Indian J. pure appl. Math., 17 (9) : 1145-1153 September 1986

FREE CONVECTION FLOW ON A NONISOTHERMAL FLAT PLATE AS INFLUENCED BY SUCTION OR BLOWING UNDER NONUNIFORM GRAVITY

M. A. MANSOUR AND F. M. HADY

Mathematics Department, Faculty of Science, Assiut University, Assiut, Egypt

(*Received* 15 *October* 1985)

The effect of blowing and suction on free convection boundary layers developed on a nonisothermal flat plate under nonuniform gravity are considered. A numerical solution is obtained for various values of poroisty parameter F_w and temperature distributions. The skin-friction and heat transfer rate on the wall at selected values of the distance from the leading edge and for some values of F_w are presented.

1. INTRODUCTION

Previous work on the effects of suction and blowing on free convection boundary layers has been confined almost entirely to the case of a heated vertical plate. Eichhorn[1] considered the power law variation in plate temperature and transpiration velocity which yields a similarity solution of the boundary-layer equations. Sparrow and Cess[2] discussed the case of constant plate temperature and transpiration velocity. They[2] obtained series expansions for temperature and velocity in powers of $x^{1/4}$ where x is the distance measured from the leading edge. Merkin[3] extended the problem by obtaining asymptotic expansions, i. e. as $x \to \infty$, for temperature and velocity as influenced by both the blowing and the suction. The series expansions for small j given in Sparrow and Cess[2] were then joined to the asymptotic solution[3] by a numerical solution of the boundary-layer equations.

In a recent paper, Clarke[4] expanded the analysis presented in Eichhorn[1] by obtaining further approximations to the solution of the full Navier-Stokes equations for large, but finite, Grashof number. In ref. [4] the density variations were considered, where as in Eiechhorn[1], Sparrow and Cess[2], Merkin[3] the density variations were assumed important only in producing the buoyancy force.

Kao *et al.*[5] presented a solution for free convection along a vertical plate with arbitrary wall temperature variations in the absence of mass transfer at the wall. Aroesty *et al.*[6] and Merkin[7] considered the effect of blowing and suction through a body having a general shape.

The purpose of this paper is to consider the effects of blowing and suction on the steady nonisothermal laminar free convection flows along (i) an infinite cold porous

plate rotating at speed ω rads^{-1} in a radial plane with its leading edge being at a distance x_0 from the axis of rotation, and (ii) a finite hot porous plate of length x_0, rotating at speed ω rads^{-1} in a radial plane about the line $x = 0$, assuming the gravity varies linearly with the distance x. The temperature distributions over the plate are also assumed to be polynomial of first and second degree, respectively. Numerical solutions were adopted and results for the skin-friction and heat transfer rate are presented for different poroisty parameter F_w (-0.43, -0.2. 0.0, 0.2 and 0.43) with Prandtl numbers Pr 0.7 and 1.0.

T he analogous problem of a free convection boundary layer on a vertical impermeable flat pate under nonuniform gravity has been treated by, Pop and Na[8], and similar cases to those given in Pop and Na[8] are considered to solve the present problem.

2. Problem Formulation

In the present analysis, an infinite cold plate rotating at speed ω rads^{-1} in a radial plane with its leading edge beginning at a distance x_0 from the axis of rotations, and a finite hot plate of length x_0, rotating at speed ω rads^{-1} in a radial plane about the line $x = 0$ subject to a nonuniform gravity field $g(x)$ are considered. The main effort in the present study is confined to the cases where the plate is porous and the plates temperature distributions $T_w(x)$ have specific forms.

Let the coordinates be chosen such that the x-coordinate is equal to the distance measu red from the leading edge of the plate and the y-coordinate is the distance measured along the direction normal to the plate. The flow is assumed to be steady and incompressible. The boundary layer approximations are adopted along with the requirement that $\beta (T_w - T_\infty) \ll 1$ (large Taylor number). Thus the equations governing the conservation of mass, momentum and energy can be written in a dimensionless from as:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad \ldots(1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = g(x) S_w(x)\theta + \frac{\partial^2 u}{\partial y^2} \qquad \ldots(2)$$

$$u\left[\frac{\partial \theta}{\partial x} + \theta \frac{d \ln S_w(x)}{dx}\right] + v\frac{\partial \theta}{\partial y} = \frac{1}{P_r}\frac{\partial^2 \theta}{\partial y^2} \qquad \ldots(3)$$

with the boundary conditions

$$\left.\begin{aligned} &y = 0,\ u = 0,\ v = \pm v_w(x),\ \theta = 1 \\ &y \to \infty, u = 0,\ \theta = 0. \end{aligned}\right] \qquad \ldots(4)$$

