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MHD ACCELERATED MOTION ON A BODY PLACED SYMMETRICAL TO THE FLOW IN THE PRESENCE OF TRANSVERSE MAGNETIC FIELD FIXED RELATIVE TO THE BODY

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The growth of the boundary layer in an accelerated flow of an electrically conducting fluid past a symmetrically placed body in the presence of uniform transverse magnetic field fixed relative to the body has been studied. The boundary layer equation has been solved by using a method previously developed by Pozzi, based on expressing the unknown velocity in term of an error function and on using differential and integral relations obtained from the balance equation. As examples, the impulsive flow past a circular cylinder and uniformly accelerated flow over a flat plate are considered. It is found that the effect of the magnetic field is to decelerate the fluid motion which results in an earlier boundary layer separation in the impulsive flow past a circular cylinder. The results show a good agreement with the numerical data available in the literature.

NOMENCLATURE

- B_0 = strength of the transverse applied magnetic field;
- C_f = coefficient of skin-friction;
- C_m = coefficient of magnetic drag;
- f = function introduced in equation (2.5)
- U_∞ = dimensional velocity at the edge of bounday layer in the absence of the magnetic field;
- U_0 = characteristic velocity;
- m = non-dimensional magnetic parameter = $\frac{\sigma_e}{\rho} B_0^2 T_0$;
- X, Y = dimensional coordinates parallel and normal to the body;
- T = dimesionsal time;
- U, V = dimensional components of velocity along and perpendicular to the body;

z = variable introduced in equation (2.5);

t = non-dimensional time $t = t(s)$;

T_0 = characteristic time;

h = scaling function of the coordinate normal to the body;

x, y = non-dimensional coordinates parallel and normal to the body;

u, v = non-dimensional components of velocity along and perpendicular to the body;

u_∞ = non-dimensional velocity at the edge of the boundary layer in the absence of the magnetic field;

S_{sp} = distance traversed by outer flow before separation impulsive flow over a circular cylinder;

t_{sp} = time at separation for impulsive flow over a circular cylinder;

$U(x)$ = unit function = 1 for $x > 0$; = 0 for $x < 0$.

Greek symbols

α constant in the mainstream velocity;

δ_1 dimensional dispalcement thickness;

δ_2 dimensional momentum thickness;

μ viscosity;

ν kinematic viscosity = μ/ρ;

ρ density;

σ_e electrical conductivity.

Subscripts

∞ condition at infinity;

0 characteristic values.

1. Introduction

The unsteady nature of a wide range of fluid flows of practical importance has received considerable attention in recent years. In many applications the ideal flow environment around the device is normally steady, but undesirable unsteady effects arise either due to self-induced motions of the body, or due to fluctuations or non-uniformities in the surrounding fluid. Generally, two categories of unsteady flow problems have been studied extensively, viz.(i) boundary layers which are created when a body starts from rest either impulsively or with acceleration, some of these studies are made by Blasius[3], Goldstein and Rosenhead[6], Goertler[7], Nigam[15], Schuh[22], Illingworth[9], and Wadhwa[30], and (ii) boundary layers which are periodic due to fluctuations of the stream of oscillations of the body, where the contribution of Schlichting[21], Lin[13], Lighthill[12], Ghosh[8], Segel[23] and Roy[19] are noteworthy, Stuart[25], Riley[20], Wirtz[29] and Mc-Croskey[14] have reviewed some of the researches carried out by the earlier authors. Socio and Pozzi[26] have studied in detail an approximate integral method for solving the unsteady laminar boundary layer equation, which is applicable to a wide range of initial and boundary conditions. Rossow[15] has studied the boundary layer flow in the impulsive motion of a viscous incompressible electrically conducting fluid in the presence of an applied magnetic fields normal to a flat plate, which was later extended by Kakutani[10] and Ong and Nicholls[16] to the case of an oscillating flat plate. Soundalgekar[24], Pop[17] and many others, including the recent work of Bansal and Dave[4], have studied the compact solution for MHD boundary layer flow near an accelerated flat plate.

