TWO DIAMETRAL CRACKS IN A POINT LOADED CIRCULAR DISC*

by RANJIT S. DHALIWAL, *Department of Mathematics and Statistics, The University of Calgary, Alberta, Canada T2N 1N4*

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In this paper the problem of determining the stress intensity factors and the crack energy of two symmetrical cracks on a diameter of a finite circular disc has been solved by the use of Mellin transform technique. Numerical results for these quantities have been tabulated.

1. INTRODUCTION

Recently Dhaliwal and Singh (1977) have considered the problem of determining the stress intensity factors and the strain energy for two symmetrical diametral cracks, in a finite circular disc, subjected to an arbitrary pressure, which is symmetric about the line of the crack. They considered in detail the plane strain case of a constant pressure. In this paper, we will obtain the stress intensity factors and the strain energy for two symmetrical diametral cracks when the disk has concentrated loads on the circumference as shown in Figs. 1(a) and 1(b). Similar problems with one radial crack have been considered by Tweed *et al.* (1972), Rooke and Tweed (1973) and Libatskii and Kovchick (1967).

2. BASIC RESULTS

Consider a circular disc of unit radius containing a pair of symmetrical cracks defined in plane polar coordinates (r, θ) by $\theta = 0, \pi, 0 < a \leqslant r \leqslant b < 1$ which are subjected to an internal pressure $p_0 f(r)$. The basic results for this problem have been obtained by Dhaliwal and Singh (1977). The stress intensity factors at $r = a, b$ denoted respectively by K_a, K_b are given by

$$K_a = p_0 \sqrt{\frac{l}{2}} \sum_{n=1}^{N} (-1)^n B_n \qquad \ldots(1)$$

$$K_b = p_0 \sqrt{\frac{l}{2}} \sum_{n=1}^{N} B_n \qquad \ldots(2)$$

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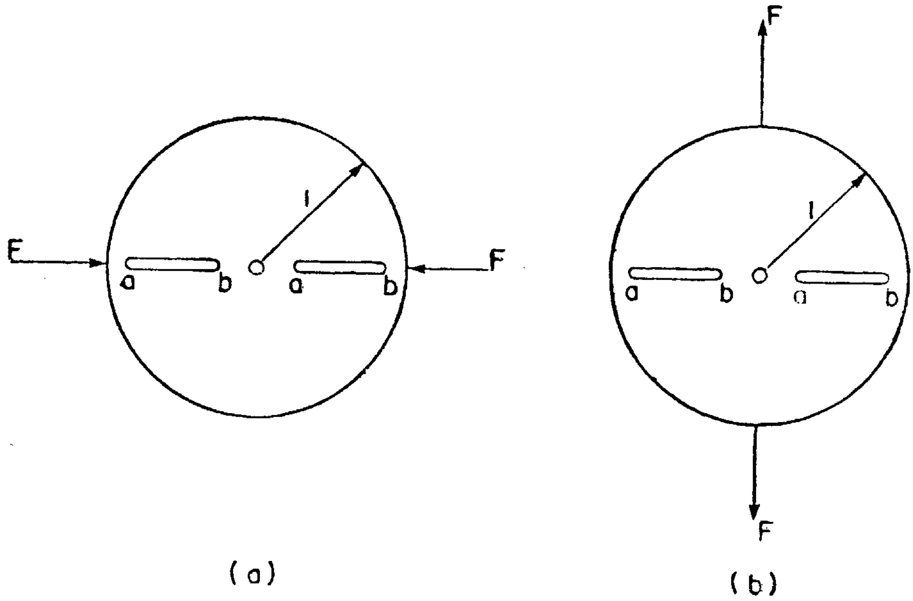

FIG. 1.

and the crack energy W and the shape of the crack $v(r, 0)$ are given by

$$W = p_0 \int_a^b f(r)\, v(r, 0)\, dr \qquad \ldots(3)$$

$$V(\rho) = v(r, 0)\left(\frac{p_0 l}{E_1}\right) = \sum_{n=1}^{N} \frac{B_n}{n} \sin(n \cos^{-1}\rho),\ a < r < b \qquad \ldots(4)$$

where

$$l = b - a,\ E_1 = \frac{E}{(1 - \nu^2)}; \qquad \ldots(5)$$

E and ν being respectively the Young's modulus and the Poisson's ratio of the elastic material.

