A MODEL FOR MICROPOLAR FLUID FILM MECHANISM WITH REFERENCE TO HUMAN JOINTS

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(Received 10 March 1987)

This paper analyses the variation of pressure and load capacity with reference to load bearing human joints by introducing a continuously varying porosity model for lower plate instead of usual uniform mono or multi-layered models studied so far. A micropolar fluid film is taken as a lubricant. By suitable choice of non-dimensional porosity variation parameter $\alpha$; it is shown that trends of variations are fairly in agreement with those recorded in earlier investigations.

NOTATION

$a$ = Characteristic length of bearing,

$\beta$ = film thickness,

$\beta_0$ = initial film thickness,

$\bar{\beta}$ = dimensional film thickness, $\beta/\beta_0$

$\alpha$ = porosity parameter of variation

$\bar{\alpha}$ = non-dimensional porosity parameter of variation, $a\alpha$

$H$ = thickness of cartilage,

$K_0$ = porosity of cartilage,

$\bar{K}_0$ = dimensionless porosity parameter, $K_0/\beta_0^2$

$l = (\gamma/4\mu)^{1/2}$

$L = \beta_0/l$

$N = \text{coupling number } (\chi/(2\mu + \chi))^{1/2}$

$p$ = pressure in fluid film region

$\bar{p}$ = non-dimensional pressure in fluid film region, $2p \beta_0^3/\mu \nu a^2$, 
MICROPOLAR FLUID

\( u, v \) = velocity components in fluid film region,

\( \tilde{u}, \tilde{v} \) = velocity components in porous matrix,

\( v \) = velocity of approach,

\( W \) = load capacity,

\( \tilde{W} \) = dimensionless load capacity \( 3W \beta^2 \rho_0 / \mu v a^2 \),

\( x \) = \( x \)-coordinate

\( y \) = \( y \)-coordinate

\( \mu \) = Newtonian viscosity coefficient

\( \gamma, \chi \) = viscosity coefficient for micropolar fluid,

\( \nu \) = micro-rotation velocity.

1. INTRODUCTION

Within the last five decades sufficient thought has been given to the study of lubrication mechanism in human joints but the recent studies have brought out a fairly clear picture of this process. The human joint may be visualised as a class of mechanical bearing because the fluid in the cavity between two mating bones is believed to act as lubricant. The human joints in this context may be described as a system consisting of two mating bones covered with cartilage with synovial fluid between them. Jones\(^1\) observed that a fluid film region was predominant mode of lubrication mechanism. The studies of Ogston and Stanier\(^2\) have brought out the visco-elastic character of the synovial fluid. Various types of lubrication mechanism are believed to occur in the human joints like hydrodynamic\(^3\), boundary\(^4\), weeping\(^5\) and mixed lubrication\(^6\). Dintenfass\(^7\) found that synovial fluid is non-Newtonian due to the presence of hyaluronic acid (a long chain polymer) molecules and its viscosity decreases with increasing shear rate. This view was experimentally supported by Bloch and Dintenfass\(^8\), Maroudas\(^9\) and Dowson\(^10\). Further more the studies of Dowson\(^10\) and Mow\(^11\) confirmed that the synovial fluid acts as lubricant. Eringen\(^12\) formulated the theory of micropolar fluids, which has been used by many authors under various physical situations. As the micropolar fluid may be considered for polymers, it can be taken for the synovial fluid consisting a long chain polymers.

Cartilage is basically a two-phase deformable porous material which can absorb or give out fluid owing to the established pressure gradient by either squeeze film action of the synovial fluid or consolidation of the solid matrix by tissue deformation. The studies of Clark\(^13\), Torizilli and Mow\(^14\)\(^15\) have pointed out that cartilage is a three layered porous medium consisting of a superficial tangential zone, a middle zone and a deep zone. Nigam \textit{et al.}\(^16\) investigated the effect of the variation of porosity in the
upper most layer of the cartilage which according to them plays a predominant role in the self adjusting nature of the human joint, taking a three layered porous medium. Sinha\textsuperscript{17} has considered the problem of lubrication of two approaching surfaces, one of which is covered with a layer of porous material and investigated the influence of porosity of the cartilage, film thickness, the thickness of the porous pad on axial pressure and load bearing capacity. He\textsuperscript{18} also investigated the influence of magnetic field on squeeze film lubrication with reference to human joints and noted that magnetotherapy can be of significant use in the treatment of diseases of human joints. Recently Tandon and Rakesh\textsuperscript{19} studied the lubrication mechanism occurring in knee joint replacement under restricted motion.

