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MHD FLOW WITH HEAT AND MASS TRANSFER DUE TO A POINT SINK

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The magnetofluid dynamic steady incompressible laminar boundary layer flow for a point sink with an applied magnetic field and mass transfer has been studied. The two-point boundary-value problem governed by self-similar equations has been solved numerically. It is observed that the magnetic field increases the skin friction, but reduces the heat transfer and mass flux diffusion. However, the skin friction, heat transfer and mass flux diffusion increase due to suction and the effect of injection is just opposite. Prandtl and Schmidt numbers affect the temperature and concentration, respectively.

1. INTRODUCTION

MHD flow problems with magnetic field find applications in vortex chambers, power generators, nuclear reactions, evolution of rotating magnetic stars, geophysical fluid dynamics etc. The study of the boundary-layer flow of an electrically conducting fluid on a cone due to a point sink with an applied magnetic field is relevant in the study of conical nozzle or diffuser flow problems[1]. The foregoing problem in the absence of the magnetic field, mass flux diffusion and the heat transfer has been studied in the past[2].

The aim of this study is to extend the aforementioned analysis to include the effects of the magnetic field, mass flux diffusion and heat transfer. The governing equations with two-point boundary conditions have been solved numerically using a shooting method[3]. The results have been compared with those available in the literature.

2. GOVERNING EQUATIONS

Let us consider the steady laminar incompressible axisymmetric boundary layer flow of an electrically conducting fluid in a circular cone with a hole at the vertex (see the inset of Fig. 1.) The hole can be regarded as a three-dimensional sink. A

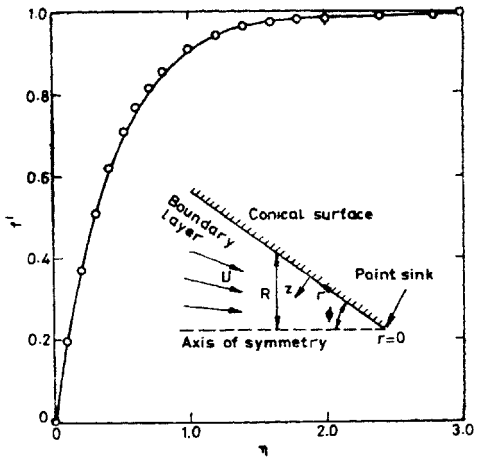

FIG. 1. Comparison of velocity profile for $M = f_w = 0$. ———, Present calculation; 0, Rosenhead[2].

magnetic field B_0 fixed relative to the fluid is applied in the z direction. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected in comparison with the applied magnetic field. The wall and the free stream are maintained at a constant temperature and concentration. The Hall effect and the dissipation terms are neglected. The effect of mass transfer (suction and injection) has been included in the analysis. It is assumed that the injected gas possesses the same physical properties as the boundary layer gas and has a static temperature equal to the wall temperature. Both gases are assumed to be perfect gases. The boundary layer equations under the foregoing assumptions are :

$$(ru)_r + (rw)_z = 0 \qquad \ldots(1)$$

$$uu_r + wu_z = -\rho^{-1} p_r + \nu u_{zz} - \rho^{-1} \sigma B_0^2 u \qquad \ldots(2)$$

$$uT_r + wT_z = \alpha T_{zz} \qquad \ldots(3)$$

$$uC_r + wC_z = DC_{zz} \qquad \ldots(4)$$

where

$$-\rho^{-1} p_r = UU_r + \rho^{-1} \sigma B_0^2 U,\ U = -m/r^2,\ m > 0. \qquad \ldots(5)$$

The boundary conditions are given by

$$u(r, 0) = 0,\ w(r, 0) = w_w,\ T(r, 0) = T_w,\ C(r, 0) = C_w$$

$$u(r, \infty) = U,\ T(r, \infty) = T_\infty,\ C(r, \infty) = C_\infty. \qquad \ldots(6)$$

Applying the following transformations

$$\eta = m^{\frac{1}{2}} z/(2\nu r^3)^{\frac{1}{2}}$$

$$ru = \psi_z,\ rw = -\psi_r,\ \psi = -(2\nu mr)^{\frac{1}{2}} f \qquad ...(7a)$$

$$u = Uf'(\eta),\ w = (m\nu/2r^3)^{\frac{1}{2}}(f - 3\eta f')$$

$$(T - T_\infty)/(T_w - T_\infty) = g(\eta),\ (C - C_\infty)/(C_w - C_\infty) = G(\eta) \qquad ...(7b)$$

$$M = 2\sigma B_0^2 r^3/(m\rho),\ Pr = \nu/\alpha,\ Sc = \nu/D$$

$$f_w = w_w (2r^3/m\nu)^{\frac{1}{2}}$$

to eqns. (1) to (4), we find eqn. (1) is satisfied identically and eqns. (2) to (4) requce to self-similar equations given by

