Indan J. pure appl. Math., **20** (10) : 1022-1031, October 1989

THE AXISYMMETRIC CAUCHY-POISSON PROBLEM IN A STRATIFIED LIQUID

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(*Received* 27 *July* 1988)

An initial value investigation is made of the axisymmetric free surface response of a non-rotating stratified liquid of fiuite or infinite depth to the initial displacement of the free surface. Based upon the Boussinesq approximation, the problem is solved by the joint Laplace and Hankel transforms and the integral solution is obtained. It is shown that the solution represents the linear superposition of the dispersive waves. Special attention is given to the governing dispersion relations corresponding to small or large frequency and the Brunt Vaisala frequency parameter. The asymptotic analysis of the integral solution is carried out by the stationary phase method to describe the solution for large time and distance from the origin of disturbance. It is shown that the asymptotic solution consists of the classical free surface gravity waves and the internal waves.

1. INTRODUCTION

Historically, Cauchy (1789-1857) and Poisson (1781-1840) first initiated the study of surface waves produced in deep water by a local disturbances on the free surface. These studies were published independently in two classical memoirs by Poisson[7] and Cauchy[2]. Subsequent investigations into the classical Cauchy-Poisson wave problem for an inviscid liquid was continued by many others including Lord Kelvin[4], Burnside[1] and Lamb[5]. In order to determine solutions of the linearized Cauchy-Poisson problem by a simpler method, Kelvin[4] developed the method of stationary phase and applied it to obtain the asymptotic solution of the wave motions due to concentrated surface elevation in deep water. Some of these solutions associated with various physical and/or geometric configurations are available in Lamb's treatise on Hydrodynamics[6].

In recent years, several authors including Debnath[3], Nikitin and Podrezov[8] and Nikitin and Potetyunko[9] have given considerable attention to the Cauchy-Poisson problems in an incompressible homogeneous inviscid or viscous liquid. The two- or three-dimensional or axisymmetric free surface response of the liquid to either initial displacement of the free surface or an initial pressure impulse applied to the free surface has also been determined by asymptotic methods. In spite of these studies, the Cauchy-

Poisson wave problem in a non-rotating stratified liquid has not yet been investigated. The main purpose of this paper is to study such a problem which seems to be important and interesting from a mathematical as well as a physical point of view.

This paper is concerned with the initial value investigation of the axisymmetric free surface response of a non-rotating stratified liquid of finite or infinite depth. Based upon the Boussinesq approximation, the problem is solved by the joint use of the Laplace and Hankel transforms. The integral solution seems to be the linear superposition of the dispersive waves. Special attention is given to the governing dispersion relations corresponding to small or large the Brunt Vaisala frequency parameter. The asymptotic analysis of the integral solution is carried out by the stationary phase approximation to determine the solution for large time and distance from the source of the disturbance. The asymptotic solution is found to consist of the classical free surface gravity waves and the internal waves.

2. Mathematical Formulation of the Problem

We consider the axisymmetric Cauchy-Poisson wave problem in an inviscid incompressible stratified liquid of finite or infinite depth. The problem will be studied under the Boussinesq approximation and the density field of the undisturbed liquid is assumed to be in the form

$$\rho_0 = \rho_{00} \exp(-\beta z), \quad \beta > 0 \qquad \ldots(2.1)$$

where ρ_{00} and β are constants. The Brunt Vaisala frequency N is defined by

$$N = \left\{ -\frac{g}{\rho_0}\frac{d\rho_0}{dz} \right\}^{1/2} \qquad \ldots(2.2)$$

which is real when the mean density distribution is stable ($d\rho_0/dz < 0$) and it remains constant throughout the flow field because of (2.1).

With the cylindrical polar coordinates (r, θ, z), we consider a semi-infinite body of liquid bounded by $0 \leqslant r < \infty$ and $-h < z \leqslant 0$. The equilibrium state to be perturbed is the state of rest so the distribution of pressure and density is the hydrostatic equilibrium state given by $p = p_0(z)$ and $\rho = \rho_0(z)$ with $dp_0 = \rho_0 g dz$ where g is the acceleration due to gravity. Invoking the axial symmetry and linearization, the velocity components without swirling motion ($v = 0$), u, w and the small perturbation and pressure p' are governed by the equations of motion

$$\rho_0 \frac{\partial}{\partial t}(u, w) = -\left(\frac{\partial}{\partial r}, \frac{\partial}{\partial z}\right) p' - (0, g\rho') \qquad \ldots(2.3ab)$$

where ρ' is the density perturbation.