However, one finds that little efforts have been made to study the unsteady hydromagnetic boundary layer flow past bodies of arbitrary shape. In this direction the works of Askovic[1], Askovic and Askovic[2], Singh and Jalal[27], Singh *et al.*[28] and Kumari and Nath[11] are of considerable interest. Recently Dave[5] has studied the growth of the boundary layer in an accelerated motion past a symmetrically place body in the presence of a transverse magnetic field fixed to the body by the method of successive approximations.

In the present paper we have studied the MHD boundary layer flow due to an unsteady external flow whose acceleration is governed by power law, on a body placed symmetrically to the flow in the presence of an external applied transverse magnetic field fixed relative to the body. A simple integral method suggested by Socio and Pozzi[26] in the case of OFD flow is modified for solving the governing equations of MHD flow over a body. The distance traversed by the boundary layer before separation occurs is determined for an impulsive flow over a circular cylinder. It is found that in the present case the effect of the magnetic field is to decelerate the fluid motion which results in an earlier boundary layer separation and thus to destablize the flow. As a second example, uniformly accelerated flow over a flat plate is considered. In both cases the results are in close agreement with the available results of Dave[5].

2. Formulation of the Problem

The problem may be regarded as one in which a velocity is accelerated in the mainstream, the body placed symmetrically to the flow and remaining at rest, and

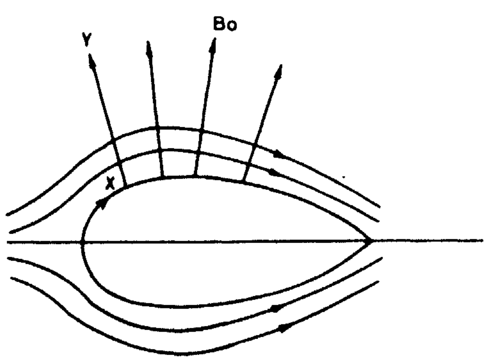

Fig.1. MHD boundary layer on a body placed symmetrical to the flow

B_0 is the external applied uniform transverse magnetic field fixed relative to the body. The X-axis along its length, the Y-axis perpendicular to it (Fig. 1) and let the motion begins at $T = 0$. If U and V are components of fluid velocity along and perpendicular to the contour of the body, then the dimensional MHD-boundry layer equations may be written as

$$\frac{\partial U}{\partial X} + \frac{\partial V}{\partial Y} = 0 \qquad \text{... (2.1)}$$

$$\frac{\partial U}{\partial T} + U\frac{\partial U}{\partial X} + V\frac{\partial U}{\partial Y} = -\frac{1}{\rho}\frac{\partial P}{\partial X} + \nu\frac{\partial^2 U}{\partial Y^2} - \frac{B_0 J_z}{\rho} \qquad \text{... (2.2)}$$

where

$$-\frac{1}{\rho}\frac{\partial P}{\partial X} = \frac{\partial U_\infty}{\partial X} + U_\infty\frac{\partial U_\infty}{\partial X}\left(\frac{B_0 J_z}{\rho}\right)_{U=U_\infty} \qquad \text{... (2.3)}$$

and J_z, in the absence of externally applied electric field,

$$J_z = \sigma U B_0. \qquad \text{... (2.4)}$$

Introducing the non-dimensional quantities as

$$u = \frac{U}{U_0}, \quad v = V\left(\frac{T_0}{\nu}\right)^{1/2}, \quad x = \frac{X}{T_0 U_0}, \quad t = \frac{T}{T_0}$$

$$y = Y(T_0 \nu)^{-1/2}, \quad p = \frac{P}{\rho U_0^2}, \quad u_\infty = \frac{U_\infty}{U_0}, \quad m = \frac{\sigma_e}{\rho} B_0^2 T_0.$$

The MHD-boundary layer equations (2.1) to (2.4) may be written in the non-dimensional form as

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial u_\infty}{\partial t} + u_\infty\frac{\partial u_\infty}{\partial x} + \frac{\partial^2 u}{\partial y^2} - mu \qquad \text{... (2.1')}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0. \qquad \text{... (2.2')}$$