$$f''' - ff'' + 4(1 - f'^2) + M(1 - f') = 0 \qquad ...(8)$$

$$Pr^{-1} g'' - fg' = 0 \qquad ...(9)$$

$$Sc^{-1} G'' - fG = 0. \qquad ...(10)$$

The boundary conditions (6) reduce to

$$f = f_w,\ f' = 0,\ g = G = 1 \text{ at } \eta = 0$$

$$f' = 1,\ g = G = 0 \text{ as } \eta \to \infty. \qquad ...(11)$$

Here r and z are the distances along and perpendicular to the cone; R is the radius of the cone ($R = r \sin \varphi$); φ is the semi-vertical angle of the cone; u and w are the velocity components along r and z directions, respectively; ψ and f are the dimensional and dimensionless stream functions, respectively p is the static pressure; c and T are concentration and temperature, respectively; g and G are the dimensionless temperature and concentration, respectively; η is the similarity variable; ρ, ν and σ are the density, kinematic viscosity and electrical, conductivity respectively; B_0 is the manetic field; α and D are the thermal diffusivity and binary diffusion coefficient, respectively; U is the inviscid flow velocity; m is the strength of the point sink; Pr and Sc are Prandtl number and Schmidt numbers, respectively; M is the magnetic parameter; f_w is the mass transfer parameter; the subscripts r and z denote derivatives with respect to r and z, respectively; the subscripts w and ∞ denote conditions at the wall and in the free stream, respectively; and prime denotes derivatives with respect to η.

It may be remarked that the boundary layer approximation is not valid in the immediate neighbourhood of the hole[2]. Also the mass transfer parameter f_w will be a constant if the velocity normal to the wall w_w varies as $r^{-3/2}$ as $m\nu$ is a constant. Also $f_w \gtrless 0$ according to whether it is injection or suction. For $M = 0$ (i.e. in the absence of the magnetic field), eqns. (8) to (10) reduce to those of non-hydromagnetic case. However so far only the velocity field in the absence of the magnetic field has been studied i.e. eqn. (8) with $M = 0$ has been solved. It is appropriate to point out here that a similar boundary layer solution for a point sink for $M > 0$ does not exist.

The local skin-friction coefficient can be expressed in the form

$$C_f = 2\tau_w/\rho U^2 = 2^{\frac{1}{2}}\, \mathrm{Re}_r^{-\frac{1}{2}}\, f_w'' \qquad \ldots(12a)$$

where

$$\tau_w = -\mu\,(u_z)_w,\ \mathrm{Re}_r = m/(r\nu). \qquad \ldots(12b)$$

The local heat-transfer coefficient in terms of Nusselt number is given by

$$Nu = rq_w/[k\,(T_w - T_\infty)] = -\,2^{-\frac{1}{2}}\,\mathrm{Re}_r^{\frac{1}{2}}\, g_w' \qquad \ldots(13a)$$

where

$$q_w = -\,k(T_z)_w. \qquad \ldots(13b)$$

Similarly, the local mass flux of the diffusing species in terms of Sherwood number can be expressed as

$$Sh = r\bar{m}_w/[\rho D\,(C_w - C_\infty)] = -\,2^{-\frac{1}{2}}\,\mathrm{Re}_r^{\frac{1}{2}}\, G_w' \qquad \ldots(14a)$$

where

$$\bar{m}_w = -\,\rho\, D\,(C_z)_w. \qquad \ldots(14b)$$

Here τ_w and q_w are the shear stress and heat transfer rate at the wall, respectively; Re_r is the local Reynolds number; k and $\bar{m}$ are the thermal conductivity and mass flux of diffusing species, respectively; μ is the viscosity; C_f, Nu and Sh are the skin friction coefficient, Nusselt number and Sherwood number, respectively.

3. Results and Discussions

The set of equations (8) to (10) under boundary conditions (11) has been solved numerically on a high speed computer (CDC CYBER 205). An outline of the numerical method used here is presented in Soundalgekar *et al.*[3]. Hence, for the sake of brevity, it is not described again here. We have considered the effect of step size $\Delta\eta$ and the edge of the boundary layer η_∞ on the solution to optimize them and the results presented here are independent of $\Delta\eta$ and η_∞ at least up to 5th decimal place. Computations have been carried out for several values of the parameters M, f_w, Pr and Sc. However, the results are presented only for some representative values of these parameter.

In order to assess the accuracy of our method, we have compared our f' and skin-friction parameter f_w'' for $M = f_w = 0$ with those of Rosenhead[2]. The results are found to be in excellent agreement. The comparison (f') is given in Fig. 1. The

skin friction parameter f_w'' is found to be 2.2728 as compared to 2.273 given in Ref. 2.