In view of the Boussinesq approximatian, we take ρ_0 as constant in (2.3ab) and introduce the acceleration potential $\chi \equiv p'/\rho_0 + gz$ and we can rewrite (2.3ab) in terms of χ as

$$\frac{\partial}{\partial t}(u, w) = -\left(\frac{\partial}{\partial r}, \frac{\partial}{\partial z}\right)(\chi - gz) - (0, g\rho'/\rho_0). \qquad \ldots(2.4ab)$$

The condition of incompressibility of the liquid is

$$\frac{\partial \rho'}{\partial t} + w\frac{\partial \rho_0}{\partial z} = 0. \qquad \ldots(2.5)$$

The equation of continuity is

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0. \qquad \ldots(2.6)$$

Introducing the free surface displacement function $\eta\,(r, t)$, the free surface boundary conditions are

$$\chi = g\eta, \; w = \eta_t \qquad \text{on } z = 0, \; t > 0. \qquad \ldots(2.7ab)$$

The bottom boundary condition is

$$\frac{\partial \chi}{\partial z} \to 0 \text{ as } z \to -h \text{ or } z \to -\infty. \qquad \ldots(2.8)$$

We further assume that χ and its derivations vanish as $r \to \infty$ and/or $z \to -h$.

The flow is generated in the liquid by the action of the free surface displacement at $t = 0$ so that the initial conditions are

$$u = v = \chi = 0, \; \eta\,(r, t) = \eta_0\,(r) \quad \text{at } t = 0, \; 0 \leqslant r < \infty. \qquad \ldots(2.9)$$

We further assume that $\eta_0\,(r)$ is sufficiently small so that the problem can be treated within the scope of the linearized theory.

3. The Solution of the Problem and the Dispersion Relation

It is convenient to reduce the initial value problem governed by the differential system (2.4ab) – (2.10ab) to a boundary value problem by using the Laplace transform

$$\bar{\chi}\,(r, z, s) = \int_0^\infty e^{-st}\,\chi\,(r, z, t)\,dt. \qquad \ldots(3.1)$$

In view of this transformation, the system (2.4ab) – (2.10ab) reduces to the boundary value problem

$$\bar{\chi}_{rr} + \frac{\bar{\chi}_r}{r} + \lambda^2 \bar{\chi}_{zz} = 0, \; 0 \leqslant r < \infty, \; -h \leqslant z \leqslant 0 \qquad \ldots(3.2)$$

where

$$\lambda^2 = \frac{s^2}{s^2 + N^2} \qquad \ldots(3.3)$$

$$\bar{\chi} = g\bar{\eta}, \quad \frac{s^2}{g}\bar{\chi} + \lambda^2 \bar{\chi}_z = s\eta_0(r) \qquad \text{on } z = 0 \qquad \ldots(3.4ab)$$

$$\bar{\chi}_z \to 0 \text{ as } z \to -h \text{ or } z \to -\infty. \qquad \ldots(3.5)$$

To solve the boundary value problem (3.2) – (3.5) we apply the Hankel transformation of zero order

$$\tilde{\chi}(k, z, s) = \int_0^\infty r J_0(kr)\, \bar{\chi}(r, z, s)\, dr. \qquad \ldots(3.6)$$

In view of this transformation, the system (3.2) – (3.5) reduces to the form

$$\tilde{\chi}_{zz} = \frac{k^2}{\lambda^2}\tilde{\chi}, \quad -h \leqslant z \leqslant 0 \text{ or } -\infty < z \leqslant 0 \qquad \ldots(3.7)$$

$$\tilde{\chi} = g\tilde{\eta}, \quad \frac{s^2}{g}\tilde{\chi} + \lambda^2 \tilde{\chi}_z = s\tilde{\eta}_0(k) \text{ on } z = 0 \qquad \ldots(3.8ab)$$

$$\tilde{\chi}_z \to 0 \text{ as } z \to -h \text{ or } z \to -\infty. \qquad \ldots(3.9)$$

