ON THE GEOMETRY OF RELATIVISTIC ELECTROMAGNETIC FLUID FLOWS I

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The purpose of this paper is to express the equations governing a relativistic electromagnetic fluid in the intrinsic form and to apply these equations to study a class of special flows, problems of possible relevance to galactic cosmogony, gravitational collapse, pulsars and sunspots which are the sheet of magnetic fields.

1. INTRODUCTION

Let us introduce the vectors u^α, a^α, b^α, n^α ($\alpha = 1, 2, 3, 4$) at a point on the observer's world line such that u^α is unit time-like vector tangential to world line and a^α, b^α, n^α are space-like unit vectors satisfying the conditions

$$u^\alpha u_\alpha = 1; \;\; a^\alpha a_\alpha = b^\alpha b_\alpha = n^\alpha n_\alpha = -1 \qquad \text{...(1.1)}$$

$$u^\alpha a_\alpha = u^\alpha b_\alpha = u^\alpha n_\alpha = a^\alpha b_\alpha = a^\alpha n_\alpha = b^\alpha n_\alpha = 0 \qquad \text{...(1.2)}$$

in a four dimensional space whose space-time metric is of signature $(+, -, -, -)$.

Let us take u^α as the velocity vector of the fluid, a^α the unit magnetic field, b^α the unit electric field and n^α the unit electromagnetic energy-flux vector.

Following Glass (1975) we define γ^α_β as the projection operator onto the 3-space quotient to the stream lines congruences u^α such that

$$\gamma_{\alpha\beta} = g_{\alpha\beta} - u_\alpha u_\beta. \qquad \text{...(1.3)}$$

Projection operator will be abbreviated by $\perp$ which projects all free indices as under

$$\perp A^{\beta\nu}_{\alpha\beta} = \gamma^\mu_\alpha \gamma^\nu_\lambda A^{\beta\lambda}_{\mu\beta}.$$

The kinematical quantities which characterize the stream lines are the acceleration, the expansion, the shear and vorticity bi-vector (Ehlers 1962) which are given by

$$D_u u^\mu = u^\alpha u^\mu_{;\alpha} \qquad \text{...(1.4)}$$

$$\theta = u^\alpha_{;\alpha} \qquad \text{...(1.5)}$$

$$\sigma_{\alpha\beta} = \perp [u_{(\alpha;\beta)} - \tfrac{1}{3}\theta\gamma_{\alpha\beta}] \qquad ...(1.6)$$

$$w_{\alpha\beta} = \perp u_{[\alpha;\beta]} \qquad ...(1.7)$$

where round bracket and rectangular bracket denote symmetric and skew symmetric part respectively and semi-colon denotes covariant differentiation.

It follows from (1.3) – (1.7) that

$$u_{\alpha;\beta} = \sigma_{\alpha\beta} + w_{\alpha\beta} + \tfrac{1}{3}\theta\gamma_{\alpha\beta} + D_u u_\alpha u_\beta. \qquad ...(1.8)$$

The vorticity vector is given by

$$w^\alpha = \tfrac{1}{2}\eta^{\alpha\beta\gamma\delta} u_\beta u_{\gamma;\delta} \qquad ...(1.9)$$

where $\eta^{\alpha\beta\gamma\delta}$ is the permutation tensor.

$$\mathcal{L}_u \gamma_{\mu\nu} = 2\sigma_{\mu\nu} + \tfrac{2}{3}\theta\gamma_{\mu\nu} \qquad ...(1.10)$$

where $\mathcal{L}_u$ denotes Lie-derivative with respect to u^α.