The unknown B_n $(n = 1, 2, \ldots N)$ are to be determined from the linear algebraic equations

$$\frac{\pi^2}{2} B_{j+1} + \sum_{n=1}^{N} h_{nj} B_n = L_j,\ j = 0, 1, 2, \ldots, N - 1 \qquad \ldots(6)$$

where

$$h_{nj} = \int_{-1}^{1} g_n(t)\, U_j(t)\, (1 - t^2)^{-1/2}\, dt \qquad \text{...(7)}$$

$$L_j = \int_{-1}^{1} L(t)\, U_j(t)\, (1 - t^2)^{-1/2}\, dt \qquad \text{...(8)}$$

$$g_n(\rho) = \int_{-1}^{1} K(\rho, t)\, T_n(t)\, (1 - t^2)^{-1/2}\, dt \qquad \text{...(9)}$$

$$L(\rho) = -\pi f(r),\ K(\rho, \tau) = \frac{b - a}{2}\, k(r, t) \qquad \text{...(10)}$$

$$\begin{aligned} k(r, t) = {} & (t + r)^{-1} + 2(t + t^{-1}) + 2t^3\, [4(1 - r^2t^2)^{-3} \\ & - (1 - r^2t^2)^{-2} - (1 - r^2t^2)^{-1}] - 2t^{-1}\, [4(1 - r^2t^2)^{-3} \\ & - 3(1 - r^2t^2)^{-2}] \end{aligned} \qquad \text{...(11)}$$

$$t = \frac{b + a}{2} + \frac{b - a}{2}\tau,\ \ r = \frac{b + a}{2} + \frac{b - a}{2}\rho \qquad \text{...(12)}$$

and $T_n(t)$ and $U_n(t)$ are Chebyshev polynomials of the first and second kind, respectively.

3. Point Loaded Disc

We will consider two types of loading shown in Figs. 1(a) and 1(b).

Case (a) : Compressive Loading

For this type of loading the pressure on the crack is given by (Muskhelishvili 1953)

$$\sigma_\theta(r, \theta)\,|_{\theta=0,\pi} = -\, p_0 f(r) = -\, \frac{F}{\pi} \qquad \text{...(13)}$$

where F is the magnitude of the concentrated load. The solution of this problem of constant internal pressure on the crack has been obtained by Dhaliwal and Singh (1977)

Case (b): Tensile Loading

For this type of loading the pressure on the crack is given by (Muskhelishvili 1953)

$$\sigma_\theta(r, \theta)\,|_{\theta=0,\pi} = \frac{F}{\pi}\left[\frac{4}{(1 + r^2)^2} - 1\right] \qquad \text{...(14)}$$

TABLE I

Values of $10\ K_a/p_0$

$a \setminus b$	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	2.078	2.943	3.560	4.064	4.426	4.685	4.862	4.996
0.2		1.873	2.579	3.059	3.399	3.673	3.797	3.922
0.3			1.618	2.192	2.558	2.800	2.956	3.068
0.4				1.331	1.777	2.045	2.207	2.309
0.5					1.038	1.366	1.546	1.645
0.6						0.761	0.981	1.085
0.7							0.511	0.637
0.8								0.295

TABLE II

Values of $10\ K_b/p_0$

$a \setminus b$	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	1.989	2.612	2.880	2.917	2.812	2.655	2.560	2.722
0.2		1.753	2.202	2.333	2.281	2.145	2.035	2.150
0.3			1.478	1.784	1.829	1.726	1.617	1.688
0.4				1.184	1.373	1.348	1.255	1.284
0.5					0.897	0.993	0.939	0.933
0.6						0.633	0.660	0.636
0.7							0.402	0.395
0.8								0.208

TABLE III

Values of W/W_0

$a \setminus b$	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.1	4.017	3.703	3.315	2.894	2.477	2.088	1.737	1.407
0.2		3.226	2.805	2.384	1.989	1.635	1.329	1.055
0.3			2.356	1.948	1.581	1.264	0.999	0.774
0.4				1.556	1.225	0.948	0.724	0.543
0.5					0.921	0.686	0.502	0.360
0.6						0.478	0.331	0.223
0.7							0.205	0.126
0.8								0.062

