RELATIVISTIC MAGNETOFLUIDS AND SYMMETRY MAPPINGS—I

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This paper reflects the general behaviour of the relativistic magnetofluid in terms of the kinematical parameters associated with the streamlines and magnetic field lines. Further, we have explored the conditions on the magnetofluids involving the symmetry properties admitted by the magnetofluid space-times.

1. INTRODUCTION

The modern investigations in astronomy and astro-physics have stimulated interest in relativistic matter fields more general than the familiar perfect fluid models. Lichnerowicz (1967) initiated fundamental studies of more general hydrodynamical matter field solutions of Einstein's field equations and later extended his investigation to relativistic magnetohydrodynamics (RMHD). His RMHD field equations are used by Yodzis (1971), Banerji (1974) and Date (1976) to infer the magnetic effect in galactic cosmogony, gravitational collapse, and pulsar theory. Recently Esposito and Glass (1977) have shown that the fluid acceleration and the magnetic field are orthogonal when the magnetic field is 'frozen-in' and lies in the surfaces of constant pressure.

Glass (1975) investigated several aspects of the shear free perfect fluids. Some of the new results obtained by him led directly to the formulation of certain interesting local conservation expressions. Oliver and Davis (1976) generalized the conservation expression obtained by Glass (1975) employing symmetry methods and pointed out that certain symmetry methods are fundamental in this type of investigation. The groups of motions and Ricci collineations admitted by the magnetofluid space-times have been studied by Shaha (1974), Asgekar and Date (1977). Recently Prasad (1978) has made a comprehensive study in the area of local conservation laws involving the symmetry properties admitted by the electromagnetic fluid space-times.

The purpose of this paper is to explore the various consequences more general than the results obtained by Shaha (1974) and Asgekar and Date (1977) in the area of local conservation laws based upon the existence of symmetry mappings admitted by the relativistic magnetofluid space-times employing the methods demonstrated by Oliver and Davis (1976), Davis (1974), Davis *et al.* (1976).

2. Kinematical Parameters and Field Equations

(a) *Kinematical Parameters*

The metric of 4-dimensional Riemannian manifold V_4 is

$$ds^2 = g_{ij}\, dx^i\, dx^j, \ (i, j = 0, 1, 2, 3) \qquad ...(2.1)$$

where x^i are arbitrary coordinates and g_{ij} are the gravitational potentials. The signature of the metric is $(+, -, -, -)$.

The time-like curves are given by

$$x^i = x^i(\eta^\alpha, s), \ (\alpha = 1, 2, 3) \qquad ...(2.2)$$

where η^α are Lagrangian coordinates of the fluid elements and s is parameter along the world-line. The unit 4-velocity vector tangential to the world line is

$$u^i = \frac{dx^i}{ds} \ (\eta^\alpha \text{ fixed}) \qquad ...(2.3)$$

with $u^i u_i = +1$. The projection tensor is defined as

$$\gamma_{ij} = g_{ij} - u_i u_j. \qquad ...(2.4)$$

The covariant derivative of the 4-velocity u_i associated with the congruence of time-like curves can be decomposed following Ehlers (1962) as follows :

$$u_{i;j} = \sigma_{ij} + w_{ij} + \theta\gamma_{ij} + Du_i u_j \qquad ...(2.5)$$

where σ_{ij}, w_{ij}, θ and Du_i denote the shear tensor, the rotation tensor, the scalar expansion parameter and the fluid acceleration vector associated with the congruence of time-like curves and they are defined (Esposito and Glass 1977) as

$$\sigma_{ij} = \perp [u_{(i;j)} - \theta\gamma_{ij}] \qquad ...(2.6)$$

$$w_{ij} = \perp [u_{[i;j]}] \qquad ...(2.7)$$

$$3\theta = u^i_{;i} \qquad ...(2.8)$$

and

$$Du^i = u^i_{;j} u^j \qquad ...(2.9)$$

respectively. Here $\perp$ is the projection operator, semi-colon denotes the covariant differentiation and D stands for absolute differentiation along the stream lines.

