# ON THE STRONG *SH* MOTION IN TRANSVERSELY ISOTROPIC LAYER OF NON-UNIFORM THICKNESS LYING OVER AN ISOTROPIC ELASTIC MATERIAL DUE TO EXPLOSIONS

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In this paper the dispersion equation of Love waves in transversely isotropic layer of non-uniform thickness lying over an isotropic elastic material due to explosions has been found by the use of a Green's function. As a special case the dispersion equation of Love waves when the layer is isotropic has been deduced. It has been observed that the displacement of the surface of the layer with given thickness becomes infinite if a certain relation exists between the layer shear wavenumber $k_1$ and one of the roots of the dispersion equation.

## INTRODUCTION

Displacements of Love waves generated by a point source in a layered medium have been studied earlier by Sezawa (1935) and Sato (1952) by the method of successive reflections at the boundaries. In this paper the same problem has been worked out by the use of a Green's function. Green's functions have been extensively discussed by Friedman (1962). Covert (1958) derived a method for finding the appropriate Green's function of built-up bodies. The use of Green's functions to find the displacements due to passage of Love waves gives us a powerful tool to find displacements when the shape of the layer is different from the space between two parallel planes.

In this problem the layer has been taken to be transversely isotropic and of non-uniform thickness. The thickness of the layer is assumed to vary linearly along the length of the layer and is assumed to be positive.

The upper boundary of the layer is sloping, the magnitude of the slope $\theta$ being small. $\theta^2$ and higher power of $\theta$ are neglected in writing down the coordinates of the image points. The upper boundary of the lower homogeneous medium is assumed to be horizontal. The source of disturbance is taken at the origin in the lower isotropic medium to avoid complexity. A dispersion equation of Love waves has been obtained which reduces to the usual dispersion equation when the layer is isotropic, i.e., when $L = N = \mu_1$, where $L$, $N$ are elastic constants and $\mu_1$ being the modulus of rigidity in the layer. The displacement on the surface obtained is found to be very large when

$$f_n = \frac{(k_1^2 h^2 - n^2\pi^2)^{1/2}}{h}.$$

The "singing phenomenon" can be cited as a related physically observed effect. In offshore seismic prospecting, seismic records sometimes assume the form of a sine wave or a simple combination of sine waves. The dominant frequency on the singing records is the third harmonic of a fundamental whose wavelength is four times the water depth. Ghosh (1961) ascribed the cause of this phenomenon to be the slope of the sedimentary layer near the observational points. He obtained the pressure in the water layer for a two dimensional point source in the same medium in a sedimentary layer with a slope and proved that the singing is caused by waves of wavelength equal to four times the water depth. Dasgupta (1962) also discussed the singing phenomenon and concluded that the singing phenomenon is more pronounced due to detonation inside the ocean bed.

## Solution of the Problem

The origin is taken at the source on the interface of the transversely isotropic layer denoted by medium 1 and the lower isotropic medium denoted by medium 2 (Fig. 1). The $x$-axis is taken along the interface and $z$-axis downwards at right angles to the interface through the point of detonation 0. Let $h$ be the height of the layer above the source point.

The equation of the transverse horizontal displacement in the medium 1 is

$$\nabla_1^2 V = \frac{\rho}{N} \frac{\partial^2 V}{\partial t^2} \qquad \ldots(1)$$

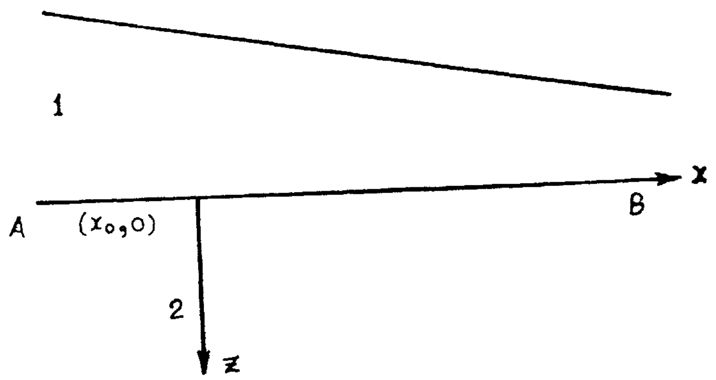


Fig. 1.