The quantities are defined in the nomenclature. Further, in the absence of magnetic field $u_\infty = w(x)\, t^\alpha$. The initial and boundary conditions and the compatibility condition are

$$u(x, y, m, t) = 0 \quad \text{for } t \le 0$$

$$u(x, 0, m, t) = v(x, 0, m, t) = 0; \; t > 0$$

$$\lim_{y \to \infty} u(x, y, m, t) = u_\infty(x, m, t)$$

$$= t^\alpha w(x)\left\{1 - \frac{mt}{1+\alpha} + \frac{m^2 t^2}{(1+\alpha)(2+\alpha)} - \dots\right\}$$

$$+ t^{2\alpha+1} w(x)\frac{dw(x)}{dx}\left\{\frac{mt}{(1+\alpha)^2} - \frac{2m^2 t^2}{(1+\alpha)^2(2+\alpha)} + \dots\right\}. \qquad \text{... (2.3')}$$

The condition at infinity is modified because of the presence of the magnetic field.

Besides these boundary conditions (2.3′) we shall need some initial conditions which will be discussed later.

An integral formulation of the problem is obtained by integration of eqns. (2.1) – (2.2) as

$$\left[\int_0^\infty (u_\infty - u)\; dy\right]_t + \left[\int_0^\infty (u_\infty^2 - u^2)\; dy\right]_x - u_\infty \left[\int_0^\infty (u_\infty - u)\; dy\right]_x$$

$$= \left(\frac{\partial u}{\partial y}\right)_{y=0} + m \int_0^\infty u\; dy \qquad \ldots (2.4')$$

Here the subscripts denote the respective partial derivatives.

The above integro-differential equation may be reduced to a partial differential equation of Lagrange's form if we make the following assumption for the velocity profile :

$$u = u_\infty\; f(z) \quad \text{with } z = \frac{y}{h(x, m, t)}. \qquad \ldots (2.5')$$

Thus the equation (2.4) reduces to

$$\alpha_1 H_t + u_\infty\; \alpha_2 H_x = 2\, u_\infty^2\, \alpha_3 - 2\, H\, (\alpha_1 + \alpha_2)\, (u_\infty)_x + 2\, m\, H \alpha_4, \qquad \ldots (2.6')$$

where

$$H = h^2 u_\infty^2\,, \quad \alpha_1 = \int_0^\infty (1 - f)\; dz\,, \quad \alpha_2 = \int_0^\infty f\,(1 - f)\; dz$$

$$\alpha_3 = f'(0)\,, \quad \alpha_4 = \int_0^\infty f\; dz.$$

3. Method of Solution

Equation (2.6) is a first-order, linear partial differential equation in the unknown scale function $h(x, m, t)$ which corresponds the boundary layer thickness and the corresponding auxiliary equations of the above equation are

$$\frac{dt}{\alpha_1} = \frac{dx}{u_\infty\, \alpha_2} = \frac{dH}{\{2\, u_\infty^2\; \alpha_3 - 2H\, (\alpha_1 + \alpha_2)\, (u_\infty)_x + 2\, m\, H \alpha_4\}} \qquad \ldots (3.1)$$

Let $\phi_1\,(x, m, t, H) = c_1$ and $\phi_2\,(x, m, t, H) = c_2$ be two independent integrals of equation (3.1). The general solution can then be expressed in the form

$$\phi_2 = \Phi\,(\phi_1) \qquad \ldots (3.2)$$

with Φ is the function to be determined from the initial conditions to be associated to the boundary conditions (2.3).