In a similar fashion we define $\overset{*}{\gamma}{}^\alpha_\beta$ as the projection operator onto the 3-space quotient to the magnetic field lines congruences a^α such that

$$\overset{*}{\gamma}_{\alpha\beta} = g_{\alpha\beta} + a_\alpha a_\beta. \qquad ...(1.11)$$

The kinematical quantities which characterize the magnetic field lines are the magnetic flux, the expansion, the shear and the vorticity bivector (Date 1972) which are given by

$$D_a a^\alpha = a^\beta a^\alpha_{;\beta} \qquad ...(1.12)$$

$$\overset{*}{\theta} = a^\alpha_{;\alpha} \qquad ...(1.13)$$

$$\overset{*}{\sigma}_{\alpha\beta} = \overset{*}{\perp} [a_{(\alpha;\beta)} - \tfrac{1}{3}\overset{*}{\theta}\,\overset{*}{\gamma}_{\alpha\beta}] \qquad ...(1.14)$$

$$\overset{*}{w}_{\alpha\beta} = \overset{*}{\perp} a_{[\alpha;\beta]} \qquad ..(1.15)$$

where $\overset{*}{\perp}$ is projection operator.

It follows from (1.11) – (1.15) that

$$a_{\alpha;\beta} = \overset{*}{\sigma}_{\alpha\beta} + \overset{*}{w}_{\alpha\beta} + \tfrac{1}{3}\overset{*}{\theta}\,\overset{*}{\gamma}_{\alpha\beta} - D_a a_\alpha \,.\, a_\beta. \qquad ...(1.16)$$

The vorticity vector is given by

$$\overset{*}{w}{}^\alpha = \tfrac{1}{2}\eta^{\alpha\beta\gamma\delta} a_\beta a_{\gamma;\delta} \qquad ...(1.17)$$

$$\mathcal{L}_a \overset{*}{\gamma}_{\mu\nu} = 2\overset{*}{\sigma}_{\mu\nu} + \tfrac{2}{3}\overset{*}{\theta}\,\overset{*}{\gamma}_{\mu\nu} \qquad ...(1.18)$$

where $\mathcal{L}_a$ denotes Lie-derivative with respect to a^α.

We define $\overset{\vee}{\gamma}{}^{\alpha}_{\beta}$ as the projection operator onto the 3-space quotient to the electric field lines congruences b^{α} such that

$$\overset{\vee}{\gamma}_{\alpha\beta} = g_{\alpha\beta} + b_{\alpha} b_{\beta}. \qquad \ldots(1.19)$$

The kinematical quantities which characterize the electric field lines are the electric flux, the expansion, the shear and the vorticity bivector (Prasad and Ojha 1977) which are given by

$$D_b b^{\alpha} = b^{\beta} b^{\alpha}_{;\beta} \qquad \ldots(1.20)$$

$$\overset{\vee}{\theta} = b^{\alpha}_{;\alpha} \qquad \ldots(1.21)$$

$$\overset{\vee}{\sigma}_{\alpha\beta} = \overset{\vee}{\perp} [b_{(\alpha;\beta)} - \tfrac{1}{3} \overset{\vee}{\theta} \overset{\vee}{\gamma}_{\alpha\beta}] \qquad \ldots(1.22)$$

$$\overset{\vee}{w}_{\alpha\beta} = \overset{\vee}{\perp} b_{[\alpha;\beta]} \qquad \ldots(1.23)$$

where $\overset{\vee}{\perp}$ is the projection operator.

It follows from (1.19) – (1.23) that

$$b_{\alpha;\beta} = \overset{\vee}{\sigma}_{\alpha\beta} + \overset{\vee}{w}_{\alpha\beta} + \tfrac{1}{3} \overset{\vee}{\theta} \overset{\vee}{\gamma}_{\alpha\beta} - D_b b_{\alpha} b_{\beta}. \qquad \ldots(1.24)$$

The vorticity vector is given by

$$\overset{\vee}{w}{}^{\alpha} = \tfrac{1}{2} \eta^{\alpha\beta\gamma\delta} b_{\beta} b_{\gamma;\delta} \qquad \ldots(1.25)$$

$$\mathcal{L}_b \overset{\vee}{\gamma}_{\alpha\beta} = 2\overset{\vee}{\sigma}_{\alpha\beta} + \tfrac{2}{3} \overset{\vee}{\theta} \overset{\vee}{\gamma}_{\alpha\beta} \qquad \ldots(1.26)$$

where $\mathcal{L}_b$ denotes Lie-derivative with respect to b^{α}.