Here u and v are the velocity components along the x, y coordinates, $S_w(x)$ is the wall temperature, θ the temperature, Pr the Prandtl number and $v_w(x)$ with minus sign corresponds to suction and with plus sign $-$ to injection'

Using the coordinate transformation

$$\xi = x, \ \eta = \frac{S_w (x)^{1/4}}{(4x)^{1/4}} y \qquad \ldots(5)$$

and introducing new dependent variables F and G such that

$$F(\xi,\eta) = \frac{\psi}{(64)^{1/4} x^{3/4} S_w (x)^{1/4}}, \ G(\xi,\eta) = \theta. \qquad \ldots(6)$$

These transformations reduce the boundary-layer equations to:

$$F''' + [3 + P(\xi)] FF'' - 2[1 + P(\xi)] F'^2 + g(\xi) G$$
$$= 4\xi \left(F' \frac{\partial F'}{\partial \xi} - F'' \frac{\partial F}{\partial \xi} \right) \qquad \ldots(7)$$

$$\frac{1}{Pr} G'' + [3 + P(\xi)] FG' - 4P(\xi) F' G = 4\xi \left(F' \frac{\partial G}{\partial \xi} - G' \frac{\partial F}{\partial \xi} \right) \qquad \ldots(8)$$

with the boundary conditions

$$\left.\begin{aligned} &F'(\xi, 0) = \mp F_w = \text{const.}, \ F'(\xi, 0) = 0, \ G(\xi, 0) = 1 \\ &F^{-}(\xi, 0) = 0, \ G(\xi, \infty) = 0 \end{aligned}\right] \qquad \ldots(9)$$

where the primes denote differentiation with respect to η, and the function $P(\xi)$ of the variable surface temperature is defined by

$$P(\xi) = \frac{\xi}{S_w(\xi)} \frac{dS_w}{d\xi}. \qquad \ldots(10)$$

The physical quantities of primary interest are the local skin-friction coefficient C_f and the local Nusselt number Nu which can be written respectively, in the form

$$C_f = \frac{\tau_w}{\rho \left(\frac{\nu}{u} \right)^2}, \ Nu = \frac{h_w x}{k}. \qquad \ldots(11)$$

From the definition of wall skin-friction $\tau_w = \mu \left(\frac{\partial u}{\partial y} \right) |_{y=0}$ and the local heat transfer coefficient $h_w = q_w/(T_w - T_\infty)$, where $q_w = -k (\partial T/\partial y)_{y=0}$, C_f and Nu are expressed as follows

$$C_f = 4 \left(\frac{Gr_x}{4} \right)^{3/4} F''(\xi, 0),$$

$$Nu = -\left(\frac{Gr_x}{4} \right)^{1/4} G'(\xi, 0). \qquad \ldots(12)$$

3. Numerical Solution

In order to solve the differential equations (7) and (8), along with boundary condition given by equation (9), we assume that

$$F = F_0(\eta) + \xi F_1(\eta) + \xi^2 F_2(\eta) + \ldots \quad \ldots(13)$$

$$G = G_0(\eta) + \xi G_1(\eta) + \xi^2 G_2(\eta) + \ldots \quad \ldots(14)$$

Upon substituting expressions (13) - (14) into the differential equations (7)-(9) and equating the coefficients of like powers of ξ, we obtain a set of ordinary differential equations. These are not reproduced here in the interest of conserving space.

The governing equations for the velocity and temperature functions have been solved on the computer using the fourth-order, Runge-Kutta numerical integration procedure in conjunction with shooting techniques.

The gravity $g(\xi)$ is assumed to follow the form

$$g(\xi) = 1 \pm \xi, \quad 0 \leqslant \xi \leqslant 1 \quad \ldots(15)$$

where the positive sign is taken for the cold plate and negative sign for the hot plate, respectively.