The space-like curves are defined by

$$x^i = x^i(\overset{*}{\eta}{}^\alpha, s^*), \quad (\alpha = 1, 2, 3) \qquad ...(2.10)$$

where $\overset{*}{\eta}^{\gamma}$ take constant values for a particular space-like curve and s^* is a parameter along this curve. The unit 4-vector tangential to the space-like curve is given by

$$n^i = \frac{dx^i}{ds^*}, \; (\overset{*}{\eta}^{\alpha} \text{ fixed}) \qquad \ldots(2.11)$$

with $n^i n_i = -1$.

The covariant derivative of n_i is decomposed according to Greenberg (1971) as follows :

$$n_{i;j} = \overset{*}{\sigma}_{ij} + \overset{*}{w}_{ij} + \overset{**}{\theta}\gamma_{ij} - \overset{*}{D}n_i n_j + Dn_i u_j$$

$$- (Dn_k u^k)\; u_i u_j + (\overset{*}{D}n_k u^k)\; u_i u_j + n_{k;j} u^k u_i \qquad \ldots(2.12)$$

where $\overset{*}{\sigma}_{ij}$, $\overset{*}{w}_{ij}$ and $\overset{*}{\theta}$ denote the shear, rotation and expansion associated with the congruence of space-like curves respectively. They are defined by

$$\overset{*}{\sigma}_{ij} = \overset{*}{\gamma}{}_i^k \overset{*}{\gamma}{}_j^l \; n_{(k;l)} - \overset{**}{\theta}\gamma_{ij} \qquad \ldots(2.13)$$

$$\overset{*}{w}_{ij} = \overset{*}{\gamma}{}_i^k \overset{*}{\gamma}{}_j^l \; n_{[k;l]} \qquad \ldots(2.14)$$

and

$$\overset{*}{\theta} = \tfrac{1}{2}(n^i_{\;;i} - n_{j;k} u^j u^k) \qquad \ldots(2.15)$$

where $\overset{*}{\gamma}_{ij}$ is the projection tensor defined as

$$\overset{*}{\gamma}_{ij} = g_{ij} - u_i u_j + n_i n_j. \qquad \ldots(2.16)$$

$\overset{*}{D}n^i = n^i_{\;;j} n^j$ can be interpreted as the normal curvature vector of the congruence of space-like curves. D^* denotes absolute differentiation along this curve.

We can apply the above mentioned theory of the kinematical parameters associated with the congruence of space-like curves for the magnetic field lines (magnetic field tubes) following the concluding remarks made by Greenberg (1971) and interpret the kinematical parameters $\overset{*}{\sigma}_{ij}$, $\overset{*}{w}_{ij}$, $\overset{*}{\theta}$ as the shear, rotation and expansion of the magnetic field tubes respectively.

(b) *The Field Equations*

The field equations for the relativistic magnetofluid are

$$R_{ij} - \tfrac{1}{2}Rg_{ij} = kT_{ij} \qquad \ldots(2.17)$$

where k is a gravitational constant, R_{ij} is the usual Ricci tensor and T_{ij} is the stress-energy-momentum tensor defined by (Date 1976)

$$T_{ij} = (\overset{*}{\rho} + p^*)\, u_i u_j - p^* g_{ij} + \nu\sigma_{ij} + q_i u_j + q_j u_i - \mu h_i h_j \qquad \text{...(2.18)}$$

which characterizes the thermally conducting, viscous, compressible fluid with infinite electrical conductivity and constant magnetic permeability. $\overset{*}{\rho}$, p^*, ν ($\geqslant 0$), q_i and h_i are the total energy density of the magnetofluid, the total pressure (hydrodynamic + magnetic pressure), the coefficient of shear viscosity, the heat energy-flux vector and the magnetic field vector respectively. $\overset{*}{\rho}$ and p^* are defined by the expression

$$\overset{*}{\rho} = \rho + \tfrac{1}{2}\mu \mid h \mid^2 ; p^* = p + \tfrac{1}{2}\mu \mid h \mid^2 \qquad \text{...(2.19)}$$