We define tensor fields

$$U_{\alpha\beta} = u_{\alpha;\beta} - U_{\alpha} u_{\beta} \qquad \ldots(1.27)$$

$$A_{\alpha\beta} = a_{\alpha;\beta} + A_{\alpha} a_{\beta} \qquad \ldots(1.28)$$

$$B_{\alpha\beta} = b_{\alpha;\beta} + B_{\alpha} b_{\beta} \qquad \ldots(1.29)$$

which respectively lie locally in the subspaces normal to u^{α}, a^{α} and b^{α}. The vectors U_{α}, A_{α} and B_{α} are the curvature vectors of the congruences generated by streamlines, magnetic field lines and electric field lines respectively. The congruences generated by u_{α}, $U_{\alpha\beta}$ are time-like and those generated by a_{α}, $A_{\alpha\beta}$ and b_{α}, $B_{\alpha\beta}$ are space-like congruences. Contracting (1.27), (1.28) and (1.29) by u^{β}, a^{β} and b^{β} respectively we get

$$D_u u^{\alpha} = U^{\alpha}; \; D_a a^{\alpha} = A^{\alpha}; \; D_b b^{\alpha} = B^{\alpha}.$$

Theorem 1.1 — The Lie derivative of the projection operator onto the 3-space quotient to the streamlines, magnetic field lines and electric field lines vanishes respectively if the fluid, magnetic field and electric field are free from shear and expansion.

PROOF: By virtue of (1.10), (1.18) and (1.26), we get the result.

2. FIELD EQUATIONS

We assume that the matter distribution in space-time of general relativity is a thermally conducting, viscous, compressible, non-inductive charged self-gravitating fluid. Thus the energy-momentum tensor is given by Date (1973)

$$T^{\alpha\beta} = \hat{\rho} u^\alpha u^\beta - \hat{p} g^{\alpha\beta} - (\lambda e^\alpha e^\beta + \mu h^\alpha h^\beta) + \nu\sigma^{\alpha\beta} + (q^\alpha - V^\alpha) u^\beta + (q^\beta - V^\beta) u^\alpha \quad \text{...(2.1)}$$

where e^α, h^α, q^α and V^α denote the electric field vector, the magnetic field vector, the heat energy flux vector and the electromagnetic energy flux vector respectively and

$$\hat{\rho} = \rho + p - \beta\theta + \lambda \mid e \mid^2 + \mu \mid h \mid^2 \quad \text{...(2.2)}$$

$$\hat{p} = p - \beta\theta + \tfrac{1}{2} (\lambda \mid e \mid^2 + \mu \mid h \mid^2) \quad \text{...(2.3)}$$

where ρ, p and β denote the matter energy density, isotropic pressure of the fluid and coefficient of bulk viscosity respectively. Units in which $8\pi G$ (gravitational constant) $= c = 1$ will be used.

The Ricci identity for any vector u^α is

$$u^\alpha_{;\alpha\beta} - u^\alpha_{;\beta\alpha} = R_{\alpha\beta} u^\alpha \quad \text{...(2.4)}$$

and the Einstein's field equations are

$$- T^{\alpha\beta} = R^{\alpha\beta} - \tfrac{1}{2} R g^{\alpha\beta} + \Lambda g^{\alpha\beta} \quad \text{...(2.5)}$$

where $R^{\alpha\beta}$ is the Ricci symmetric tensor and Λ is the cosmological constant.