From the first two members of equation (3.1) one has immediately

$$\phi_1 = x - \frac{\alpha_2}{\alpha_1} \int_0^t u_x \, dt. \qquad \text{... (3.3)}$$

To determine ϕ_2, one considers the first and last members of equation (3.1), which on integration give

$$H = \exp\left(-\int_0^t \{A(u_x)_x - Bm\} \, dt\right)$$

$$\left[\frac{2\alpha_3}{\alpha_1} \int_0^t u_x^2 \left\{\exp\left(\int_0^t \{A(u_x)_x - Bm\} \, dt\right)\right\} dt + C_2\right] \qquad \text{... (3.4)}$$

where

$$A = 2\left(1 + \frac{\alpha_2}{\alpha_1}\right), \quad B = \frac{2\alpha_4}{\alpha_1}. \qquad \text{... (3.5)}$$

Finally,

$$\phi_2 = H \exp\left(\int_0^t \{A(u_x)_x - Bm\} \, dt\right)$$

$$- \frac{2\alpha_3}{\alpha_1} \int_0^t u_x^2 \left\{\exp\left(\int_0^t \{A(u_x)_x - Bm\} \, dt\right)\right\} dt. \qquad \text{... (3.6)}$$

The general integral (3.2) can then be expressed as

$$H = \exp\left(-\int_0^t \{A(u_x)_x - Bm\} \, dt\right) \left[\phi(m, t) + \Phi(x - s)\right] \qquad \text{... (3.7)}$$

with

$$\phi(m, t) = \frac{2\alpha_3}{\alpha_1^t} \int_0^t u_x^2 \left\{\exp\left(\int_0^t \{A(u_x)_x - Bm\} \, dt\right)\right\} dt \qquad \text{... (3.8)}$$

and

$$s = \frac{\alpha_2}{\alpha_1} \int_0^t u_x \, dt \qquad \text{... (3.9)}$$

which implies that $t = t(s)$.

The function Φ in eqn. (3.7) can be obtained with the help of following conditions :

$$\hat{X}(x, m) = h^2(x, m, 0) \qquad \text{... (3.10)}$$

$$\hat{T}(m, t) = h^2(0, m, t). \qquad \text{... (3.11)}$$

The function $\Phi\ (x-s)$, in view of the initial conditions (3.10) and (3.11), may be written in terms of another function $\psi\,(s-x)$ as

$$\Phi\,(x-s) = [\,\hat{X}\,(x-s,m)\ u_x^2\ (x,m,0) + U(s-x)$$

$$-\ \{\hat{X}\,(x-s,m)\ u_x^2\ (x,m,0) + \psi(s-x)\}\,] \qquad \ldots\ (3.12)$$

Where $U(x)$ is the unit function ($=0$ for $x<0$; $=1$ for $x>0$).

We now observe the following properties of the function Φ

(i) for $x > s,\ s-x < 0$

$$\Phi\ (x-s) = \hat{X}\,(x-s,m)\,u_x^2\,(x,m,0) \qquad \ldots\ (3.13)$$

(ii) for $x < s,\ s-x > 0$

$$\Phi\ (x-s) = \psi\,(s-x)$$

or $$\Phi\ (-s) = \psi\,(s) \qquad \ldots\ (3.14)$$

putting $x = 0$ in eqn. (3.7) and using the condition (3.11), we get

$$\hat{T} = (\,u_x^2\,)_{x=0} = \exp\left(-\int_0^t \{A\,((u_x)_x) - Bm\}\ dt\right)[\psi\,(m,t) + \Phi\,(-s)\,].$$

By using (3.14)

$$\psi\,(s) = \hat{T}\ (\,u_x^2\,)_{x=0}\ \exp\left(\int_0^t \{A\,((u_x)_x) - Bm\}\ dt\right) - \phi\,(m,t). \qquad \ldots\ (3.15)$$

Equation (3.7), with the help of eqn. (3.12) may now be written as

$$H = \exp\left(-\int_0^t \{A\,(u_x)_x - Bm\}\ dt\right)[\,\phi\,(m,t) + \hat{X}\,(x-s,m)\ u_x^2\ (x,m,0)$$

$$+\ U(s-x)\,\{-\hat{X}\,(x-s,m)\ u_x^2\ (x,m,0) + \psi\,(s-x)\}\,] \qquad \ldots\ (3.16)$$

where the function $\phi\,(m,t)$ and $\psi\,(s)$ are defined by eqn. (3.8) and eqn. (3.15) respectively. The above expression shows two different regions (i) the MHD Rayleigh region ($x > s$) and (ii) MHD Blasius region ($x < s$).