where

$$\nabla_1^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z_1^2} \qquad ...(2)$$

and

$$z = \sqrt{\frac{L}{N}} \cdot z_1.$$

For the lower isotropic medium 2 the equation is

$$\nabla^2 V = \frac{\rho_2}{\mu} \frac{\partial^2 V}{\partial t^2} \qquad ...(3)$$

where

$$\nabla^2 \equiv \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}. \qquad ...(4)$$

Taking the time dependence proportional to $e^{i\omega t}$, equations for media 1 and 2 are

$$\nabla_1^2 V_1 + k_1^2 V_1 = 0 \qquad ...(5)$$

and

$$\nabla^2 V_2 + k_2^2 V_2 = 0 \qquad ...(6)$$

where

$$\left.\begin{aligned} k_1^2 &= \frac{\omega^2 \rho}{N} \\ k_2^2 &= \frac{\omega^2 \rho_2}{\mu}. \end{aligned}\right\} \qquad ...(7)$$

The boundary conditions are at the free surface

$$N \frac{\partial V_1}{\partial x} \sin \theta - L \frac{\partial V_1}{\partial z} \cos \theta = 0 \qquad ...(8)$$

and at the interface $z = 0$,

$$V_1 = V_2 \qquad ...(9)$$

and

$$L \frac{\partial V_1}{\partial z} = \mu \frac{\partial V_2}{\partial z}. \qquad ...(10)$$

We propose to solve eqns. (5) and (6) under prescribed boundary conditions (8), (9) and (10) by the use of Green's functions technique. We are interested in the displacement on the surface and so we shall try to find $G(x, z/0, 0)$ for the body (1) viz., the upper layer. The method of finding Green's function of such composite media was indicated by Covert (1958) and has been successfully applied by Ghosh (1961, 1963), Dasgupta (1962), and Bhattacharyya (1962).

Let $G_1$, $G_2$ be the Green's function for media 1 and 2 under the boundary conditions $\frac{\partial G_1}{\partial n_1} = \frac{\partial G_2}{\partial n_2} = 0$ at the interface and $G_1 = 0$ at the free surface, $n_1$, $n_2$ correspond to the normal drawn outwards from the upper and lower medium respectively. The Green's function $G_1(r/r_0)$ for the upper medium satisfies the inhomogeneous Helmholtz equation

$$\nabla^2 G_1(r/r_0) + k^2 G_1(r/r_0) = -4\pi\delta(r - r_0) \qquad \ldots(11)$$

where $G_1(r/r_0)$ is the value of the Green's function at $r(x, z)$ for a unit point source at $r_0(x_0, y_0)$. Similarly we have the equation for $G_2$.

We have for a general source distribution of densities $\rho_1^1(r)$ and $\rho_2^1(r)$ in the upper and lower medium respectively, (cf. Morse and Feshback 1953)

$$V_1(r) = \int G_1(r/r_0)\,\rho_1^1(r_0)\,dV_{01} + \frac{1}{4\pi}\int_{AB} G_1(r/r_s)\,\frac{\partial V_1(r_s)}{\partial n_1}\,dS_{01} \qquad \ldots(12)$$

$$V_2(r) = \int G_2(r/r_0)\,\rho_2^1(r_0)\,dV_{02} + \frac{1}{4\pi}\int_{AB} G_2(r/r_s)\,\frac{\partial V_2(r_s)}{\partial n_2}\,dS_{02} \qquad \ldots(13)$$

where $r_s(x_s, 0)$ is a point on the interface and integration is taken over all the points $(x_s, 0)$ on the interface. The conditions $\frac{\partial G_1}{\partial n_1} = \frac{\partial G_2}{\partial n_2} = 0$ at the interface are employed in (12) and (13).

Using the boundary conditions (9) and (10) we have at a point $r_1(x_1, 0)$ on $AB$

$$\frac{1}{4\pi}\int_{AB}\left[G_1(r_1/r_s) + \frac{L}{\mu}\,G_2(r_1/r_s)\right]\frac{\partial V_1}{\partial n_1}\,dx_s + \int G_1(r_1/r_0)\,\rho_1^1(r_0)\,dV_{01}$$
$$- \int G_2(r_1/r_0)\,\rho_2^1(r_0)\,dV_{02} = 0. \qquad \ldots(14)$$

If the point source lies very near the origin but in the medium 2, then $\rho_1^1 = 0$, $\rho_2^1 = \delta(r - r_0)$. Here $r_0(0, 0)$ denotes the source point which is the origin. Under these circumstances $V_1$ and $V_2$ become the Green's function in the respective bodies. Then eqn. (14) becomes

$$G_2(r_1/0) = \frac{1}{4\pi}\int_{AB}\left[G_1(r_1/r_s) + \frac{L}{\mu}\,G_2(r_1/r_s)\right]\frac{\partial G(r_s/0)}{\partial z}\,dx_s \qquad \ldots(15)$$

where $G$ is the proper Green's function for body 1 corresponding to the source in the medium 2.