The equations defining the thermodynamical variables are as follows:

$$T_0 dS = di + pd\left(\frac{1}{\rho_0}\right) \quad \text{...(2.6)}$$

$$q^\alpha = - K(T_{0,\beta} - T_0 D_u u_\beta) \gamma^{\alpha\beta} \quad \text{...(2.7)}$$

$$S^\alpha = \rho_0 S u^\alpha + \frac{1}{T_0} q^\alpha \quad \text{...(2.8)}$$

where ρ_0 is the proper matter density, i the internal energy density of the fluid, T_0 the rest temperature, K the heat conduction coefficient and S^α the entropy flux vector.

The energy-momentum tensor (2.1) satisfies $T^{\alpha\beta}_{;\beta} = 0$ therefore using (1.5), (1.8), (1.12), (1.13), (1.20) and (1.21), we get

$$(D_u \hat{\rho})\, u^\alpha + \hat{\rho} D_u u^\alpha + \hat{\rho}\theta u^\alpha - \hat{p}_{,\beta} g^{\alpha\beta} - D_b(\lambda \mid e \mid^2)\, b^\alpha$$
$$- (\lambda \mid e \mid^2)\, D_b b^\alpha - (\lambda \mid e \mid^2)\, \check{\theta} b^\alpha - D_a(\mu \mid h \mid^2)\, a^\alpha$$
$$- \mu \mid h \mid^2 D_a a^\alpha - \mu \mid h \mid^2 \overset{*}{\theta} a^\alpha + (\nu\sigma^{\alpha\beta})_{;\beta} + D_u(q^\alpha - \nu^\alpha)$$
$$+ \theta(q^\alpha - \nu^\alpha) + (q^\beta - \nu^\beta)_{;\beta} u^\alpha + (q^\beta - \nu^\beta)\,(w^\alpha_\beta + \sigma^\alpha_\beta$$
$$+ \tfrac{1}{3}\,\theta\,\gamma^\alpha_\beta = 0 \qquad \ldots(2.9)$$

where $D_u \hat{\rho} = \hat{\rho}_{,\alpha} u^\alpha$ ... and comma denotes partial differentiation.

The mass conservation law (Misner and Sharp 1969) is

$$(\rho_0 u^\alpha)_{;\alpha} = 0. \qquad \ldots(2.10)$$

Contracting (2.9) by u_α and using (1.30), (2.6), (2.7), (2.8) and (2.10), we have

$$D_u(\lambda \mid e \mid^2 + \mu \mid h \mid^2) = 4\nu\sigma^2 + 2\beta\theta^2 - 4\theta(\lambda \mid e \mid^2 + \mu \mid h \mid^2)$$
$$- 2\eta T_0 - 2(\lambda \mid e \mid^2 b^\alpha b^\beta + \mu \mid h \mid^2 a^\alpha a^\beta)\,\sigma_{\alpha\beta}$$
$$+ \frac{2 \mid q \mid^2}{K T_0} + 2\zeta - 2 V^\alpha U_\alpha \qquad \ldots(2.11)$$

where $\eta = S^\alpha_{;\alpha}$; $\zeta = V^\alpha_{;\alpha}$, η is generation of entropy and ζ the generation of electromagnetic energy flux. Equation (2.11) is also obtained by Date (1973) in different form. The differential identity (2.11) is governing the variation of electromagnetic energy density (or pressure) along streamlines and it depends upon the curvature, expansion and shear associated with streamlines.

We deduce the following theorems :

Theorem 2.1 — The heat energy flux vector balances the electromagnetic energy flux vector if the expansion, rotation and shear of the fluid vanish.

PROOF : By straightforward calculations from (1.8), (2.1), (2.4) and (2.5), we get

$$(q_\alpha - V_\alpha) = w^\beta_{\alpha;\beta} - \sigma^\beta_{\alpha;\beta} + \tfrac{1}{3}\,\theta_{,\beta}\,\gamma^\beta_\alpha - D_u\theta u_\alpha$$
$$+ 2(w^2 - \sigma^2)\, u_\alpha + D_u u^\beta(w_{\alpha\beta} + \sigma_{\alpha\beta}) + \theta_{,\alpha} \qquad \ldots(2.12)$$

where $2w^2 = w_{\alpha\beta} w^{\alpha\beta}$ and $2\sigma^2 = \sigma_{\alpha\beta} \sigma^{\alpha\beta}$,

which shows that $q_\alpha = V_\alpha$ when $\theta = w_{\alpha\beta} = \sigma_{\alpha\beta} = 0$.