4. Applications

In order to apply this method, two steps are necessary. First, the functional relation in eqn. (2.5) is to be expressed in explicit form, then $h(x, m, t)$, the scale parameter of the velocity profile, is to be evaluated from eqn. (3.16). The asymptotic behaviour of the exact solution in the absence of the magnetic field for an accelerated motion over a body suggests the choice of $f(z)$ as the error function

$$f(z) = 1 - \frac{\text{erfc}\,(z+a)}{\text{erfc}\ a} \qquad \ldots\ (4.1)$$

where 'a' is given by

$$a = \frac{\pi^{1/2}}{4} h_0^2 \left(\frac{1}{u_\infty} \frac{\partial u_\infty}{\partial t} + \frac{\partial u_\infty}{\partial x} \right) \qquad \ldots (4.2)$$

and

$$\alpha_1 = \frac{\alpha_3}{2} - a\,; \quad \alpha_2 = \left(\frac{2}{\pi}\right)^{1/2} \left(\frac{\operatorname{erfc}(\sqrt{2}\, a)}{\operatorname{erfc}^2 a} \right) - \frac{\alpha_3}{2}$$

$$\alpha_3 = \frac{2\, e^{-a^2}}{\sqrt{\pi}\, \operatorname{erfc} a}\,; \quad \alpha_4 = 2 - \frac{\alpha_3}{2}. \qquad \ldots (4.3)$$

Now from eqn. (2.5), in view of eqn. (4.1), we write

$$u = u_\infty \left(1 - \frac{\operatorname{erfc}(z + a)}{\operatorname{erfc} a} \right). \qquad \ldots (4.4)$$

Determination of $H(x, m, t)$

Using the boundary conditions (2.3) in equation (3.16), we get

$$H = \exp\left(-\int_0^t \{A\,(u_\infty)_t - Bm\}\, dt \right) [\phi\,(m, t) + U\,(s-x)\,\psi\,(s-x)\,]. \qquad \ldots (4.5)$$

Equation (4.5) reduces to

$$H = \exp\left(-\int_0^t \{A\,(u_\infty)_t - Bm\}\, dt \right) \phi\,(m, t)\,; \quad x > s \qquad \ldots (4.6)$$

and

$$H = \exp\left(-\int_0^t \{A\,(u_\infty)_t - Bm\}\, dt \right) [\phi\,(m, t) + \psi\,(s-x)\,]\,; \; x < s. \qquad \ldots (4.7)$$

Further from eqn. (3.9), we obtain

$$s = \frac{\alpha_2}{\alpha_1} \left[\frac{w\, t^{\alpha+1}}{\alpha+1} \left\{ 1 - \frac{mt}{\alpha+2} + \frac{m^2 t^2}{(2+\alpha)(3+\alpha)} - \ldots \right\} w \frac{dw}{dx} t^{2\alpha+2} + \left\{ \frac{mt}{(1+\alpha)^2 (2\alpha+3)} - \frac{2m^2 t^2}{(1+\alpha)^2 (2+\alpha)(2\alpha+4)} + \ldots \right\} \right] \qquad \ldots (4.8)$$

where $\phi\,(m, t)$ is given by equation (3.8) $\psi\,(s-x)$ from (3.15), and α_i's from (4.3). Equations (4.6) (4.7) show that the flow has a singularity at the junction of the two regions.

Examples

(a) For uniformly accelerated flow over a flat plate

$$\alpha = 1, \quad w(x) = \text{constant} = 1. \qquad \ldots (4.9)$$

Therefore, the boundary condition (2.3) reduces to

$$u_\infty\ (x, m, t) = u_\infty\ (m, t) = \frac{1}{m}\ [\ 1 - e^{-mt}\ ]. \quad \text{... (4.10)}$$