From eqn. (12) we get the Green's function for the upper layer as

$$G = \frac{1}{4\pi} \int_{AB} G_1(r/r_s) \frac{\partial G(r_s/0)}{\partial z} \, dx_s. \qquad \text{...(16)}$$

From the integral eqn. (15), $\frac{\partial G(r_s/0)}{\partial z}$ can be obtained and after substitution of the value of it in eqn. (16) we determine the Green's function for the upper layer completely.

If a source be situated at the origin in the medium 2, the Green's function $G_1$ for the upper layer corresponding to the boundary conditions $\frac{\partial G_1}{\partial n_1} = 0$ at the interface and $G_1 = 0$ at the upper boundary is obtained by the method of reflection in the form

$$\begin{aligned} G_1(x, z/0, 0) = 2i[&H_0^{(1)}\{k_1 \sqrt{x^2 + z^2}\} + H_0^{(1)}\{k_1 \sqrt{(x - 2h\theta)^2 + (z + 2h)^2}\} \\ &+ H_0^{(1)}\{k_1 \sqrt{(x - 2h\theta)^2 + (z - 2h)^2}\} \\ &+ H_0^{(1)}\{k_1 \sqrt{(x - 8h\theta)^2 + (z - 4h)^2}\} \\ &+ H_0^{(1)}\{k_1 \sqrt{(x - 8h\theta)^2 + (z - 4h)^2}\} + \ldots], \end{aligned}$$

where the reflected points are $(2h\theta, -2h)$, $(2h\theta, 2h)$, $(8h\theta, -4h)$, $(8h\theta, 4h)$, $(18h\theta, -6h)$, $(18h\theta, 6h)$, ... to the first powers in $\theta$ and $H_0^{(1)}$ is the Hankel function of the first kind of order zero. Using the integral representation of $H_0^{(1)}(k_1 r/r_0)$ in the form

$$H_0^{(1)}(k_1 r/r_0) = \frac{1}{\pi i} \int_{-\infty}^{\infty} \frac{\exp\,(if(x - x_0) - \alpha(y - y_0))}{\alpha} \, df$$

where $y - y_0 > 0$ and $\alpha^2 = f^2 - k_1^2$ (cf. Morse and Feshback, 1953, p. 823),

we get,

$$G_1(x, z/0, 0) = \frac{2}{\pi} \left[ \int_{-\infty}^{\infty} \frac{\exp\left(ifx + \sqrt{\frac{N}{L}}\,\alpha z\right)}{\alpha} \, df \right.$$

$$+ \int_{-\infty}^{\infty} \frac{\exp\left(if(x - 2h\theta) - \alpha\left(\sqrt{\frac{N}{L}}\, z + 2h\right)\right)}{\alpha} \, df +$$

(*equation continued on p.* 533)

$$+\int_{-\infty}^{\infty}\frac{\exp\left(if(x-2h\theta)-\alpha\left(2h-\sqrt{\frac{N}{L}}z\right)\right)}{a}\,df$$

$$\left.+\int_{-\infty}^{\infty}\frac{\exp\left(if(x-8h\theta)-\alpha\left(\sqrt{\frac{N}{L}}z+4h\right)\right)}{\alpha}\,df+\ldots\right],$$

which when expanded up to first powers of θ (assuming $|f\theta|$ to be small always) becomes

$$G_1(x,z/0,0)=\frac{2}{\pi}\int_{-\infty}^{\infty}\left[\frac{\exp\left(\alpha\left(\sqrt{\frac{N}{L}}\right)z\right)+\exp\left(-\alpha\left(\sqrt{\frac{N}{L}}z+2h\right)\right)}{1-e^{-2h\alpha}}\right.$$

$$-2ifh\theta\frac{e^{-2h\alpha}(1+e^{-2h\alpha})}{(1-e^{-2h\alpha})^3}\cdot\left\{\exp\left(\alpha\sqrt{\frac{N}{L}}z\right)\right.$$

$$\left.\left.+\exp\left(-\alpha\sqrt{\frac{N}{L}}z\right)\right\}\right]\frac{e^{ifx}}{\alpha}\,df. \qquad \ldots(17)$$