The solutions for $\tilde{\chi}$ and $\tilde{\eta}$ in a liquid of finite depth are

$$\tilde{\chi}(k, z, s) = \frac{g s \tilde{\eta}_0(k) \cosh \frac{k}{\lambda}(z + h)}{s^2 \cosh\left(\frac{kh}{\lambda}\right) + gk\lambda \sinh\left(\frac{kh}{\lambda}\right)} \qquad \ldots(3.10)$$

$$\tilde{\eta}(k, s) = \frac{s\tilde{\eta}_0(k)}{s^2 \cosh\left(\frac{kh}{\lambda}\right) + gk\lambda \sinh\left(\frac{kh}{\lambda}\right)}. \qquad \ldots(3.11)$$

The inverse Laplace-Hankel transforms gives the solutions in a liquid of finite depth

$$\chi(r, z, t) = \frac{g}{2\pi i}\int_{c-i\infty}^{c+i\infty} s e^{st}\, ds \int_0^\infty \frac{\tilde{\eta}_0(k)\, k J_0(kr) \cosh\frac{k}{\lambda}(z + h)\, dk}{s^2 \cosh\left(\frac{kh}{\lambda}\right) + gk\lambda \sinh\left(\frac{kh}{\lambda}\right)} \qquad \ldots(3.12)$$

$$\eta(r, t) = \frac{1}{2\pi i}\int_{s-i\infty}^{c+i\infty} s e^{st}\, ds \int_0^\infty \frac{\tilde{\eta}_0(k)\, k J_0(kr)\, dk}{s_2 \cosh\left(\frac{kh}{\lambda}\right) + gk\lambda \sinh\left(\frac{kh}{\lambda}\right)} \qquad \ldots(3.13)$$

The solutions for $\tilde{\chi}$ and $\tilde{\eta}$ in a liquid of infinite depth are given

$$\tilde{\chi}\ (k, z, s) = \frac{sg\ \tilde{\eta}_0\ (k)}{s^2 + gk\lambda} e^{kz/\lambda} \qquad \ldots(3.14)$$

$$\tilde{\eta}\ (k, s) = \frac{s\ \tilde{\eta}_0\ (k)}{s^2 + gk\lambda}. \qquad \ldots(3.15)$$

Application of the inverse Laplace-Hankel transformations gives

$$\chi = \frac{g}{2\pi i} \int_{c-i\infty}^{c+i\infty} s\, e^{st}\, ds \int_0^\infty \frac{\tilde{\eta}_0\ (k)}{(s^2 + gk\lambda)} \exp\left(\frac{k\, z}{\lambda}\right) kJ_0\ (kr)\, dk \qquad \ldots(3.16)$$

$$\eta = \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} s\, e^{st}\, ds \int_0^\infty \frac{k\tilde{\eta}_0\ (k)\ J_0\ (kr)\, dk}{(s^2 + gk\lambda)} \qquad \ldots(3.17a)$$

$$= \frac{1}{2\pi i} \int_{c-i\infty}^{c+i\infty} \left(s + \frac{N^2}{s}\right) e^{st}\, ds \int_0^\infty \frac{k\ \tilde{\eta}_0\ (k)\ J_0\ (kr)\, dk}{\left(s^2 + N^2 + \frac{gk}{\lambda}\right)} \qquad \ldots(3.17b)$$

where Re $(\lambda) > 0$ that is necessary for a meaningful dispersion relation.

We guarantee the restriction Re $(\lambda) > 0$ imposed in solutions (3.16) — (3.17) by selecting an appropriate branch cut along Im (s) between the branch points at $s = \pm iN$. It is easy to check that the integrand in (3.17) has two imaginary poles at $s = \pm i\omega$ outside the branch cut $(\omega > N)$. Consequently, substituting $s = \pm iN$. $\lambda^2 = (1 - N^2\, \omega^{-2})$ in the denominator of the integrand of the solutions and equating the resulting denominator to zero, we obtain the implicit dispersion relation

$$\omega^2 = N^2 + \frac{gk}{\lambda}, \lambda = \left(1 - \frac{N^2}{\omega^2}\right)^{-1/2}. \qquad \ldots(3.18ab)$$

Eliminating of λ from (3.18ab) and expanding of this result for small N^2, we obtain

$$\omega^2 = \frac{N^2}{2} + g\, k + O\left(\frac{N^4}{4g^2\, k^2}\right) \text{ as } N^2 \to 0. \qquad \ldots(3.19)$$

In the limit of $N^2 \to 0$, (3.18ab) yields the well known dispersion relation $\omega^2 = g\, k$ for the deep water waves in the absence of stratification.