Using (4.10) and (3.8) in eqn. (4.6), we finally get

$$\frac{h^2}{t} = \frac{2\alpha_3}{\alpha_1}\ \frac{e^{Bmt}}{(1-e^{-mt})^2}\ \left[\ \frac{1}{Bmt}\{\ 1 - e^{-Bmt}\ \} + \frac{1}{(B+2)\ mt}\ \{\ 1 - e^{-(B+2)\ mt}\ \}\right.$$

$$\left. - \frac{2}{(1+B)\ mt}\ \{\ 1 - e^{-(1+B)\ mt}\ \}\ \right] \quad \text{... (4.11)}$$

and equation (4.8) gives

$$s = \frac{\alpha_2{}'}{\alpha_1}\left[\ \frac{t^2}{2}\ \left\{\ 1 - \frac{mt}{3} + \frac{m^2\ t^2}{3.4} + \ldots\ \right\}\ \right]. \quad \text{... (4.12)}$$

(b) For impulsive flow over a circular cylinder of radius 'r'

$$\alpha = 0 \quad \text{... (4.13)}$$

therefore, the boundary condition (2.3) becomes

$$u_\infty\ (x, m, t) = w(x)\ e^{-mt} + t\ \frac{dw}{dx}\ mt\ (1 + mt)^{-1}. \quad \text{... (4.14)}$$

Using eqns. (4.14) and (3.8) in (4.6), we find

$$H = \frac{2\alpha_3}{\alpha_1}\ \exp\left(-\int_0^t\ \{A\ (u_\infty)_x\ -\ Bm\ \}\ dt\right)$$

$$\left[\ \int_0^t\ u_\infty^2\ \exp\left(\ \int_0^t\ \{A\ (u_\infty)_x\ -\ Bm\ \}\ dt\right)\ dt\ \right] \quad \text{... (4.15)}$$

where u_∞ is given by (4.14).

At the point of separation, we put $\left(\dfrac{\partial u}{\partial y}\right)_{y=0} = 0,\ t\ =\ t_{sp}$ and the distance covered before the separation is given by (Dave[5])

$$S_{sp} = \int_0^{t_{sp}}\ 1\ dt_{sp} = t_{sp} \quad \text{... (4.16)}$$

and for a circular cylinder of 'r' in the neighbourhood of rear stagnation point, we have

$$\left(-\ \frac{dw\ (x)}{dx}\right)\ S_{sp} = \frac{2}{r}\ . \quad \text{... (4.17)}$$

On simplification, equation (4.15) given in view of (4.17)

$$h^2 = \frac{2\alpha_3}{\alpha_1} \exp\left(\frac{2A}{r}\, t_{sp} - m\left\{\frac{2A}{r}\, t_{sp}^2 + \frac{4A}{3r^2}\, t_{sp}^2 - (B-2)\, t_{sp}\right\}\right)$$

$$\times \int_0^{t_{sp}} \exp\left(-\frac{2A}{r}\, t_{sp} - m\left\{\frac{2A}{r}\, t_{sp}^2 + \frac{4A}{3r^2}\, t_{sp}^3 - (B+2)\, t_{sp}\right\}\right) dt_{sp}. \qquad (4.18)$$

Although the flow for other values of α may be obtained in a similar manner but the analysis become increasingly complicated.

5. Characteristic Boundary Layer Parameters

(1) *Coefficient of Skin-friction*

The dimensionless coefficient of skin-friction C_f for the accelerated motion over a body, in view of (2.5) and (4.1) is given by

$$C_f = \frac{\mu \left(\frac{\partial U}{\partial Y}\right)_{Y=0}}{\rho\, U_0^2 / 2} \qquad \ldots (5.1)$$

$$\frac{1}{2\, t^{\alpha - 1/2}}\, C_f^* = \frac{1}{t^{\alpha - 1/2}} \left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\alpha_3\, u_\infty}{h t^{\alpha - 1/2}} = \left(\frac{2\, e^{-a^2}}{\sqrt{\pi}\ \mathrm{erfc}\ a}\right) \frac{u_\infty}{h\, t^{\alpha - 1/2}} \qquad \ldots (5.2)$$