To calculate $G_1(x, z/x_s, 0)$ we substitute for $h$, $h-x_s\theta$ and for $x$, $x-x_s$ in eqn. (17), so that neglecting $\theta^2$ and higher powers we get

$G_1(x, z/x_s, 0) =$

$$\frac{2}{\pi}\int_{-\infty}^{\infty}\frac{\exp\left(if(x-x_s)\right)}{\alpha}\left[\frac{\exp\left(\alpha\,.\sqrt{\frac{N}{L}}z\right)+\exp\left(-\alpha\left(\sqrt{\frac{N}{L}}z+2h\right)\right)}{1-e^{-2h\alpha}}\right.$$

$$+2\alpha\,.\frac{x_s\theta}{(1-e^{-2h\alpha})^2}\cdot e^{-2h\alpha}\left\{\exp\left(\alpha\sqrt{\frac{N}{L}}z\right)+\exp\left(-\alpha\sqrt{\frac{N}{L}}z\right)\right\}$$

$$\left.-2ifh\theta\left\{\exp\left(\alpha\sqrt{\frac{N}{L}}z\right)+\exp\left(-\alpha\sqrt{\frac{N}{L}}z\right)\right\}\cdot\frac{e^{-2h\alpha}(1+e^{-2h\alpha})}{(1-e^{-2h\alpha})^3}\right]df.$$

But $x_s\theta$ is small compared to $h$ for all $x_s$ for which there is significant contribution to the value of the required Green's function. We therefore ignore the term containing $x_s\theta$ and thereby obtain

$G_1(x, z/x_s, 0) =$

$$\frac{2}{\pi}\int_{-\infty}^{\infty}\frac{e^{if(x-x_s)}}{\alpha}\left[\frac{\exp\left(\alpha\sqrt{\frac{N}{L}}\,z\right)+\exp\left(-\alpha\left(\sqrt{\frac{N}{L}}\,z+2h\right)\right)}{1-e^{-2h\alpha}}\right.$$

$$\left.-\,2ifh\theta\left\{\exp\left(\alpha\sqrt{\frac{N}{L}}\,z\right)+\exp\left(-\alpha\sqrt{\frac{N}{L}}\,z\right)\right\}\cdot\frac{e^{-2h\alpha}(1+e^{2h\alpha})}{(1-e^{-2h\alpha})^3}\right]df \qquad ...(18)$$

so that $G_1(x, z/x_s, 0)$ at a point $(x_1, 0)$ on the interface is

$$G_1(x_1, 0/x_s, 0)=\frac{2}{\pi}\int_{-\infty}^{\infty}\frac{\exp(if(x_1-x_s))}{\alpha}\left[\frac{1+e^{-2h\alpha}}{1-e^{-2h\alpha}}-4ifh\theta\times\frac{e^{-2h\alpha}(1+e^{-2h\alpha})}{(1-e^{-2h\alpha})^3}\right]df. \qquad ...(19)$$

Now, if a source be situated at a point $(x_s, 0)$ on the boundary

$$G_2(x, z/x_s, 0)=\frac{2}{\pi}\int_{-\infty}^{\infty}\frac{\exp(if(x-x_s)-\beta z)}{\beta}\,df$$

where $\beta^2 = f^2 - k_2^2$, from which $G_2(x, z/x_s, 0)$ at a point $(x_1, 0)$ on the interface is given by

$$G_2(x_1, 0/x_s, 0)=\frac{2}{\pi}\int_{-\infty}^{\infty}\frac{\exp(if(x_1-x_s))}{\beta}\,df. \qquad ...(20)$$

