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SUCTION FLOW ALONG A CIRCULAR SURFACE

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The suction velocity and the skin-friction of a viscous flow along the surface of a circular ditch have been calculated.

1. INTRODUCTION

Stuart[1] dealt with the steady flows of viscous incompressible fluids in general and in particular also. He considered the suction flows round the corners and outside of a circular cylinder. Satya Prakash[2] considered the flow along parabolic corners and channels with suction and injection. Radhey Shiam[3] has also treated some problems of suction flows along surfaces – specifically elliptical, semi-elliptical and triangular on the same lines. In this paper suction flow along a circular surface has been considered.

2. FORMULATION OF THE PROBLEM

Let x, y, z be the cartesian coordinates with velocity components (u, v w), z denoting the coordinate parallel to the length of the ditch, t denoting time. Since the ditch is extending from minus infinity to plus infinity, the flow may be taken independent of z. Therefore, the equations of motion are

$$\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\upsilon\nabla^2 u \qquad \ldots (1)$$

$$\frac{\partial v}{\partial t}+u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\upsilon\nabla^2 v \qquad \ldots (2)$$

$$\frac{\partial w}{\partial t}+u\frac{\partial w}{\partial x}+v\frac{\partial w}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial z}+\upsilon\nabla^2 w \qquad \ldots (3)$$

3. SOLUTION OF THE PROBLEM

Following Stuart[1], we take for steady motion, u and v as velocity components, derived from some potential function $\phi(x,y)$, thus

$$u=-\frac{\partial\phi}{\partial x}$$

$$v = -\frac{\partial\phi}{\phi y} \qquad \ldots (4)$$

$$\nabla_2\phi = 0$$

and, substituting the above and making further use of $p = p(x,y)$ in eqns. (1), (2) and (3), we obtain,

$$\frac{\partial\phi}{\partial x}\frac{\partial^2}{\partial x^2}\frac{\partial\phi^2}{} + \frac{\partial\phi}{\partial y}\frac{\partial^2\phi}{\partial x\,\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} \qquad \ldots (5)$$

$$\frac{\partial\phi}{\partial x}\frac{\partial^2\phi}{\partial y\,\partial x} + \frac{\partial\phi}{\partial y}\frac{\partial^2\phi}{\partial y^2} = -\frac{1}{\rho}\frac{\partial p}{\partial y} \qquad \ldots (6)$$

and

$$\frac{\partial\phi}{\partial x}\frac{\partial w}{\partial x} - \frac{\partial w}{\partial y}\frac{\partial\phi}{\partial y} = \upsilon\nabla^2 w. \qquad \ldots (7)$$

From eqns. (5) and (6) we get by integration,

$$\frac{P}{\rho} = -\frac{1}{2}(\phi_x^2 + \phi_y^2).$$

The solution of (7) is,

$$w = A + Be_{-\phi/\upsilon} \qquad \ldots (8)$$

where A and B are constants.
A suitable form of eqn. (8) is,

$$\pm\frac{w}{W_0} = 1 - e^{-\phi/\upsilon} \qquad \ldots (9)$$

where W_0 is the external stream parallel to the length of the ditch.

Now, if we taken an infinite length of the dietch situated in the fluid in such a way that it's length is along with axis of z, then, the boundary conditions would be:

(I) $w = 0$, on the surface of the ditch,

(II) $w = \pm W_0$, at infinite distance from the surface of the ditch, and $\partial\phi/\partial n$ if non zero, would be the velocity of suction on the surface of the ditch.

If we take a transformation[4],

$$Z = ic \cot \zeta / 2$$

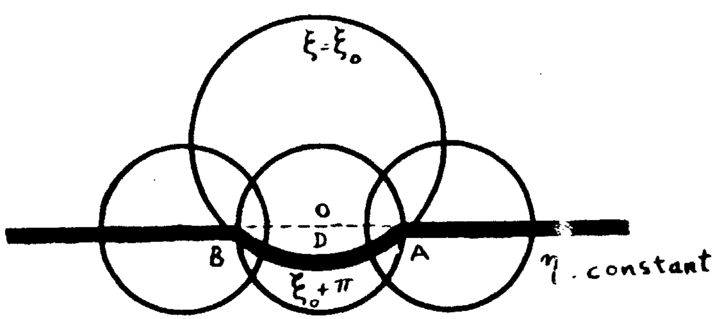
where c is the parameter of transformation, we obtain,

$$\frac{c\,sinh\,\eta}{cosh\,\eta - cos\,\xi}$$

$$y = \frac{c\,sin\,\xi}{cosh\,\eta - cos\,\xi}$$

where $Z = x + iy$ and $\zeta = \xi + i\eta$. In coaxial system of coordinate, A and B are the limiting points of the system. P describes a circle passing through the point A and B.

ξ and η are called coaxial coordinates. $\xi = \xi_0 + \pi$ is a circle with a centre $[0,\ c\ \cot(\xi_0 + \pi]$ and radius $c\ \mathrm{cosec}(\xi_0 + \pi)$. $\eta = \eta_0$ is a circle with centre $(c\ \coth \eta_0,\ 0)$ and radius $c\ \mathrm{cosec}\ \eta_0$, $\xi = \xi_0 + \pi$ is an arc of a circle through A and B for which $y < 1$, which represents the ditch ADB in Fig. 1.
We take



$$\phi = \beta(\xi - \xi_0 - \pi)$$

(where β is an arbitrary positive constant). and making use of

$$w = W_0(1 - e^{-\phi/\nu})$$

we note ϕ satisfies the laplace equation

$$\frac{\partial^2 \phi}{\partial \xi^2} + \frac{\partial^2 \phi}{\partial \eta^2} = 0$$

The conditions of the problem (I) and (II) become:

$$\phi = 0 \text{ for } \xi = \xi_0 + \pi \text{ and } \phi = \infty \text{ for } \xi = \infty$$

The suction velocity along the surface of the ditch is $h\frac{\partial\phi}{\partial\xi}$

where, $h = \left|\frac{\partial\zeta}{\partial z}\right|$.

Since $\left|\frac{\partial\zeta}{\partial z}\right| = \left(\frac{\cosh\eta - \cos\xi}{c}\right)$.

The suction velocity becomes

$$h\left|\frac{\partial\phi}{\partial\xi}\right|_{\xi=\xi_0+\pi} = \beta\left(\frac{\cosh\eta - \cos\xi}{c}\right)_{\xi_0+\pi}$$

$$= \frac{\beta}{c}\left\{\cosh\eta - \cos(\xi_0 + \pi)\right\}$$

$$= \frac{\beta}{c}\left(\cosh\eta + \cos\xi_0\right) \qquad \ldots (10)$$

The skin-friction is $\mu h\frac{\partial w}{\partial\xi}$ and since

$$w = W_0(1 - e^{-\beta(\xi-\xi_0-\pi)/v})$$

Its value on the surface of the ditch is given by

$$\left(\mu h\frac{\partial w}{\partial\xi}\right)_{\xi=\xi_0+\pi} = \mu\frac{W_0\beta}{v}\left\{\frac{\cosh\eta - \cos(\xi_0+\pi)}{c}\right\}$$

$$= \mu\frac{W_0\beta}{vc}\left(\cosh\eta + \cos\xi_0\right) \qquad \ldots (11)$$

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