4. Asymptotic Analysis of the Free Surface Displacement Function

In the absence of stratification $(N = 0)$, the initial value problem formulated in previous sections reduces to the classical Cauchy-Poisson problem which has the solu-

tion (Lamb[6], p. 432) in the similarity form

$$\eta(r, t) = \frac{1}{\sqrt{2\pi r^2}} \xi F(\xi), \quad \xi = \frac{g t^2}{4r} \qquad \ldots(4.1ab)$$

where $F(\xi) = \cos \xi$.

Invoking $\eta_0(r) = a\,\delta(r)/r$ so that $\tilde{\eta}_0(k) = a$, and introducing non-dimensional free-surface displacement function $\eta^* = r^2/a\,\eta(r, t)$, we obtain from (3.17b) that

$$\eta^*(r, t) = \frac{r^2}{2\pi i} \int_{-i\infty}^{t\infty} \left(s + \frac{N^2}{s}\right) e^{st}\, ds \int_0^\infty \frac{k\, J_0(kr)\, dk}{\left(s^2 + N^2 + \frac{g\,k}{\lambda}\right)} \qquad \ldots(4.2)$$

where the path of integration in the complex s-plane passes to the right of both the poles at $s = \pm i\omega$ and the branch cut between $s = \pm iN$, $\omega > N$.

It is, in principle, possible to develop $\eta^*(r, t)$ as a power series in time t by expanding its Laplace transform function about $s = \infty$. The power series representation is quite complicated compared to the classical solution given in Lamb (1932), and it does not admit to a wave interpretation. Consequently, we write the solution only to the first approximation

$$\eta^*(r, t) = r^2 \int_0^\infty k\, J_0(kr)\, kd \frac{1}{2\pi i} \int_{-i\infty}^{t\infty} \left[\left(\frac{1}{s} - \frac{gk}{s^3}\right) + O\left(\omega_0^2\, s^{-5}, N^2 \omega_0 \times s^{-5}\right)\right] e^{st}\, ds \qquad \ldots(4.3)$$

where $\omega_0^2 = gk$.

We next introduce non-dimensional quantities defined by

$$\kappa \equiv \frac{g t^2}{4r}, \quad \alpha \equiv \frac{N^2 r}{g}, \quad \tau = (\kappa\lambda)^{1/2} = Nt \qquad \ldots(4.4abc)$$

a change of variable $kr = \zeta$ and then perform term by term Laplace inversion to obtain the solution

$$\eta^*(r, t) = \int_0^\infty [1 - 2\kappa\zeta + O\{\kappa^2 \zeta^2, \kappa\, \zeta\tau^2\}]\, \zeta\, J_0(\zeta)\, d\zeta. \qquad \ldots(4.5)$$

This integral can be evaluated by using standard integrals of Bessel functions to obtain

$$\eta^*(r, t) = \frac{2\Gamma(1)}{\Gamma(0)} - 2\kappa \frac{2^2\, \Gamma(\frac{1}{2} + 1)}{\Gamma(-\frac{1}{2})} + O(\kappa^2, \kappa\tau^2)$$

$$\sim \frac{g t^2}{2r} + O(\kappa^2, \kappa\tau^2). \qquad \ldots(4.6)$$

This corresponds to classical similarity from of the solution with gt^2/r as the similarity variable.

In order to determine the wave structure in the stratified liquids, the asymptotic solution for sufficiently large time is of special interest. It is then convenient to deform the path of integration of the s-integral in (4.2) into three closed loops separately encircling the poles and the branch cut. At the same time, we identify the combined contribution of the poles as η_1^* and the contribution of the branch points as η_2^*. We next appl y the laplace inversion to write down the integral solution for as η_1^* in the form

$$\eta_1^* (r, t) = r^2 \int_0^\infty \cos \omega t \, J_0 (kr) \, k dk \qquad ...(4.7)$$

where $\omega^2 = N^2 + \frac{gk}{\lambda}$ which becomes, in the limit $N^2 \to 0$

$$\omega^2 \sim g k + \frac{N^2}{2} + O (N^4). \qquad ...(4.8)$$

In view of this limiting expression combined with a change of variable $kr = \xi/\lambda$ we obtain

$$\eta_1^* = \int_0^\infty \frac{\cos \omega t}{\lambda^2} J_0 \left(\frac{\xi}{\lambda} \right) \xi \, d \xi \qquad .. (4.9)$$