Theorem 2.2 — The heat energy flux vector becomes orthogonal to magnetic field vector if the expansion, rotation and shear of the magnetic field vanish.

PROOF : The equations (1.16), (2.1), (2.5) and the Ricci identity for a^α similar to (2.4) yield

$$q_\alpha a^\alpha = D_a a^\alpha (\overset{*}{w}_{\beta\alpha} + \overset{*}{\sigma}_{\beta\alpha} + \tfrac{1}{3} \overset{*}{\theta} g_{\beta\alpha}) u^\beta$$
$$+ \tfrac{1}{3} \overset{*}{\theta} u^\beta a^\alpha (\overset{*}{w}_{\beta\alpha} + \overset{*}{\sigma}_{\beta\alpha}) - \tfrac{2}{3} D_u \overset{*}{\theta} - 2(\overset{*}{w}^2 - \overset{*}{\sigma}^2) \qquad \ldots(2.13)$$

which shows that $q_\alpha a^\alpha = 0$ when $\overset{*}{\theta} = \overset{*}{w}_{\alpha\beta} = \overset{*}{\sigma}_{\alpha\beta} = 0$.

Theorem 2.3 — The heat energy flux vector becomes orthogonal to the electric field if the expansion, rotation and shear of the electric field vanish.

The proof follows the pattern of the proof of Theorem (2.2).

Let us define a similar kinematical relation for n^α as (1.16)

$$n_{\alpha;\beta} = \underset{\vee}{\sigma}_{\alpha\beta} + \underset{\vee}{w}_{\alpha\beta} + \tfrac{1}{3} \underset{\vee\vee}{\theta\gamma}_{\alpha\beta} - D_n n_\alpha n_\beta. \qquad \ldots(2.14)$$

Here the parameters have their usual meaning.

Theorem 2.4 — The heat energy flux vector can never be orthogonal to the electromagnetic energy flux vector due to vanishing of kinematical parameters associated with unit electromagnetic energy flux vector.

The proof follows the pattern of the proof of Theorem 2.2.

Theorem 2.5 — The variation of the magnetic pressure is balanced by total pressure due to viscous fluid and electric field along the magnetic field lines if the heat energy flux vector is orthogonal to magnetic field and balances electromagnetic flux vector simultaneously.

PROOF : We have from Theorems 2.1 and 2.2

$$q^\alpha = V^\alpha \Rightarrow \theta = w_{\alpha\beta} = \sigma_{\alpha\beta} = 0$$

$$q^\alpha a_\alpha = 0 \Rightarrow \overset{*}{\theta} = \overset{*}{w}_{\alpha\beta} = \overset{*}{\sigma}_{\alpha\beta} = 0$$

$$u^\alpha a_\alpha = 0 \Rightarrow D_u u^\alpha a_\alpha + u^\alpha u^\beta a_{\alpha;\beta} = 0.$$

Using (1.16), we get

$$u^\alpha a_\alpha = 0 \Rightarrow D_u u^\alpha a_\alpha = 0.$$

Similarly

$$b^{\alpha}a_{\alpha} = 0 \Rightarrow D_b b^{\alpha} a_{\alpha} = 0.$$

Contracting (2.9) by a_{α} and using $D_u u^{\alpha} a_{\alpha} = 0$; $D_b b^{\alpha} a_{\alpha} = 0$, we get

$$D_{\alpha}(\tfrac{1}{2}\mu \mid h \mid ^2) = D_a(p - \beta\theta + \tfrac{1}{2}\lambda \mid e \mid ^2) \qquad ...(2.15)$$

which proves the statement.