Substituting the values of $G_1(x_1, 0/x_s, 0)$, $G_2(x_1, 0/x_s, 0)$ and $G_2(x_1, 0/0, 0)$ in eqn. (15) we obtain

$$\frac{1}{4\pi}\int_{-\infty}^{\infty}\left[\int_{-\infty}^{\infty}\frac{\exp(if(x_1-x_s))}{\alpha}\left\{\frac{1+e^{-2h\alpha}}{1-e^{-2h\alpha}}-4ifh\theta\cdot\frac{e^{-2h\alpha}(1+e^{-2h\alpha})}{(1-e^{-2h\alpha})^3}\right\}df\right.$$

$$\left.+\frac{L}{\mu}\int_{-\infty}^{\infty}\frac{\exp(if(x_1-x_s))}{\beta}\,df\right]\frac{\partial G(x_s, 0/0, 0)}{\partial z}\,dx_s=\int_{-\infty}^{\infty}\frac{e^{ifx_1}}{\beta}\,df.$$

Then,

$$\frac{1}{4\pi}\int_{-\infty}^{\infty}e^{-ifx_s}\left[\frac{1}{\alpha}\left\{\frac{1+e^{2h\alpha}}{1-e^{-2h\alpha}}-4ifh\theta\,\frac{e^{-2h\alpha}(1+e^{-2h\alpha})}{(1-e^{-2h\alpha})^3}\right\}\right.$$

$$\left.+\frac{L}{\mu\beta}\right]\frac{\partial G(x_s, 0/0, 0)}{\partial z}\,dx_s=\frac{1}{\beta}\cdot$$

Or,

$$\frac{1}{4\pi}\int_{-\infty}^{\infty} e^{-ifx_s}\,.\,\frac{\partial G(x_s,\,0/0,\,0)}{\partial z}\;dx_s$$

$$=\frac{1}{\beta\left[\dfrac{L}{\mu\beta}+\dfrac{1+e^{-2h\alpha}}{\alpha(1-e^{-2h\alpha})}-4ifh\theta\,.\,\dfrac{e^{-2h\alpha}\,(1+e^{-2h\alpha})}{\alpha(1-e^{-2h\alpha})^3}\right]}.$$

Taking Fourier inverse transform we get,

$$\frac{\partial G(x_s,\,0/0,\,0)}{\partial z}=2\int_{-\infty}^{\infty}\frac{e^{ifx_s}\,df}{\beta\left[\dfrac{L}{\mu\beta}+\dfrac{1+e^{-2h\alpha}}{\alpha(1-e^{-2h\alpha})}-4ifh\theta\,.\,\dfrac{e^{-2h\alpha}\,(1+e^{-2h\alpha})}{\alpha(1-e^{-2h\alpha})^3}\right]}$$

$$=2\int_{-\infty}^{\infty}\frac{e^{ifx_s}}{\beta\left[\dfrac{L}{\mu\beta}+\dfrac{1+e^{-2h\alpha}}{\alpha(1-e^{-2h\alpha})}\right]}\,df$$

$$+\,2\int_{-\infty}^{\infty}4ifh\theta\,.\,\frac{e^{ifx_s}\,e^{-2h\alpha}\,(1+e^{-2h\alpha})}{\beta\alpha(1-e^{-2h\alpha})^3\left[\dfrac{L}{\mu\beta}+\dfrac{1+e^{-2h\alpha}}{\alpha(1-e^{-2h\alpha})^2}\right]^2}\,df$$

where $\theta^2$ and higher powers of $\theta$ have been neglected. Substituting the values of

$$G_1(x,\,z/x_s,\,0),\quad\frac{\partial G(x_s,\,0/0,\,0)}{\partial z}$$

from eqns. (18) and (21) in eqn. (16) we find

$G(x,\,z/0,\,0)=$

$$\frac{1}{\pi^2}\int_{-\infty}^{\infty}dx_s\left[\iint_{-\infty}^{\infty}\frac{\exp\left(\alpha\sqrt{\dfrac{N}{L}}\,z\right)+\exp\left(-\alpha\left(\sqrt{\dfrac{N}{L}}\,z+2h\right)\right)}{\alpha(1-e^{-2h\alpha})}\right.$$

$$\times\frac{e^{ifx}\,.\exp\left(i(f'-f)\,x_s\right)\,df\,df'}{\beta'\left[\dfrac{L}{\mu\beta'}+\dfrac{1-e^{-2h\alpha'}}{\alpha'(1-e^{-2h\alpha'})}\right]}+\iint_{-\infty}^{\infty}4if\,.\,h\theta\,.\,e^{ifx}\,.\,\exp\left(i(f'-f)\,x_s\right)$$

$$\times\frac{\exp\left(\alpha\sqrt{\dfrac{N}{L}}\,z\right)+\exp\left(-\alpha\left(\sqrt{\dfrac{N}{L}}\,z+2h\right)\right)}{\alpha(1-e^{-2h\alpha})}\times$$