4. Results and Discussion

4.1 *Isothermal Porous Plate*

In this case S_w = constant. The series solutions by Lienhard *et al.*[9] for the parameters C_f and Nu can be written in the form

$$C_f = 4\left(\frac{Gr_x}{4}\right)^{3/4}\left[\sum_{n=0}^{\infty}(-1)^{nl}\ \frac{F_n^a(0)\xi^n}{n!}\right] \quad \ldots(16)$$

$$Nu = \left(\frac{Gr_x}{4}\right)^{1/4}\left[-\sum_{n=0}^{\infty}(-1)^{nl}\ \frac{\theta_n(0)}{n!}\,\xi_n\right] \quad \ldots(17)$$

where $l = 0$ for positive sign in equation (15) as $l = 1$ for minus sign in the same equation. For comparison purpose, the calculates skin-friction and Nusselt number for cold and hot plates are given in Tables I and II respectively at some porosity parameter F_w.

Figure 1 presents the dimensionless velocity F' and the dimensionless temperature G for the isothermal plate (S_w = const.) at different values of F_w. It is quite clear from this figure that an increase in porosity parameter F_w leads to a decrease in F'. However, near the plate ($\eta \leqslant 0.4$) the temperature shows a decrease as F_w increases.

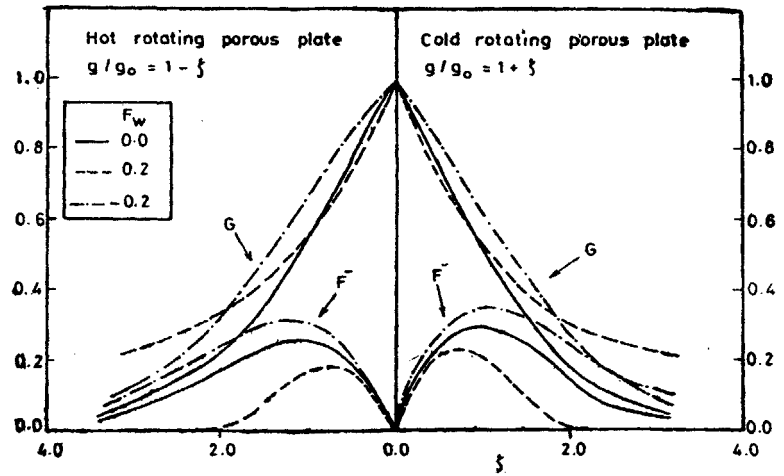

FIG. 1. Temperature and velocity functions for the free convection along rotating isothermal porous plates with $Pr = 0.7$ and $\xi = 0.2$.

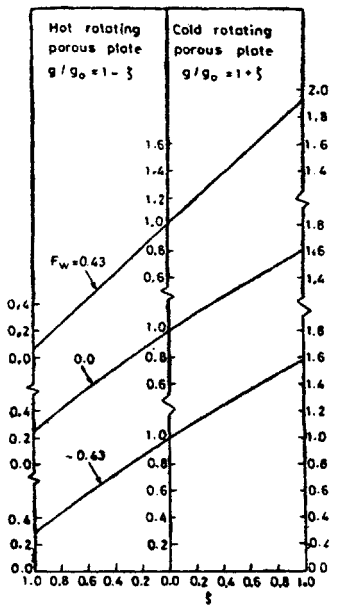

FIG. 2. Comparison of $F''(\xi, 0)/F''(0,0)$ for the free convention along rotating isothermal porous plates, $Pr = 0.7$.

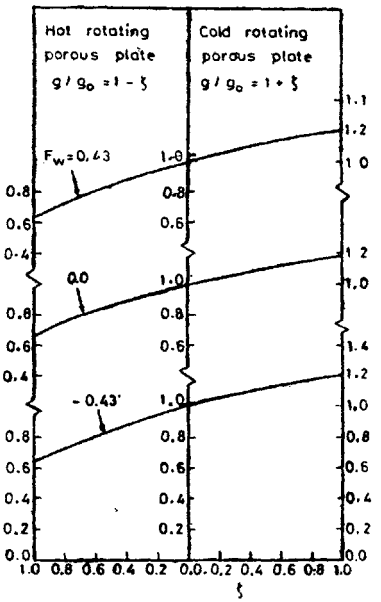

FIG. 3. Comparison of $G'(\xi,0)/G'(0,0)$ for the free convention along rotating isothermal porous plates, $Pr = 0.7$.