In this paper, the superficial division of cartilage matrix into three distinct layers is replaced by a continuously varying porosity matrix. The proposed model assumes flow of a fluid in a porous matrix of continuously varying porosity with a squeeze film of a micropolar fluid between two approaching surfaces. The mathematical analysis of the problem has been done by taking the continuity of pressure and velocities at the inter-face of the fluid film and the porous layer.

2. Mathematical Formulation

Referring to Fig. 1, the proposed model is conceived as a flow model of a squeeze film lubrication between two approaching surfaces with micropolar fluid and flow of

![Diagram](image-url)
viscous fluid in a continuous porous matrix with variable porosity. The porous matrix is fixed and upper surface is rigid and moves with uniform velocity \( V \) in the negative direction of \( y \)-axis as shown in the Fig. 1.

### 2.1 Fluid Film Region

Following Eringen\(^{12} \) the field equations for micropolar fluid may be reduced to the following form,

\[
\frac{\partial^2 v}{\partial y^2} - m^2 \frac{\partial v}{\partial y} = \frac{m^2}{2\mu} \frac{\partial p}{\partial x} \quad \text{...(1)}
\]

\[
0 = \gamma \frac{\partial^2 v}{\partial y^2} - 2xv - \chi \frac{\partial u}{\partial y} \quad \text{...(2)}
\]

and equation of continuity is

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad \text{...(3)}
\]

where

\[
m = \frac{N}{l} = \left( \frac{4\mu\chi}{\nu (2\mu + \chi)} \right)^{1/2}
\]

\[
N = \left( \frac{\chi}{2\mu + \chi} \right)^{1/2} \text{ and } l = \left( \frac{\gamma}{4\mu} \right)^{1/2}.
\]

### 2.2 Porous Region

Following Darcy’s law the flow of a viscous fluid in a porous matrix is governed by

\[
\ddot{u} = - \frac{K}{u} \frac{d\rho}{dx} \quad \text{...(4)}
\]

\[
\ddot{v} = - \frac{K}{\mu} \frac{d\rho}{dy} \quad \text{...(5)}
\]

and equation of continuity is

\[
\frac{\partial \ddot{u}}{\partial x} + \frac{\partial \ddot{v}}{\partial y} = 0 \quad \text{...(6)}
\]

where

\[
K = K_0 e^{-\alpha y}.
\]

### 2.3 Boundary conditions for Fluid Film

The boundary conditions is the fluid film region are
\[ p = 0 \text{ at } x = \pm a \]
\[ v = 0 \text{ at } y = 0, \beta \]
\[ u = 0 \text{ at } y = \beta \]
\[ u = \bar{u} \text{ at } y = 0 \]
\[ v = \bar{v} \text{ at } y = 0 \]
\[ \text{and} \]
\[ v = V \text{ at } y = \beta \]

2.4. Boundary Conditions for porous matrix

The boundary condition for porous matrix are
\[ \dot{p} = 0 \text{ at } x = \pm a \]
\[ \frac{\partial \bar{p}}{\partial y} = 0 \text{ at } y = -H. \]

2.5 Matching Condition

On the interface, \( y = 0 \),
\[ p(x) = \dot{p}(x, 0). \]

3. Solutions

The geometry of the model and the frames of reference are shown in Fig. 1. In what follows the conventional assumptions of lubricant theory are assumed.

3.1 Porous region

The lubricant in the porous region is an incompressible Newtonian fluid. Using the value of \( \bar{u} \) and \( \bar{v} \) from eqns. (4) and (5) in eqn. (6) we get
\[ \frac{\partial^2 \bar{p}}{\partial x^2} + \frac{\partial^2 \bar{p}}{\partial y^2} - \alpha \frac{\partial \bar{p}}{\partial x} = 0. \]

Integrating eqn. (10) with respect to \( y \) over the porous matrix thickness \( H \), we get
\[ \left( \frac{\partial \bar{p}}{\partial y} \right)_{y=0} = -H \frac{\partial^2 \bar{p}}{\partial x^2} + \alpha H \frac{\partial \bar{p}}{\partial x} \]
assuming that \( H \) is small.