$$= \int_0^\infty \frac{1}{\lambda^2} \cos [(4\kappa)^{1/2} \left(\xi + \frac{N^2 r}{g} \right)^{1/2}] J_0 [\xi + \frac{N^2 r}{2g} + O \left(\frac{N^2 r}{g} \right.$$

$$\left. \times \xi^{-1} \right)] \xi \, d \xi \qquad ...(4.10)$$

where

$$\omega^2 = N^2 \frac{g \xi}{r}. \qquad .. (4.11)$$

The exact evaluation of the integral in (4.10) is a formidable task. Hence the stationary phase method is employed to determine the dominant contribution of the integral for $4\kappa \gg 1, \alpha$. we then replace $J_0 (z)$ by its asymptotic formula

$$J_0 (z) \sim \left(\frac{2}{\pi z} \right) \cos \left(z - \frac{\pi}{4} \right) z \to \infty. \qquad .. (4.12)$$

In the integral (4.10) to obtain

$$\eta_1^* = \left(\frac{2}{\pi}\right)^{1/2}\int_0^\infty [\sqrt{\xi} + \frac{3}{2}\frac{N^1 r}{g\sqrt{\xi}} + O(\xi^{-3/2}, \frac{N^4 r^2}{g}\xi^{-3/2}))$$

$$X \cos [\sqrt{4\kappa}\left(\xi + \frac{N^2 r)}{g}\right)^{1/2}] \cos [\xi + \frac{N^2 r}{2g} - \frac{\pi}{4}]d\xi. \quad ...(4.13)$$

We next express the product of cosines by the sum of the cosines of the sum and difference of the two arguments so that (4.13) becomes

$$\eta_1^* \sim \frac{1}{\sqrt{2\pi}}\int^\infty \left(\sqrt{\xi} + \frac{3\alpha}{2\sqrt{\xi}}\right)[\cos \{(4\kappa)^{1/2}(\xi + \alpha)^{1/2} + \xi + \frac{\alpha}{2}$$

$$- \frac{\pi}{4}\} + \cos \{(4\kappa)^{1/2}(\xi + \alpha)^{1/2} - \xi - \frac{\alpha}{2} + \frac{\pi}{4}\}] d\xi. \quad ...(4.14)$$

We next apply the stationary phase method for large 4κ to obtain significant contributions of the two integrals in (4.14) to η_1^*. The first integral does not have any stationary point and so it has no contributions. But the second integral has a stationary point at $\xi_1 = \kappa - \alpha$ which makes a significant to this integral for large 4κ. Hence evaluation of the contribution from $\xi_1 = \kappa - \alpha$ to the integral gives

$$\eta_1^* \sim \left(\sqrt{\xi_1} + \frac{3\alpha}{2\sqrt{\xi_1}}\right)(2\kappa)^{1/2} \cos \left(\kappa + \frac{\alpha}{2}\right). \quad ...(4.15)$$

In terms of the original variables with $4\kappa > \alpha$, this result has the representation

$$\eta_1^* \sim \frac{1}{\sqrt{2}}\left(\frac{gt^2}{2r} + \frac{N^2 r}{g}\right)\left(\frac{gt^2}{4r} - \frac{N^2 r}{g}\right)^{-1/2}\left(\frac{gt^2}{4r}\right)^{1/2}\cos$$

$$\times \left(\frac{gt^2}{4r} + \frac{N^2 r}{2g}\right). \quad ...(4.16)$$

This result corresponds to the surface waves in a stratified liquid governed by the modified dispersion relation (3.14), and is believed to be new. In the limit $N \to 0$, the result (4.16) agrees with the classical solution of the Cauchy-Poisson problem in a non-stratified liquid.

We next turn our attention to the evaluation of η_2^* and make some changes of variables

$$s = iN \sin \theta, \quad \lambda = -i \tan \theta, \quad kr = \zeta \quad ...(4.17abc)$$

in (4.2) to obtain

$$\eta_2^* = \frac{N^2}{2\pi}\int_0^{2\pi} \exp (iNt \sin \theta) \cos \theta \sin \theta \, d\theta \int_0^\infty \frac{\zeta_0 J(\zeta) d\lambda}{iN^2 \sin \theta \cos \theta - g\zeta/r}$$

.. (4.18)

where Im $\theta \to -$.