(*equation continued on p.* 536)

$$\times \frac{e^{-2h\alpha'}(1+e^{-2h\alpha'})\,df\,df'}{\alpha'\beta'(1-e^{-2h\alpha'})^3\left[\frac{L}{\mu\beta'}+\frac{1+e^{-2h\alpha'}}{\alpha'(1-e^{-2h\alpha'})}\right]^2}$$

$$-\int_{-\infty}^{\infty}\!\!\int 2ifh\theta\,.\,e^{ifx}\,.\,\exp\,(i(f'-f)\,x_s)$$

$$\times\left\{\exp\left(\alpha\sqrt{\frac{N}{L}}z\right)+\exp\left(-\alpha\sqrt{\frac{N}{L}}\,.\,z\right)\right\}$$

$$\times\frac{e^{-2h\alpha}(1+e^{-2h\alpha})}{\alpha(1-e^{-2h\alpha})^3}\,.\,\frac{df\,df'}{\beta'\left[\frac{L}{\mu\beta'}+\frac{1+e^{-2h\alpha'}}{\alpha'(1-e^{-2h\alpha'})}\right]} \qquad \ldots(22)$$

where $\alpha'^2 = f'^2 - k_1^2$ and $\beta'^2 = f'^2 - k_2^2$.

We now put $f' - f = \eta$ so that $df' = d\eta$. Using the results

$$\delta(f'-f)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\exp(i(f'-f)\,x_s)\,dx_s$$

and

$$\int g(f')\,\delta(f'-f)\,df' = g(f)$$

eqn. (22) becomes

$G(x, z/0, 0) =$

$$\frac{2}{\pi}\int_{-\infty}^{\infty}\frac{\exp\left(\alpha\sqrt{\frac{N}{L}}z\right)+\exp\left(-\alpha\left(\sqrt{\frac{N}{L}}\,z+2h\right)\right)}{\alpha(1-e^{-2h\alpha})}$$

$$\times\frac{e^{ifx}\,df}{\beta\left[\frac{L}{\mu\beta}+\frac{1+e^{-2h\alpha}}{\alpha(1-e^{-2h\alpha})}\right]}$$

$$-\frac{2}{\pi}\int_{-\infty}^{\infty}2ifh\theta\,.\,\frac{\left\{\exp\left(\alpha\sqrt{\frac{N}{L}}z\right)+\exp\left(-\alpha\sqrt{\frac{N}{L}}z\right)\right\}e^{-2h\alpha}(1+e^{-2h\alpha})}{\alpha(1-e^{-2h\alpha})^3}$$

$$\times\frac{e^{ifx}\,df}{\mu\left[\frac{L}{\mu\beta}+\frac{1-e^{-2h\alpha}}{\alpha(1-e^{-2h\alpha})}\right]}+$$

*(equation continued on p. 537)*

$$+\frac{2}{\pi}\int_{-\infty}^{\infty} 4ifh\theta\,.\,\frac{\exp\left(\alpha\sqrt{\frac{N}{L}}z\right)+\exp\left(-\alpha\left(\sqrt{\frac{N}{L}}z+2h\right)\right)}{\alpha(1-e^{-2h\alpha})}$$

$$\times\frac{e^{-2h\alpha}\,(1+e^{-2h\alpha})\,e^{ifx}\,df}{\alpha\beta(1-e^{-2h\alpha})^3\left[\frac{L}{\mu\beta}+\frac{1+e^{-2h\alpha}}{\alpha(1-e^{-2h\alpha})}\right]^2} \qquad \ldots(23)$$

which when simplified gives the result

$G(x, z/0, 0) =$

$$\frac{2\mu}{\pi}\int_{-\infty}^{\infty} e^{ifx}.\frac{\exp\left(\alpha\left(\sqrt{\frac{N}{L}}z+h\right)\right)+\exp\left(-\alpha\left(\sqrt{\frac{N}{L}}z+h\right)\right)}{L\alpha(e^{h\alpha}-e^{-h\alpha})+\mu\beta(e^{h\alpha}+e^{-h\alpha})}\,df$$