3.2 Fluid Film Region of eqn. (1) under boundary conditions \( v = 0 \) at \( y = 0, \beta \), is
\[ v = -\frac{C}{\sinh m\beta} \{ \sinh m\beta (\beta - y) - \sinh m\beta + \sinh my \} \]
\[ + \frac{1}{2\mu} \frac{dp}{dx} \left\{ \frac{\beta \sinh my}{\sinh m\beta} - y \right\}. \]
Substituting this value of \( v \) in equation (2), we get
\[
\frac{\partial u}{\partial y} = \frac{y}{\mu} \frac{dp}{dx} - \frac{Dm\beta \sinh my}{2\mu \sinh m\beta} \frac{dp}{dx}
\]
\[- \frac{C}{\sinh m\beta} \{2 \sinh m\beta - Dm \sinh m (\beta - y) - Dm \sinh my\} \]
\[
\ldots(14)
\]
where
\[
D = \frac{2}{m} - \frac{\gamma m}{\gamma}.
\]
Integrating eqn. (14) with respect to \( y \), under boundary conditions we get
\[
u = \frac{y^2}{2\mu} \frac{dp}{dx} - \frac{K_0}{\mu} \frac{dp}{dx} + \frac{D\beta}{2\mu} \frac{dp}{dx} \left(1 - \cosh my\right) \frac{\sinh m\beta}{\sinh m\beta}
\]
\[- \frac{C}{\sinh m\beta} \{2y \sinh m\beta + D \cosh m(\beta - y) - D \cosh my
\]
\[- D \left( \cosh m\beta - 1\right) \]
\[
\ldots(15)
\]
where
\[
C = - \frac{\beta}{2\mu} \frac{dp}{dx} \left(\sinh m\beta \left(\beta - \frac{2K_0}{\beta}\right) - D \left(\cosh m\beta - 1\right) \right)
\]
\[2 \left( D \left(\cosh m\beta - 1\right) - \beta \sinh m\beta \right)\].
Integrating eqn. (3) with respect to \( y \) we have
\[
\frac{i}{\partial x} \int_0^\beta udy = - \frac{K_0}{\mu} \left\{ - H \frac{d^2 p}{dx^2} + a H \frac{dp}{dx} \right\} + V. \]
\[
\ldots(16)
\]
Substituting the value of from eqn. (15) in eqn. (16) we get
\[
\frac{d^2 p}{dx^2} + a \frac{dp}{dx} = b_0 \]
\[
\ldots(17)
\]
where
\[
a_0 = - \frac{12 \alpha H K_0}{\beta \left( \beta^2 + 6 K_0 + \frac{12HK_0}{\beta} + \frac{6D}{m} - 3D \beta \coth \frac{m\beta}{2} \right) \}
\]
and
\[
b_0 = \frac{12 \mu v}{\beta \left( \beta^2 + 6K_0 + \frac{12HK_0}{\beta} + \frac{6D}{m} - 3D \beta \coth \frac{m\beta}{2} \right) \}
\]
On solving eqn (17) under boundary conditions, we get
\[ p = \frac{b_0a}{a_0} \left\{ - \coth a_0a + \frac{\exp(-a_0a \frac{x}{a})}{\sinh a_0a} + \frac{x}{a} \right\}. \] ... (18)

### 3.3 Load

The load carrying capacity of the joint is given by

\[ W = \int_{-a}^{a} p \, dx \]

\[ = - \frac{2b_0a}{a_0} \left\{ a \coth a_0a = \frac{1}{a_0a} \right\}. \] ... (19)

### 4. NON-DIMENSIONALIZATION

Using following non-dimensional parameter

\[ \bar{x} = \frac{x}{a}, \quad \bar{\beta} = \frac{\beta}{\beta_0}, \quad L = \frac{\beta_0}{L}, \quad \bar{K}_0 = \frac{K_0}{\beta_0^3}, \quad \bar{H} = \frac{H}{\beta_0}, \]

\[ \bar{p} = \frac{2p_0^{3}}{\mu\nu a^2}, \quad \bar{W} = \frac{3W_0}{\mu\nu a^3} \quad \bar{a} = a, \quad \bar{a} = a_0a, \quad \bar{b} = \frac{b_0}{\mu\nu}. \]

The non-dimensional pressure distribution is given by

\[ \bar{p} = \frac{2\bar{b}}{\bar{a}} \left\{ - \coth \bar{a} + \frac{\bar{a} \bar{b}}{\sinh \bar{a}} + \bar{x} \right\}. \] ... (20)