The effects of the magnetic parameter M and mass transfer f_w have been shown in Fig. 2. It is found that the skin friction, heat transfer and mass flux diffusion parameters $\left(f_w'', -g_w', -G_w'\right)$ all increase with suction ($f_w < 0$) and the effect of injection is just opposite. However the effect of the magnetic parameter M on f_w'', $-g_w'$, $-G_w'$ is slightly different. Although the skin friction parameter $\left(f_w''\right)$ increases with M, the heat transfer and mass flux diffusion parameters $\left(-g_w', -G_w'\right)$ slightly decrease. Similar effect has been observed in the case of a flat plate with magnetic field[4]. The reason for such a behaviour is that the suction ($f_w < 0$) and magnetic parameter (M) reduce the thickness of the momentum boundary layer which results in increase in skin friction. The effect of injection is just opposite. On the other hand, suction ($f_w < 0$) reduces the thermal and concentration boundary layers whereas the injection ($f_w > 0$) and the magnetic parameter (M) increase them. Consequently, the heat transfer and mass flux diffusion parameters $\left(-g_w', -G_w'\right)$ increase with suction ($f_w < 0$) and decrease with injection ($f_w > 0$) or magnetic field (M). The effect of mass transfer (f_w) on the skin friction, heat transfer and mass flux diffusion parameters $\left(f_w'', -g_w', -G_w'\right)$ is more pronounced than the magnetic parameter (M).

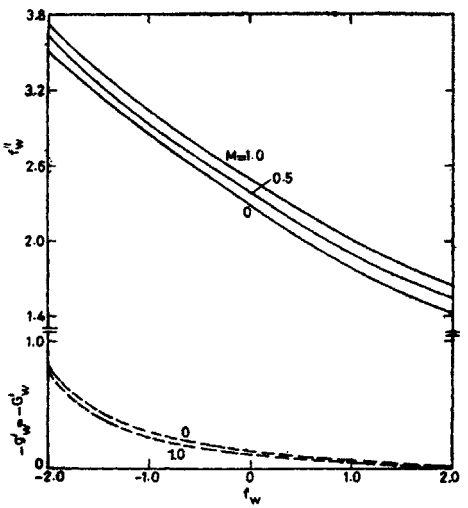

FIG. 2. Skin friction, heat transfer and mass flux diffusion parameters for $Pr = Sc = 0.7$

——, f_w''; ·····, $-g_w' = -G_w'$.

The velocity, temperature and concentration profiles (f', g, G) are shown in Figs. 3 and 4. The velocity, temperature and concentration profiles (f', g, G) become more steep due to suction ($f_w < 0$) and the reverse trend is observed for injection ($f_w > 0$). The reason for the above behaviour has been explained earlier. The profiles for injection ($f_w > 0$) are not shown here. The Prandtl number (Pr) and the Schmidt number (Sc) affect the temperature (g) and concentration (G), respectively, and for $Pr = Sc$, $g = G$. As expected, increase in Prandtl number Pr or Schmidt number Sc results in increase in heat transfer or in mass flux diffusion due to reduction in the thermal or concentration boundary layer thickness. Here only the temperature profile (g) is shown. The concentration profile (G) is similar to temperatute profile and hence it is not shown here. The thermal and concentration boundary layers are found to be thicker than the momentum boundary layer.

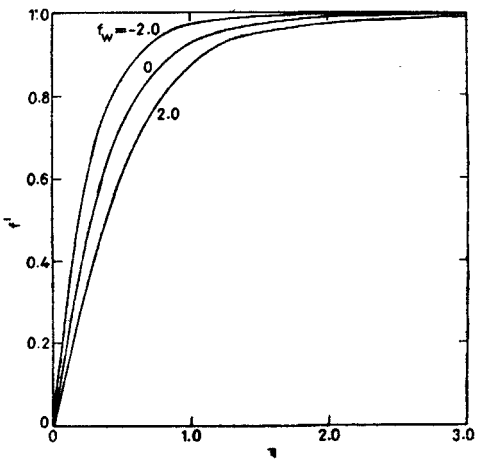

FIG. 3. Velocity profile for $M = 1.0$

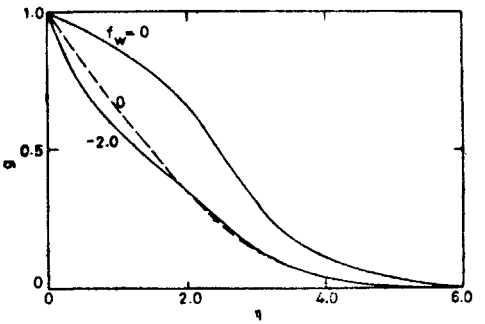

FIG. 4. Temperature profile for $M = 1.0$. ———, $Pr = 0.7$; ....., $Pr = 7.0$.

4. CONCLUSIONS

The magnetic field increases the skin friction, but reduces the heat transfer and mass flux diffusion. On the other hand, the skin friction, heat transfer and mass flux diffusion increase due to suction and the effect of injection is just opposite. Prandtl number and Schmidt number affect the temperature and concentration, respectively. The effect of mass transfer on the skin friction, heat transfer and mass flux diffusion is more pronounced than the magnetic field.

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