Theorem 2.6 — The normal curvature vectors of the streamlines and the electric field lines are orthogonal to the magnetic field lines simultaneously when heat energy flux vector is orthogonal to magnetic field.

PROOF : We have from Theorem 2.2

$$q^{\alpha}a_{\alpha} = 0 \Rightarrow \overset{*}{\theta} = \overset{*}{w}_{\alpha\beta} = \overset{*}{\sigma}_{\alpha\beta} = 0$$

due to which, we get

$$D_u u^{\alpha} . a_{\alpha} = D_b b^{\alpha} . a_{\alpha} = 0.$$

Using (1.30), we have

$$U^{\alpha}a_{\alpha} = B^{\alpha}a_{\alpha} = 0.$$

Theorem 2.7 — The normal curvature vectors of the streamlines and the magnetic field lines are orthogonal to the electric field lines simultaneously when heat energy flux vector is orthogonal to the electric field.

The proof of the Theorem 2.7 is similar to that of the Theorem 2.6.

Theorem 2.8 — The variation of electric pressure is balanced by total pressure due to viscous fluid and magnetic field along the electric field lines if the heat energy flux is orthogonal to the electric field and balances the electromagnetic energy flux vector simultaneously.

PROOF : Contracting (2.9) by b^{α} and using theorems 2.1, 2.3 and 2.7, we have

$$D_b(\tfrac{1}{2}\lambda \mid e \mid ^2) = D_b(p - \beta\theta + \tfrac{1}{2}\mu \mid h \mid ^2) \qquad ...(2.16)$$

which proves the statement.

3. MAXWELL EQUATIONS

To discuss intrinsic mechanism of Maxwell equations we shall write first and second Maxwell's electromagnetic field equations in our notations respectively as (Banerji 1974)

$$D_u B^{\alpha} + B^{\alpha}\theta - B^{\beta}_{;\beta} u^{\alpha} - B^{\beta} u^{\alpha}_{;\beta} + \eta^{\alpha\beta\gamma\delta}(u_{\gamma;\beta}e_{\delta} + u_{\gamma}e_{\delta;\beta} = 0) \qquad ...(3.1)$$

$$D_u D^{\alpha} + D^{\alpha}\theta - D^{\beta}_{;\beta}\, u^{\alpha} - D^{\beta}\, u^{\alpha}_{;\beta} + \eta^{\alpha\beta\gamma\delta}\, u_{\gamma;\beta}\, h_{\delta}$$
$$+ \eta^{\alpha\beta\gamma\delta}\, u_{\gamma}\, h_{\delta;\beta} = -\,\epsilon u^{\alpha} - k e^{\alpha} \qquad \ldots(3.2)$$

where $B^{\alpha} = \mu h^{\alpha}$; $D^{\alpha} = \lambda e^{\alpha}$; k is conductivity and ϵ the charge density.

Theorem 3.1 — When heat energy flux vector is orthogonal to the magnetic field the variation of electromagnetic field is governed by the equation

$$D_a(\mu \mid h \mid) = 2 \mid e \mid w^{\alpha} b_{\alpha}. \qquad \ldots(3.3)$$

PROOF : Putting $B^{\alpha} = \mu h^{\alpha} = \mu \mid h \mid a^{\alpha}$ and $e^{\alpha} = \mid e \mid b^{\alpha}$ in (3.1) and contracting by u_{α}, and using Theorem 2.2, we get (3.3).

Corollary 3.1 — The magnetic induction along the magnetic field lines becomes constant when rotation of the fluid is orthogonal to the electric field lines.

Theorem 3.2 — When the heat energy flux vector is orthogonal to the electric field lines the variation of electromagnetic field is governed by the equation

$$D_b(\lambda \mid e \mid) = \epsilon + 2 \mid h \mid w^{\alpha} a_{\alpha}. \qquad \ldots(3.4)$$

The proof runs on the lines of Theorem 3.1.