A simple transformation of contributions of each of the four θ-quadrants to the first quadrant combined with Im $(\theta) \to 0$ — gives

$$\eta_2^* = \frac{4N^2}{2\pi} \int_0^{\pi/2} \cos(Nt \sin\theta) \cos^2\theta \, d\theta \int_0^{\infty} \frac{\zeta J_0(\zeta) \, d\zeta}{iN^2 \sin 2\theta - 2g\zeta/r^{-1}} \quad ...(4.19)$$

After some algebraic manipulation of the integrand of the ζ-integral, it turns out that the integrand is of the order $O(\alpha^2)$ and hence the ζ-integral has no significant contributions. The final result can be written in the form

$$\eta_2^* \sim -\frac{2N^2 r}{\pi g} \int_0^{\pi/2} \cos(Nt \sin\theta) \cos^2\theta \, d\theta + O(\alpha^2)$$

$$= -\frac{2N^2 r}{\pi g} \frac{1}{Nt} J_1(Nt) + O(\alpha^2) \quad ...(4.20)$$

$$\sim -\frac{2N^2 r}{\pi g (Nt)} \left(\frac{2}{\pi Nt}\right)^{1/2} \sin\left(Nt - \frac{\pi}{4}\right) \text{ as } Nt \to \infty$$

$$= -\alpha \left(\frac{2}{\pi Nt}\right)^{3/2} \sin\left(Nt - \frac{\pi}{4}\right) \text{ as } Nt \to \infty. \quad ...(4.21)$$

This asymptotic result corresponds to waves of amplitude $O(\alpha (Nt)^{-3/2})$ and frequency N. The existence of these waves is entirely due to the density stratification, and has no antecedents in a non-stratified liquid. Ultimately, these waves will decay as $Nt \to \infty$.

5. Discussions and Conclusions

It follows from the above asymptotic analysis that the dimensional form of the free surface elevation has the following asymptotic representation

$$\eta(r, t) = \frac{a}{r^2} \eta^*(r, t)$$

$$= \frac{a}{r^2} [\eta_1^*(r, t) + \eta_2^*(r, t)]$$

$$\sim \frac{a}{\sqrt{2r^2}} \left(\frac{gt^2}{2r} + \frac{rN^2}{g}\right) \left(\frac{gt^2}{4r} - \alpha\right)^{-1/2} \left(\frac{gt^2}{4r}\right)^{1/2}$$

$$\cos\left(\frac{gt^2}{4r} + \alpha\right) - \frac{a\alpha}{r^2} \left(\frac{2}{\pi Nt}\right)^{3/2} \sin\left(Nt - \frac{\pi}{4}\right). \quad ...(5.1)$$

This solution consists of two distinct terms representing waves. The first term in (5.1) corresponds to surface waves which are qualitatively similar to those in the

classical Cauchy-Poisson problem for an inviscid non-stratified liquid. The amplitude of the surface waves is modified by stratification. But the principal effect of stratification on the classical solution is the phase shift by an amount α in the asymptotic wavetrains. The second term in (5.1) also represents waves of frequency N and amplitude $O(\alpha r^{-2} Nt)^{-3/2})$ which decays to zere as $Nt \to \infty$. These are not surface waves and their existence is eniirely due to the density stratification. They have no antecedents in a non-stratified inviscid liquid.

As an example, we mention the initial displacement produced by a surface depression in the form

$$\eta_0(r) = \eta_0 \left[1 - \left(\frac{r}{a}\right)^2\right] \exp\left(-\frac{r^2}{a^2}\right) \qquad ...(5.2)$$

where the surface depression is in the form of a cavity of constant depth η_0 with a concentric lip. The Hankel transform of $\eta_0(r)$ is

$$\tilde{\eta}_0(k) = \tfrac{1}{8}\eta_0 a^2 (ka)^2 \exp\left[-\tfrac{1}{4}(ka)^2\right]. \qquad ...(5.3)$$

As asymptotic analysis similar to the case of the initially concentrated surface disturbance can be carried out without any difficulty. Except the mathematical form of the solution, the solution and its essential features will remain unchanged. Hence further discussion of this case would bs redundant.

ACKNOWLEDGEMENT

This work was completed at the Department of Mathematics, University of Central Florida, and was partially supported by the University of Central Florida. The second author expresses her grateful thanks to the University of Calcutta for granting her leave of absenee to conduct research in the United States.

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