MHD FLUCTUATING FLOW OF VISCOELASTIC FLUID PAST A POROUS INFINITE FLAT PLATE IN SLIP FLOW REGIME

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In the present paper exact solutions of the equations describing the flow of an incompressible, electrically conducting Non-Newtonian fluid [Rivlin-Ericksen viscoelastic (1955) model] in the presence of a transverse magnetic field in slip flow regime have been obtained. It is found that the velocity profile asymptotically approaches the mean stream velocity when h_1, K and S increase and the skin friction decreases with the increase of h_1, K and S where h_1 is the rarefaction parameter, K the magnetic field parameter and S the viscoelastic parameter.

1. INTRODUCTION

The flow of a viscous imcompressible and electrically conducting fluid past an infinite porous flat plate subjected to uniform suction in the presence of a transverse magnetic field when the free stream velocity oscillates in magnitude but not in direction has been discussed by Suryaprakash Rao (1962). Later on Siddappa and Chetty (1975) extended Suryaprakash Rao's case by considering the velocity slip conditions in place of non-slip boundary conditions.

In the present paper, we have extended Siddappa and Chetty's case to viscoelastic (Rivlin-Ericksen model) fluids.

2. EQUATIONS DESCRIBING THE FLOW

Let u and v be the components of velocity in x- and y-directions respectively taken along and perpendicular to the plate. A constant magnetic field of strength H_0 is applied in the y-direction and fixed relative to the plate. Induced magnetic field may be neglected by assuming the conductivity of the fluid to be very small. Here all parameters are independent of x except the pressure since the plate is infinite.

The equations describing the flow of an incompressible, electrically conducting non-Newtonian (Rivlin-Ericksen viscoelastic model) fluid in the presence of a transverse magnetic field in slip flow regime are

$$\frac{\partial u}{\partial t} - |V_0| \frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \alpha\frac{\partial^2 u}{\partial y^2} + \beta\frac{\partial^2}{\partial y^2}\left\{\frac{\partial u}{\partial t} - |V_0|\frac{\partial u}{\partial y}\right\} - \frac{\sigma}{\rho} B_0^2 u \qquad \ldots(1)$$

$$-\frac{1}{\rho}\frac{\partial p}{\partial y}+2(2\beta+\gamma)\frac{\partial u}{\partial y}\frac{\partial^2 u}{\partial y^2}=0 \qquad \ldots(2)$$

$$\frac{\partial V}{\partial y}=0 \qquad \ldots(3)$$

$$V=-\mid V_0 \mid \qquad \ldots(4)$$

where α, β and γ are the kinematic viscosity, kinematic viscoelasticity and kinematic cross-viscosity respectively, $V_0\,(>0)$ is velocity suction into the plate, p is the pressure, ρ the density of the fluid, σ the electrical conductivity, $B_y=\mu_e H_0=B_0$ (constant) the component of electromagnetic induction.

The boundary conditions for u are

$$u=\frac{(2-f_1)}{f_1}L\left(\frac{\partial u}{\partial y}\right)=L_1\left(\frac{\partial u}{\partial y}\right) \text{ at } y=0$$

(i.e. the first order velocity slip boundary condition)
and

$$u=U(t)=U_0\{1+\epsilon\, e^{i\omega t}\} \text{ as } y\to\infty \qquad \ldots(5)$$

where

$$L=\left(\frac{\pi}{2p\rho}\right)^{1/2}\mu, \quad L_1=\frac{(2-f_1)}{f_1}L$$

L is the mean free path constant and $U(t)$ is the free stream velocity, U_0 is the mean of U and ω is the frequency of the fluctuating stream, since $y\to\infty$, $u\to U$, the partial derivatives of u with respect to y tend to zero.

Equation (1) then becomes

$$\frac{dU}{dt}=-\frac{1}{\rho}\frac{\partial p}{\partial x}-\sigma\frac{B_0^2 u}{\rho}.$$

On substituting for $-\dfrac{1}{\rho}\dfrac{\partial p}{\partial x}$ in (1), we get

$$\frac{\partial u}{\partial t}-\mid V_0\mid\frac{\partial u}{\partial y}=\frac{dU}{dt}+\alpha\frac{d^2u}{dy^2}+\beta\frac{\partial^2}{\partial y^2}\left(\frac{\partial u}{\partial t}-\mid V_0\mid\frac{\partial u}{\partial y}\right)$$

$$+\frac{\sigma}{\rho}B_0^2(U-u). \qquad \ldots(6)$$

Following the method of Stuart (1955) we assume a solution of (6) in the form

$$u=U_0\{\phi_0(y)+\epsilon\, e^{i\omega t}\,\phi_1(y)\}. \qquad \ldots(7)$$

Substituting for u and U in (6) and equating non-harmonic and harmonic terms, we get

$$\alpha\frac{d^2\phi_0}{dy^2} - \beta \mid V_0 \mid \frac{d^3\phi_0}{dy^3} + \frac{\sigma B_0^2}{\rho}(1 - \phi_0) = - \mid V_0 \mid \frac{d\phi_0}{dy} \quad ...(8)$$

and

$$i\omega(\phi_1 - 1) - \mid V_0 \mid \frac{d\phi_1}{dy} = \alpha\frac{d^2\phi_1}{dy^2} + \beta i\omega \frac{d^2\phi_1}{dy^2} - \beta \mid V_0 \mid \frac{d^3\phi_1}{dy^3} + \frac{\sigma B_0^2}{\rho}(1 - \phi_1) \quad ...(9)$$

with the boundary conditions

$$\left.\begin{array}{l} \left.\begin{array}{l} \phi_0(y) = L_1 \dfrac{d\phi_0}{dy} \\ \text{and} \\ \phi_1(y) = L_1 \dfrac{d\phi_1}{dy} \end{array}\right\} \text{ at } y = 0 \\ \text{and} \\ \phi_0 = \phi_1 = 1 \text{ as } y \to \infty. \end{array}\right\} \quad ...(10)$$