Theorem 3.3 — The rotation of electric field lines is orthogonal to stream lines when heat energy flux vector is orthogonal to the magnetic field and balances electromagnetic energy flux vector.

PROOF : Putting $B^{\alpha} = \mu \mid h \mid a^{\alpha}$ and $e^{\alpha} = \mid e \mid b^{\alpha}$ in (3.1) and contracting by b_{α}, we get

$$\mu \mid h \mid (a_{\alpha;\beta} b^{\alpha} u^{\beta} - u_{\alpha;\beta} b^{\alpha} u^{\beta}) + 2 \mid e \mid \overset{\vee}{w}{}^{\alpha} u_{\alpha} = 0 \qquad \ldots(3.5)$$

which due to theorems (2.1) and (2.2) reduces to

$$\overset{\vee}{w}{}^{\alpha} u_{\alpha} = 0. \qquad \ldots(3.6)$$

This proves the statement.

Theorem 3.4 — The rotation of the magnetic field lines is orthogonal to the streamlines when the heat energy flux vector is orthogonal to electric field and balances electromagnetic energy flux vector.

The proof follows the pattern of Theorem 3.3.

Finally we discuss two special cases of Maxwell equations. Let us suppose that the field is purely of electrical nature. Thus in this case we have the following theorem.

Theorem 3.5 — The rotation of the fluid is orthogonal to electric field lines and the rotation of electric field lines is orthogonal to the streamlines simultaneously in the absence of magnetic field.

PROOF : In the absence of magnetic field eqn. (3.1) reduces to

$$\eta^{\alpha\beta\gamma\delta}\, u_{\gamma;\beta}\, e_\delta + \eta^{\alpha\beta\gamma\delta}\, u_\gamma e_{\delta;\beta} = 0. \qquad ...(3.7)$$

Substituting $e^\alpha = |\, e \,|\, b^\alpha$ in (3.7) and contracting by u_α and b_α, we have

$$w^\alpha b_\alpha = 0 \qquad ...(3.8)$$

$$\overset{\vee}{w}{}^\alpha u_\alpha = 0 \qquad ...(3.9)$$

respectively.

When the field is purely of magnetic nature we have the following theorem.

Theorem 3.6 — The rotation of the magnetic field lines is orthogonal to the streamlines but the rotation of the fluid is not orthogonal to the magnetic field lines in the absence of electric field.

PROOF : In the absence of electric field the eqn. (3.2) yields.

$$\eta^{\alpha\beta\gamma\delta}(u_{\gamma;\beta}\, h_\delta + u_\gamma h_{\delta;\beta}) = -\, \epsilon u^\alpha. \qquad ...(3.10)$$

Putting $h_\alpha = |\, h \,|\, a_\alpha$ in (3.10) and contracting by u_α and a_α, we have

$$w^\alpha a_\alpha = -\tfrac{1}{2}\,\frac{\epsilon}{|\, h \,|} \qquad ...(3.11)$$

$$^{*}w^\alpha u_\alpha = 0 \qquad ...(3.12)$$

which proves the statement.

Remark : (3.8) is also proved by Banerji (1974) and Raychaudhuri and De (1970) and (3.11) by Som and Raychaudhuri (1968), Ellis and Stewart (1968), Yodzis (1971) and Mason (1972).

Further from the equation of continuity

$$J^\alpha_{\ ;\alpha} = 0 \qquad ...(3.13)$$

where J^α is the electric current vector, the conservation of charge has already been proved by Banerji (1974). Here we consider the conservation of ohmic current in the absence of charge. In this case eqn. (3.13) leads to the following equation

$$D_b(\log k\, |\, e \,|\,) + \overset{\vee}{\theta} = 0 \qquad ...(3.14)$$

which shows that the variation of ohmic current along the electric field lines depends upon the expansion of the electric field.

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