The non-dimensional load carrying capacity is

\[ \bar{W} = \frac{4\bar{b}}{\bar{a}} \left\{ \frac{1}{\bar{a}} - \coth \bar{a} \right\}. \] ... (21)

where

\[ \bar{a} = - \frac{12 \bar{a} \bar{H} \bar{K}_0}{\bar{\beta} \left\{ \bar{\beta}^2 + 6\bar{K}_0 + \frac{12\bar{K}_0 \bar{H}}{\bar{\beta}} + \frac{12}{L^2} - \frac{6N_{\bar{\beta}}}{L} \coth \frac{NL_{\bar{\beta}}}{2} \right\}} \]

and

\[ \bar{b} = \frac{12}{\bar{\beta} \left\{ \bar{\beta}^2 + 6\bar{K}_0 + \frac{12\bar{K}_0 \bar{H}}{\bar{\beta}} + \frac{12}{L^2} - \frac{6N_{\bar{\beta}}}{L} \coth \frac{NL_{\bar{\beta}}}{2} \right\}} \]
5. Discussion

To bring out the effects of change of shape and size of micromolecules and the concentration on bearing characteristics by using micropolar fluid as lubricant (as a substitute of synovial fluid) in a conjunction with influence of the porous nature of the cartilage. The micropolar fluid involves two dimensionless parameters $N$ and $L$. Which do not occur in Newtonian theory and are called coupling number and material characteristic length respectively. In limiting cases as $N \to 0$ or $N \to \infty$, they consider with Newtonian cases provided $\mu$ is replaced by $(\mu + \frac{1}{2} \chi)$. Also a large values of $l$ and small values of $\beta_0$ give rise to increase the effective viscosity and this is in line with the experimental evidence of Hanniker$^{20}$. Consequently the number $L$ gives the influence of the shape and size of the suspended particles and $N$ is a measure of the concentration of suspended particles.

Torzilli and Mow$^{14-15}$ have given the following data of articular cartilage by considering a three layered model for the cartilage

\[
\frac{K_1}{\mu} = 3 \times 10^{-18} \text{ cm}^4/\text{dyne} \cdot \text{sec}
\]

\[
\frac{K_2}{\mu} = 6 \times 10^{-18} \text{ cm}^4/\text{dyne} \cdot \text{sec}
\]

\[
\frac{K_3}{\mu} = 9 \times 10^{-18} \text{ cm}^4/\text{dyne} \cdot \text{sec}
\]

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**Fig. 2.** Non-dimensional axial pressure distribution for different values of shape and size index $L$. 

- $k = 0.00022$
- $\beta = 0.5$
- $N = 0.83666$
- $k_0 = 3 \times 10^{-7}$
\( H_1 = 200 \) microne, \( H_2 = 0.2 \) cm, \( H_3 = 0.28 \) cm and \( \mu = 1 \) poise.

Following Nigam et al.\(^{16}\) minimum film thickness \( \beta_0 = 10^{-5} \) and upper surface porosity of the cartilage \( K_0 = 3 \times 10^{-7} \). We have by assuming continuous variation in the porosity of the cartilage taken \( H_1 + H_2 + H_3 \) is 0.5 cm and have introduced a non-dimensional porosity parameter of variation \( \bar{a} = 0.00022 \). Since exact data for quantities \( N \) and \( L \) for synovial fluid is not known we have chosen some values for these quantities.

In Fig. 2 the variation of non-dimensional axial pressure \( \bar{p} \) is shown for different values of shape and size index \( L \) it is obvious that as \( L \) increases, axial pressure decreases i.e. when shape and size of hyaluronic acid molecules decreases, axial pressure also decreases.

Figure 3 shows that as porosity at the interface increases, pressure decreases and this happens at a faster rate. The case of \( \bar{K}_0 = 0 \) corresponds to the case of non-porous cartilage for which the pressure is maximum. These results are in agreement with the observation of Prakash and Sinha\(^{21}\).

In Fig. 4 the variation of non-dimensional load bearing capacity \( \bar{W} \) with the film thickness \( \beta \) for different values of the porosity of the interface is shown. It is noted that the increase of film thickness, decreases the load bearing capacity \( \bar{W} \) and
Fig. 4. Variation of non dimensional load carrying capacity $\bar{\theta}$ with film thickness $\bar{\beta}$ for different values of porosity $\bar{k}_o$.

The increase of interface porosity $\bar{k}_o$ decreases $\bar{W}$. It is interesting to note that by choosing porosity parameter of variation $\bar{\alpha}_H = 0.00022$ we have obtained $\bar{p}$ and $\bar{W}$ which are in close agreement with those given by Nigam et al.\textsuperscript{10}. Thus exponential law of variation of porosity as assumed here is quite efficient to replace a three layered porous matrix by a continuously varying porosity matrix.

REFERENCES