Figures 2 and 3 refer to the relative changes of the local skin-friction $F''(\xi, 0)/F''(0,0)$ and the local heat transfer $G'(\xi, 0)/G'(0,0)$ for representative values of ξ and

different values of F_w. For the cold plate the local skin-friction and the local heat transfer increase with the increase of ξ, but for the hot plate the effect of ξ is just the opposite. Also, the increase in the porosity parameter F_w leads to an increase in the local skin friction. The local heat transfer are identical. As is expected, $g(\xi)$ has a significant effect on C_f and Nu as depicted by Figs. 2 and 3.

4.2. *Nonisothermal Porous Plate*

In this case, the class of wall temperature distributions defined, for instance as

$$S_w(\xi) = a_0 + a_1 \xi + a_2 \xi^2 + \ldots \qquad \ldots(18)$$

We shall assign values to the constants a_i. Thus the problem is defined for the following cases :

$$\begin{aligned} &\text{I } S_w = 1 - \tfrac{1}{2}\xi \\ &\text{II } S_w = 1 + \tfrac{1}{2}\xi \\ &\text{III } S_w = 1 - \tfrac{1}{2}\xi + \tfrac{1}{4}\xi^2. \end{aligned} \qquad \ldots(19)$$

Figures 4 and 5 present the results for the skin-friction and heat transfer for the cases I – III evaluated at various ξ locations for different values of porosity parameter. Also, the profiles in the particular case of isothermal plates (S_w = const.) are also

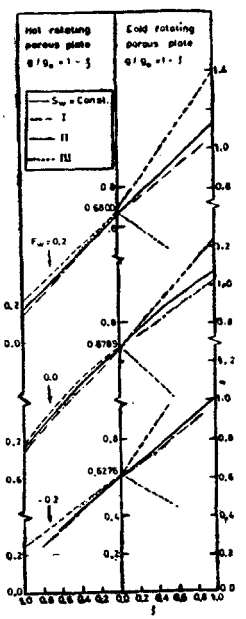

FIG. 4. Skin-fraction results for nonisothermal rotating porous plates.

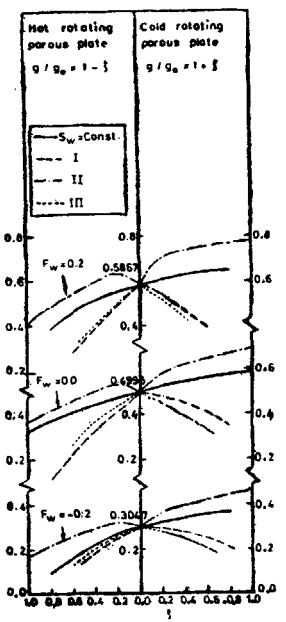

FIG. 5. Heat transfer results for nonisothermal rotating porous plates, $Pr = 0.7$.

TABLE I

Summary of numerical results for an isothermal cold rotating porous plate when $P_r = 0.7$

ξ	$C_f/4\,(Gr_x/4)^{3/4}$			$Nu/\,(Gr_x/4)^{1/4}$		
	$F_w \equiv -0.43$	0,0	0.43	—0.43	0.0	0.43
0.0	0.6630	0.6789	0.5579	0.2898	0.4995	0.8812
0.0375	0.6788	0.6958	0.5772	0.2929	0.5043	0.8875
0.1034	0.7064	0.7250	0.6111	0.2980	0.5124	0.8976
0.1488	0.7251	0.7449	0.6344	0.3015	0.5178	0.9036
0.2426	0.7632	0.7853	0.6826	0.3084	0.5283	0.9140
0.3539	0.8074	0.8319	0.7397	0.3160	0.5398	0.9224
0.4825	0.8571	0.8841	0.8056	0.3242	0.5518	0.9269
0.5713	0.8905	0.9191	0.8510	0.3294	0.5593	0.9268
0.6003	0.9013	0.9304	0.8659	0.3310	0.5616	0.9261
0.6565	0.9219	0.9519	0.8946	0.3310	0.5616	0.9261
0.7710	0.9631	0.9948	0.9530	0.3398	0.5737	0.8971
0.9451	1.0234	1.0572	1.0417	0.3475	0.5834	0.8971

TABLE II

Summary of numerical results for an isothermal hot rotating porous plate when $P_r = 0.7$

