_2 the usual topology on Y. Suppose $f : (X, Q_1, Q_2) \to (Y, P_1, P_2)$ be the identity mapping. Now $A = [0, \infty)$ is 21-coset in Y. But $f^{-1}(A) = A$ is not 21-coset in X. Therefore, f is not p-set connected. But it is clear that f is $Q_1\ P_1$ weakly continuous w.r.t. P_2 and is not p-weakly continuous.

Remark 4.7 : The converse of the Theorem 4.4 is false as is seen from the example below.

Example 4.8—Let $X = Y = R$; Q_1, Q_2 respectively denote the cofinite and discrete topology on X and let $P_1 = P_2 =$ usual topology on Y. Let $f : (X, Q_1, Q_2) \to (Y, P_1, P_2)$ be the identity map. Since (Y, P_1, P_2) is p-connected, f is p-set connected. Now consider $o \in X$ and $V = (-1, 1)$ is a P_1-neighbourhood of $f(o) = 0$ in Y. Then there cannot exist any Q_1-open neighbourhood U of o such that $f(U) \subset P_2-\text{cl}V = [-1, 1]$. Hence f is not p-weakly continuous.

Theorem 4.9—Let $f : X \to Y$ be p-set connected and Y is p-extremally disconnected, then f is p-weakly continuous.

PROOF : Let $x \in X$ and $f(x) \in V \in P_i$ $(i = 1, 2)$. Then $P_j-\text{cl}V$ is ji-coset in Y and so $(P_j-\text{cl}\ V) \cap f(X)$ is ji-coset in $f(X)$ $(i, j = 1, 2; i \neq j)$. Since f is p-set connected $f^{-1}(P_j-\text{cl}\ V \cap f(X)) = U$ (say) is ji-coset in X. Then U is a Q_i-open neighbourhood of x such that $f(U) \subset P_j-\text{cl}\ V$ $(i, j = 1, 2$ and $i \neq j)$, Hence f is p-weakly continuous. We shall sharpen the result in Theorem 4.9. For that we require the following definition.

Definition 4.10[2]—A mapping $f : (X, Q_1, Q_2) \to (Y, P_1, P_2)$ is called Q_1 P_1-almost continuous w.r.t. P_2 iff for each x of X and each P_1-neighbourhood V of $f(x)$ in Y there exists a Q_1-neighbourhood U of x in X such that $f(U) \subset P_1 - \text{int}\,(P_2-\text{cl}\ V)$. If f is, in addition, Q_2 P_2-almost continuous w.r.t. P_1, then f is called p-almost continuous.

Remark 4.11 : Since every p-almost continuous map is obviously p-weakly continuous, from Theorem 4.4 it follows that a p-almost continuous surjection is p-set connected.

Now we observe that p-weakly continuity in Theorem 4.9 can be replaced by p-almost continuity.

Theorem 4.12—Let Y be p-extremally disconnected and $f : X \to Y$ be p-set connected. Then f is p-almost continuous.

PROOF : The same proof of Theorem 4.9 can be carried over here, noticing that $P_j-\text{cl}\ V = P_i-\text{int}\,(P_j-\text{cl}\ V)$ $i, j = 1, 2$ and $i \neq j)$, since Y is p-extremally disconnected.

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