where $\quad C_f^* = \dfrac{T_0^{1/2}}{\nu^{1/2}}\, U_0\, C_f.$

(2) *Coefficient of Magnetic Drag*

The dimensionless coefficient of magnetic drag C_m is defined as

$$C_m = \frac{2}{\rho\, U_0^2} \int_0^\infty \sigma_e B_0^2\, U\, dY \qquad \ldots (5.3)$$

$$\frac{1}{2}\, t^{\alpha - 1/2}\, C_m^* = mt\, \frac{h}{\sqrt{t}}\, \alpha_4\, \frac{u_\infty}{t^\alpha}$$

$$\frac{1}{2}\, t^{\alpha - 1/2}\, C_m^* = mt\, \frac{h}{\sqrt{t}} \left(2 - \frac{e^{-a^2}}{\sqrt{\pi}\ \mathrm{erfc}\ a}\right) \frac{u_\infty}{t^\alpha} \qquad \ldots (5.4)$$

where $\quad C_m^* = \dfrac{T_0^{1/2}\, U_0}{\nu^{1/2}}\, U_0\, C_m.$

(3) *Displacement Thickness*

The dimensional displacement thickness (by definition)

$$\delta_1 = \int_0^\infty \left(1 - \frac{U}{U_\infty}\right) dY \qquad \ldots (5.5)$$

$$\frac{\delta_1^*}{\sqrt{t}} = \frac{h}{\sqrt{t}}\,\alpha_1 = \frac{h}{\sqrt{t}}\left(\frac{2\,e^{-a^2}}{\sqrt{\pi}\,\mathrm{erfc}\;a} - a\right) \qquad \ldots (5.6)$$

where

$$\delta_1^* = \frac{\delta_1}{(T_0\,\nu)^{1/2}}\,.$$

(4) *Momentum Thickness*

The dimensional momentum thickness (by definition)

$$\delta_2 = \int_0^\infty \frac{U}{U_\infty}\left(1 - \frac{U}{U_\infty}\right) dY \qquad \ldots (5.7)$$

$$\frac{\delta_2^*}{\sqrt{t}} = \frac{h}{\sqrt{t}}\left[\left(\frac{2}{\pi}\right)^{1/2}\frac{\mathrm{erfc}\;(\sqrt{2}\,a)}{\mathrm{erfc}^2\,a} - \frac{e^{-a^2}}{\sqrt{\pi}\,\mathrm{erfc}\;a}\right] \qquad \ldots (5.8)$$

where $\quad \delta_2^* = \dfrac{\delta_2}{(T_0\,\nu)^{1/2}}\,.$

The value of h is to be taken from equation (3.16) and a is given by eqn. (4.2).

6. Numerical Discussion and Conclusion

Equation (3.16) shows that when the fluid is accelerated over a body placed symmetrical to the flow in the presence of transverse magnetic field to the body, the flow field can be divided in two regions namely MHD Rayleigh region ($x > s$) and MHD Blasius region ($x < s$) with a singularity at the junction of the two regions.

Table I

Distance traversed by the boundary layer before separation occurs in an impulsive flow past a circular cylinder of radius 'r' in the presence of a transverse magnetic field fixed relative to the cylinder.

	$\alpha = 0.0$ S_p/r	
mt_{sp}	Present method	Dave[5]
0.0	0.3536077	0.351021
0.1	0.2923175	0.313217
0.2	0.2131908	0.276054

The distance traversed by the boundary layer before separation occurs for an impulsive flow on a circular cylinder of radius 'r', for small values of the magnetic interaction parameter mt_{sp} is given in Table I and the results are compared with Dave[5]. This clearly shows that the effect of the magnetic field results in an earlier separation of the bundary layer and thus to destabilize the flow.

The velocity distribution is shown graphically in Figs. 2 and 3 respectively and wherever possible the results are compared with Dave[5]. One may conlcude from these figures that the velocity decreases with the increase in the magnetic field . This shows the decelerating effect of the magnetic field on boundary layer flow and concequently the coefficient of skin-friction C_f^* decreases which is given in Table II and same is shown graphically in Fig. 4 for uniformly accelerated flow over a flat plate.