ξ	$C_f/4\,(Gr_x/4)^{3/4}$			$Nu/(Gr_x/4)^{1/4}$		
	$F_w \equiv -0.43$	0.0	0.43	—0.43	0.0	0.43
0.0	0.6630	0.6789	0.5579	0.2898	0.4995	0.8812
0.0375	0.6471	0.6619	0.5385	0.2867	0.9946	0.8743
0.1034	0.6187	0.6316	0.5046	0.2811	0.4857	0.8612
0.1488	0.5989	0.6105	0.4811	0.2772	0.4794	0.8513
0.2426	0.5575	0.5662	0.4327	0.2687	0.4657	0.8286
0.3539	0.5073	0.5123	0.3752	0.2581	0.4485	0.7978
0.4825	0.4479	0.4484	0.3085	0.2452	0.4119	0.7256
0.5713	0.4060	0.4032	0.2625	0.2359	0.4119	0.7256
0.6003	0.3922	0.3883	0.2474	0.2328	0.4068	0.7148
0.6565	0.3652	0.3591	0.2183	0.2328	0.4068	0.7148
0.7710	0.3092	0.2985	0.1587	0.2136	0.3748	0.6453
0.9451	0.2219	0.2037	0.0680	0.1928	0.3396	0.5644

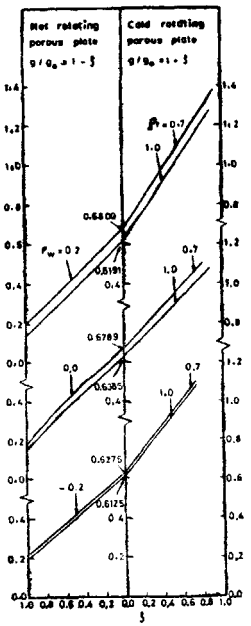

FIG. 6. Skin-friction results for nonisothermal rotating porous plates case I.

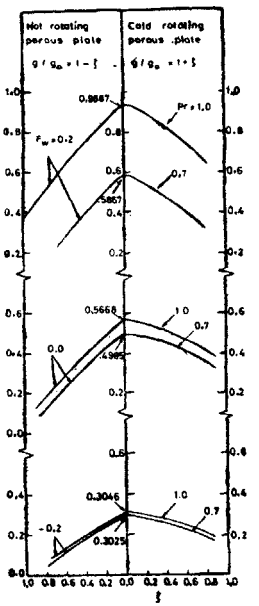

FIG. 7. Heat transfer results for nonisothermal rotating porous plates case I.

plotted at constant ξ distances in Figs. 4 and 5. For all the values of porosity parameter, the skin-friction is greater in case I than in cases II and III of eqns. (19). For the cold plate the skin-friction increases with the increase ξ, but for the hot plate the effect of increasing ξ is to decrease the skin-friction. However for case III the skin-friction decreases with increase of ξ for both the cold and hot plates. For case III also, the increase in F_w leads to an increase in the skin-friction. The results shown in Fig. 5 indicate that the heat transfer for all values of F_w is greater in case II than in cases I and III. Also, an increase in F_w leads to an increase in the heat transfer for all the studied wall temperature.

Figures 6 and 7 show the calculated values of C_f and Nu for case I at successive distances ξ for various porosity parameter with $Pr = 0.7$ and 1.0. The effect of P_r is to decrease C_f and to increase Nu as Pr increase. The effect of Pr becomes predominant at high values of F_w.

REFERENCES

1. R. Eichhorn, *J. Heat Transfer* 82c (1960), 260-63.
2. E. M. Sparrow, and R. D. Cess, *J. Heat Transfer* 81c (1960), 287-89.
3. J. H. Merkin, *Int. J. Heat Mass Transfer* 15 (1972), 989.
4. J. F. Clarke, *J. Fluid Mech*, 57 (1973), 45.

5. T. Kao, G. A. Domoto, and H. G. Elrod, *J. Heat Transfer* **99c** (1977), 72–78.
6. J. Aroesty, and J. D. Cole, Rand Corp, Memorandum RM-4620-ARPA (1965).
7. J. H. Merkin, *Int. J. Heat Mass Transfer* **18** (1975) 237.
8. I. Pop and T. Y. Na, *Int. J. Heat Mass Transfer* 26 (1983) 531–38.
9. J. Lienhard, R. Eichhorn, and V. Dihr, *Trans. Am. Soc. Mech. Engrs, Series C, J. Heat Transfer* **94** (1972), 80-86.