TABLE IV

Values of 10 $lV(\rho)/p_0$

a	ρ \ b	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	−0.75	0.612	1.216	1.793	2.323	2.789	3.182	3.489	3.669
	−0.50	0.797	1.571	2.293	2.935	3.478	3.909	4.217	4.341
	−0.25	0.886	1.732	2.501	3.159	3.690	4.084	4.327	4.342
0.1	0.00	0.911	1.763	2.516	3.135	3.607	3.929	4.097	4.027
	0.25	0.877	1.682	2.369	2.910	3.296	3.535	3.640	3.548
	0.50	0.780	1.480	2.058	2.489	2.774	2.933	2.998	2.949
	0.75	0.592	1.112	1.524	1.814	1.990	2.082	2.144	2.254
	−0.75		0.550	1.061	1.523	1.928	2.269	2.539	2.712
	−0.50		0.715	1.365	1.937	2.419	2.804	3.084	3.219
	−0.25		0.793	1.498	2.099	2.584	2.945	3.179	3.233
0.2	0.00		0.812	1.518	2.098	2.542	2.849	3.022	3.010
	0.25		0.780	1.440	1.962	2.339	2.578	2.694	2.660
	0.50		0.691	1.262	1.692	1.983	2.149	2.223	2.214
	0.75		0.523	0.943	1.245	1.433	1.531	1.586	1.680
	−0.75			0.474	0.896	1.262	1.569	1.812	1.973
	−0.50			0.613	1.146	1.591	1.945	2.204	2.343
	−0.25			0.678	1.250	1.709	2.052	2.280	2.360
0.3	0.00			0.693	1.259	1.692	1.997	2.177	2.206
	0.25			0.663	1.187	1.569	1.817	1.948	1.956
	0.50			0.586	1.034	1.342	1.525	1.613	1.631
	0.75			0.442	0.768	0.979	1.093	1.149	1.229
	−0.75				0.388	0.722	0.999	1.217	1.366
	−0.50				0.501	0.917	1.246	1.488	1.627
	−0.25				0.552	0.994	1.325	1.549	1.648
0.4	0.00				0.562	0.994	1.300	1.488	1.549
	0.25				0.537	0.932	1.194	1.340	1.379
	0.50				0.473	0.806	1.011	1.115	1.153
	0.75				0.356	0.595	0.731	0.796	0.861

(Continued)

TABLE IV (*continued*)

a	$\rho \setminus b$	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
	−0.75					0.302	0.550	0.746	0.880
	−0.50					0.388	0.694	0.919	1.055
	−0.25					0.426	0.746	0.966	1.077
0.5	0.00					0.432	0.741	0.936	1.021
	0.25					0.411	0.689	0.851	0.914
	0.50					0.361	0.592	0.714	0.766
	0.75					0.271	0.433	0.512	0.566
	−0.75						0.220	0.392	0.511
	−0.50						0.282	0.489	0.618
	−0.25						0.308	0.520	0.639
0.6	0.00						0.311	0.512	0.611
	0.25						0.294	0.471	0.551
	0.50						0.257	0.400	0.463
	0.75						0.192	0.291	0.338
	−0.75							0.147	0.249
	−0.50							0.187	0.306
	−0.25							0.203	0.321
0.7	0.00							0.203	0.311
	0.25							0.191	0.283
	0.50							0.166	0.239
	0.75							0.123	0.174
	−0.75								0.084
	−0.50								0.105
	−0.25								0.112
0.8	0.00								0.111
	0.25								0.103
	0.50								0.088
	0.75								0.064

so that, we have

$$p_0 = \frac{F}{\pi}, \quad f(r) = -1 + \frac{4}{(1 + r^2)^2}. \qquad \ldots(15)$$

From (3), we find that

$$\frac{W}{W_0} = \int_{-1}^{1} F(\rho)\, V(\rho)\, d\rho \qquad ...(16)$$

where

$$F(\rho) \equiv f(r),\; V(\rho) = \frac{\pi E_1 v(r, 0)}{(Fl)},\; W_0 = \frac{F^2 l^2}{(\pi E_1)} \qquad ...(17)$$

For $f(r)$ given by (15), eqns. (6) have been solved for $N = 20$ by evaluating the numerical values of h_{nj} and L_j from (7) and (8). Then the numerical values of K_a and K_b have been obtained from eqns. (1) and (2) for all possible combinations of values of $a = 0.1, 0.2, ..., 0.8$ and $b = 0.2, 0.3, ..., 0.9$ $(b > a)$ and are given in Tables I and II. The numerical values of $V(\rho)$ for $\rho = -0.75\ (0.25)\ 0.75$, obtained by using eqn. (4), are contained in Table IV for all possible combination of values of a and b given above. It may be noticed that $V(\rho) \equiv 0$ for $\rho = -1, 1$ for all possible values of a, b $(b > a)$. The numerical values of W/W_0 are given in Table III and these have been calculated from eqn. (16) by using the numerical values of $V(\rho)$.

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