Assuming

$$\eta = \frac{\mid V_0 \mid y}{\alpha}, \lambda = \frac{\omega\alpha}{\mid V_0 \mid^2}, K = \frac{\sigma B_0^2\alpha}{\rho \mid V_0 \mid^2}, S = \frac{\beta \mid V_0 \mid^2}{\alpha} \quad ...(11)$$

where K is the magnetic field parameter and S the viscoelastic parameter.

Using (11) eqns. (8), (9) and the boundary conditions (10) reduce to

$$S\frac{d^3\phi_0}{d\eta^3} - \frac{d^2\phi_0}{d\eta^2} - \frac{d\phi_0}{d\eta} + K\phi_0 = K \quad ...(12)$$

$$S\frac{d^3\phi_1}{d\eta^3} + (1 + i\lambda S)\frac{d^2\phi_1}{d\eta^2} + \frac{d\phi_1}{d\eta} - \phi_1(K + i\lambda) = -(K + i\lambda) \quad ...(13)$$

and

$$\left.\begin{array}{l} \left.\begin{array}{l} \phi_0(y) = h_1 \dfrac{d\phi_0}{d\eta} \\ \phi_1(y) = h_1 \dfrac{d\phi_1}{d\eta} \end{array}\right\} \text{ at } \eta = 0 \\ \text{and} \\ \phi_0 = \phi_1 = 1 \text{ as } \eta \to \infty \end{array}\right\} \quad ...(14)$$

where

$$h_1 = \frac{L_1 \mid V_0 \mid}{\alpha} = \text{Rarefaction parameter.} \quad ...(15)$$

The solution of (12) under the boundary conditions (14) is given by

$$\phi_0(\eta) = 1 - \frac{e^{-h\eta}}{1 + hh_1} \quad ...(16)$$

where h_1 is given (15) and $-h$ is the root of

$$Sr^3 - r^2 - r + K = 0.$$

Similarly the solution of (13) under the boundary conditions (14) is

$$\phi_1(\eta) = 1 - \frac{e^{-h_2 \eta}}{1 + h_1 h_2} \qquad ...(17)$$

where $-h_2$ is the root of the cubic

$$Sr^3 + (1 + Si\lambda)\, r^2 + r - (K + i\lambda) = 0.$$

Therefore, from (7) the solution for u is given by

$$u = U_0 \left\{ 1 - \frac{e^{-h\eta}}{1 + hh_1} + \epsilon\, e^{i\omega t} \left(1 - \frac{e^{-h_2 \eta}}{1 + h_1 h_2} \right) \right\}. \qquad ...(18)$$

In slip flow, because of the boundary condition, the shear stress at the wall, τ_ω, is proportional to the slip velocity at the wall and is given by

$$\tau_\omega = \left\{ \alpha \frac{\partial u}{\partial y} + \beta \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial t} \right) \right\}_{y=0}$$

$$= |V_0|\, U_0 \left[\frac{h}{1 + hh_1} + \frac{\epsilon\, e^{i\omega t}}{\alpha} \left(\frac{h_2}{1 + h_1 h_2} \right) (\alpha + i\beta\omega) \right] \qquad ...(19)$$

It is to be noted from (18) that for fixed h_1, as

$$K \rightarrow \infty,\ h,\ h_2 \rightarrow \infty.$$

Therefore,

$$u = U_0(1 + \epsilon\, e^{i\omega t}).$$

i. e. main stream velocity.

For fixed K, as $h_1 \rightarrow \infty$, we have

$$u = U_0(1 + \epsilon\, e^{i\omega t}).$$

Also for fixed h_1, as S increase, h and h_2 increase and ultimately tend to ∞.

$$\therefore\ u = U_0(1 + \epsilon\, e^{i\omega t})$$

It is evident from the above explanations that with an increase in the magnetic field parameter and viscoelastic parameter velocity increases. But in the slip flow regime the velocity asymptotically approaches the main stream velocity as per Siddappa and Chetty (1975). Also, for fixed K and S, as $h_1 \rightarrow \infty$, $\tau_\omega \rightarrow 0$ and for fixed h_1, as $K \rightarrow \infty$, $S \rightarrow \infty$, we get $\tau_\omega = 0$. This shows that the skin friction co-efficient decreases with the increase of h_1, K and S. From equation (2), the expression for pressure is given by

$$p = (2\beta + \gamma)\left(\frac{\partial u}{\partial y}\right)^2 + g(t)$$

$$= \frac{|V_0| U_0}{\alpha}(2\beta + \gamma)\left[\frac{he^{-h\eta}}{1 + hh_1} + \frac{\epsilon\, h_2 e^{i\omega t}\, e^{-h_2\eta}}{1 + h_1 h_2}\right]^2 + g(t)$$

where $g(t)$ is constant of integration.

From the expression of p, as given above, one can notice that along with elasticity, cross-viscosity of the fluid also affects the pressure.

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