TABLE II

Characteristic boundary layer parameters for uniformly accelerated flow over a flat plate in the presence of transverse magnetic field relative to the plate for $s - x < 0$

mt	$\frac{h}{\sqrt{t}}$	$\frac{1}{4\sqrt{t}} C_f^*$	$\frac{\delta_1^*}{\sqrt{t}} = \alpha_1 \frac{h}{\sqrt{t}}$	$\frac{\delta_2^*}{\sqrt{t}} = \alpha_2 \frac{h}{\sqrt{t}}$	$\frac{1}{4\sqrt{t}} C_m^*$	$\frac{1}{4\sqrt{t}} (C_f^* + C_m^*)$
0.0	1.82417	0.54024	0.72036	0.29538	0.00000	0.54024
0.1	2.05195	0.47907	0.78458	0.34927	0.09442	0.57350
0.2	2.21617	0.41921	0.85743	0.41781	0.19583	0.61504
0.3	2.61929	0.37243	0.94428	0.41807	0.29561	0.66804

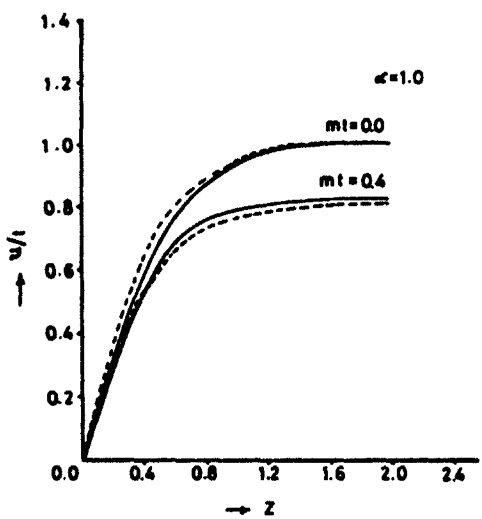

Fig.2. Velocity proiles for uniformly accelerated flow over a flat plate for $s - x < 0$; solid linies; present method; dotted lines; Dave (1989).

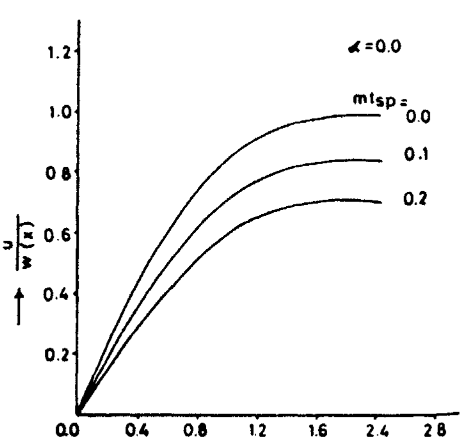

Fig.3. Velocity profiles near seperation in MHD boundary layer growth in an impulsive motion past a circular cylinder for different values of mt_{sp}.

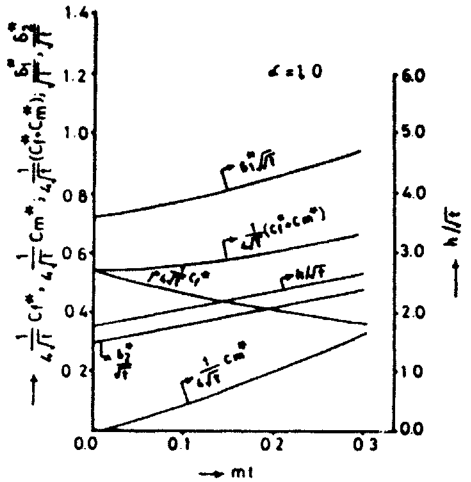

Fig.4. Characteristic boundary layer parameters for uniformly accelerated flow over a flat plate for $s - x < 0$.

For uniformly accelerated flow over a flat plate, the variation of scale factor h, displacement thickness δ_1^*, momentum thickness δ_2^* and coefficient of magnetic drag C_m^* with magnetic interaction parameter 'mt' are included in Table II and same are included in Fig. 4. The scale factor, displacement thickness, momentum thickness and coefficient of magnetic drag increase with the increase in the magnetic parameter.

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