$$+\frac{4i\mu h\theta}{\pi}\int_{-\infty}^{\infty} f\,e^{ifx}\,.\,\frac{e^{h\alpha}+e^{-h\alpha}}{e^{h\alpha}-e^{-h\alpha}}$$

$$\times\frac{\mu\beta\left\{\exp\left(\alpha\sqrt{\frac{N}{L}}z\right)-\exp\left(-\alpha\sqrt{\frac{N}{L}}z\right)\right\}-L\alpha\left\{\exp\left(\alpha\sqrt{\frac{N}{L}}z\right)+\exp\left(-\alpha\sqrt{\frac{N}{L}}z\right)\right\}}{[L\alpha(e^{h\alpha}-e^{-h\alpha})+\mu\beta\,(e^{h\alpha}+e^{-h\alpha})]^2}\,df. \qquad \ldots(24)$$

Writing $\alpha = i\alpha_1$ where $\alpha_1 = (k_1^2 - f^2)^{1/2}$ eqn. (24) becomes

$$G(x, z/0, 0) = \frac{2\mu}{\pi}\int_{-\infty}^{\infty} e^{ifx}\,.\,\frac{\cos\alpha_1\left(\sqrt{\frac{N}{L}}\,.\,z+h\right)}{\mu\beta\cos\alpha_1 h - L\alpha_1\sin\alpha_1 h}\,df$$

$$+\frac{2i\mu\theta h}{\pi}\int_{-\infty}^{\infty} f\,e^{ifx}\,.\,\frac{\cos\alpha_1 h\left(\mu\beta\sin\alpha_1\sqrt{\frac{N}{L}}z - L\alpha_1\cos\alpha_1\sqrt{\frac{N}{L}}z\right)}{\sin\alpha_1 h(\mu\beta\cos\alpha_1 h - L\alpha_1\sin\alpha_1 h)^2}\,df$$

...(25)

which is the expression for *SH* displacement at a point corresponding to a source in the lower medium but sufficiently near the origin. Integrals of this type have been evaluated by Sezawa (1935). In order to evaluate the integral we choose the contour as the real axis and infinite semi-circle in the upper half plane with necessary cuts at the branch points $f = k_1, k_2$. The solution can then be expressed as the sum of the residues of the integrands and two integrals along branch lines corresponding to the branch points $f = k_1$ and $f = k_2$. The branch line integrals are $O(x^{-3/2})$ and become negligible for large $x$. Therefore neglecting the contributions of the branch line integrals, we find for large values of $x$,

$$\frac{2\mu}{\pi}\int_{-\infty}^{\infty} e^{ifx}\frac{\cos\alpha_1\left(\sqrt{\frac{N}{L}}z+h\right)}{\mu\beta\cos\alpha_1 h - L\alpha_1\sin\alpha_1 h}\,df$$

$$= 4\mu i\sum_n e^{if_n x}\cdot\frac{\cos\alpha_{1n}\left(\sqrt{\frac{N}{L}}\cdot z+h\right)}{F'(f_n)} \qquad \ldots(26)$$

where

$$F(f) = \mu(f^2-k_2^2)^{1/2}\cos(\sqrt{k_1^2-f^2}\,.\,h) - L(k_1^2-f^2)^{1/2}\sin(\sqrt{k_1^2-f^2}\,.\,h) \qquad \ldots(27)$$

so that $f_n$ ($n = 1, 2, 3, \ldots$) are the roots of the equation

$$F(f_n) = \mu(f_n^2-k_n^2)^{1/2}\cos(\sqrt{k_1^2-f_n^2}\,.\,h) - L(k_1^2-f_n^2)^{1/2}\sin(\sqrt{k_1^2-f_n^2}\,.\,h) = 0$$

and

$$\alpha_{1n} = (k_1^2 - f_n^2)^{1/2}. \qquad \ldots(28)$$

Similarly, the second integral of (25)

$$= \frac{2i\mu\theta h}{\pi}\int_{-\infty}^{\infty} fe^{ifx}\cdot\frac{\cos\alpha_1 h\left(\mu\beta\sin\alpha_1\cdot\sqrt{\frac{N}{L}}z - L\alpha_1\cos\alpha_1\sqrt{\frac{N}{L}}z\right)}{\sin\alpha_1 h(\mu\beta\cos\alpha_1 h - L\alpha_1\sin\alpha_1 h)^2}\,df$$

$$= \frac{2i\mu\theta h}{\pi}.\,2\pi i\,.\,(\text{sum of the residues})$$

$$= -4\mu\theta h\sum_n \frac{\frac{d}{df}\left[fe^{ifx}\cot\alpha_1 h(\mu\beta\sin\alpha_1\cdot\sqrt{\frac{N}{L}}z - L\alpha_1\cos\alpha_1\cdot\sqrt{\frac{N}{L}}z)\right]\Big|_{f=f_n}}{F(f_n)^2}$$

$$+ 4\mu\theta h\sum_n \frac{f_n e^{if_n x}\,.\cot\alpha_{1n}h\left(\mu\beta_n\sin\alpha_{1n}\,.\sqrt{\frac{N}{L}}z - L\alpha_{1n}\cos\alpha_{1n}.\sqrt{\frac{N}{L}}z\right)}{[F'(f_n)]^3}$$

$$\times F''(f_n)$$

$$- 4\mu\theta h\sum_m \frac{\alpha_{1m}}{h\mu^2\beta_m^2}e^{if_m x}\left[\mu\beta_m\sin\left(\alpha_{1m}\sqrt{\frac{N}{L}}z\right) - L\alpha_{1m}\cos\left(\alpha_{1m}\sqrt{\frac{N}{L}}z\right)\right]$$

$$\ldots(29)$$

where $\alpha_{1m}$ are given by $\sin \alpha_{1m}h = 0$ giving $\alpha_{1m}h = m\pi$, $(m = 1, 2, 3, \ldots)$, and $f_m$, $\beta_m$ are the corresponding values of $f$ and $\beta$.

Since $\theta$ is small, for large values of $x$, the important contribution comes from the first summation of (29) in the form

$$-4i\mu\theta h \sum_n x f_n e^{if_n x} \cdot \frac{\cot\alpha_{1n}h\left(\mu\beta_n \sin\alpha_{1n}\sqrt{\frac{N}{L}}z - L\alpha_{1n}\cos\alpha_{1n}\sqrt{\frac{N}{L}}z\right)}{(F'(f_n))^2}$$

$$= 4i\mu\theta h \sum_n \frac{x f_n e^{if_n x}}{(F'(f_n))^2} \cdot \frac{L\alpha_{1n}}{\sin\alpha_{1n}h} \cdot \cos\alpha_{1n}\left(\sqrt{\frac{N}{L}}z + h\right)$$

using eqn. (28) in the form

$$\mu\beta_n \cos\alpha_{1n}h - L\alpha_{1n}\sin\alpha_{1n}h = 0.$$

Hence for large values of $x$,

$$G(x, z/0, 0) \simeq 4\mu i \sum_n e^{if_n x} \cdot \frac{\cos\left\{\alpha_{1n}\left(\sqrt{\frac{N}{L}}z + h\right)\right\}}{F'(f_n)}$$

$$+ 4i\mu\theta h \sum_n \frac{x f_n e^{if_n x}}{(F'(f_n))^2} \cdot \frac{L\alpha_{1n}}{\sin(\alpha_{1n}.h)} \cdot \cos\left\{\alpha_{1n}\left(\sqrt{\frac{N}{L}}z + h\right)\right\}$$

$$= 4\mu i \sum_n \frac{e^{if_n x}.\cos\left\{\alpha_{1n}\left(\sqrt{\frac{N}{L}}z + h\right)\right\}}{(F'(f_n))}\left(1 + \frac{L\alpha_{1n}x f_n\theta}{F'(f_n)\sin\alpha_{1n}h}\right). \quad \ldots(30)$$

The second term of (30) is due to the slope of the upper boundary of the layer, which is large when $\sin\alpha_{1n}h = 0$. This gives $\alpha_{1n}h = n\pi$, $(n = \pm 1, \pm 2, \ldots)$ from which we find

$$f_n = \frac{(k_1^2 h^2 - n^2\pi^2)^{1/2}}{h}. \quad \ldots(31)$$

Hence the displacement on the surface of the layer becomes infinite if (31) holds.

We observe that the modified dispersion equation is given by

$$\mu\beta_n \cos\alpha_{1n}h - L\alpha_{1n}\sin\alpha_{1n}h = 0.$$

Or,

$$\tan\alpha_{1n}h = \frac{\mu\beta_n}{L\alpha_{1n}}$$

which reduces to the standard form when $L = N = \mu_1$ (say), i.e., when the layer is isotropic

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