where ρ is the matter energy density of the fluid and μ is the magnetic permeability. The matter energy density ρ is connected with the proper matter density ρ_0 and the internal energy density i by (Lichnerowicz 1967)

$$\rho = \rho_0(1 + i) \qquad \text{...(2.20)}$$

The relations connecting the thermodynamical variables are (Lichnerowicz 1967, Eckart 1940)

$$TDS = Di + pD\left(\frac{1}{\rho_0}\right) \qquad \text{...(2.21)}$$

$$S^i = \rho_0 S u^i + \frac{q^i}{T} \qquad \text{...(2.22)}$$

$$q^i = K(T_{,j} - TDu_j)\,\gamma^{ij} \qquad \text{...(2.23)}$$

$$X = 1 + i + \frac{p}{\rho_0} \qquad \text{...(2.24)}$$

where T is the rest temperature, S is the specific entropy, S^i is the entropy-flux vector, K is the heat conduction coefficient and X is the fluid index. Units are so chosen that the velocity of light is unity.

The Maxwell field equations are (Lichnerowicz 1967)

$$(u^i B^j - u^j B^i)_{;j} = 0 \qquad \text{...(2.25)}$$

where B^i is the magnetic induction vector.

3. General Relations and Theorems

In this section we shall study the various consequences of the relativistic magnetofluid in the light of kinematical parameters associated with the congruences of time-like and space-like curves. We shall begin our discussion from the Maxwell field equations. Equation (2.25) with the help of (2.8) yields

$$DB^i + 3\theta B^i - u^i_{;j}B^j - u^i B^j_{;j} = 0. \quad ...(3.1)$$

Transvecting (3.1) with u_i, putting $B^i = | B | n^i$, and using (2.15) we get

$$D^* \ln | B | A^* = 0 \quad ...(3.2)$$

where $| B |$ is the magnitude of the magnetic induction vector and A^* is the proper area subtended by the magnetic field lines as they pass through the screen is the 2-surface dual to the surface formed by u_i and B_i. A^* is defined by (Greenberg 1971)

$$2\overset{*}{\theta} = D^* \ln A^* \quad ...(3.3)$$

where D^* stands for the absolute differentiation along the magnetic field lines.

Equation (3.2) shows that $| B | A^*$ is constant along the magnetic field tube. This statement is similar to the Kelvin-Helmholtz theorem of Newtonian theory for the vorticity of the fluid in hydrodynamics. Banerji (1974) has also obtained this result by another method. Thus magnetic fields are always 'frozen-in' when the magnetic field is of infinite conductivity.

Theorem 3.1 — The fluid acceleration and the magnetic field are orthogonal iff the relativistic magnetofluid is free from the expansion, rotation and shear, and the magnetic field tubes lie in the surfaces of constant pressure.

PROOF : The equations of the streamlines $\gamma^k_i \, T^{ij}_{;j} = 0$ yields

$$(\overset{*}{\rho} + p^*) Du_k - p^*_{,j} \gamma^j_k - \{\mu h_{k;j}h^j + (\sigma_{ij} + \theta\gamma_{ij}) h^i h^j u_k$$

$$+ \mu h^j_{;j} h_k\} + \nu\sigma^j_{k;j} + 2\nu\sigma^2 u_k + Dq_k + u_k q_j Du^j$$

$$+ 4\theta q_k + (\sigma_{jk} + w_{jk}) q^j = 0. \quad ...(3.4)$$

Substituting the resulting equation obtained by the contraction of (3.1) with u_i in the equation obtained by the contraction of (3.4) with h^k, we get

$$(\rho + p) h^k Du_k - p_{,j}h^j + \nu\sigma^j_{k;j} h^k + h^k Dq_k + 4\theta h^k q_k$$

$$+ (\sigma_{jk} + w_{jk}) q^j h^k = 0 \quad ...(3.5)$$

which is a generalization of the result obtained by Esposito and Glass (1977).

Now using (2.17) and (2.18) in the relation due to Ehlers (1962)

$$\gamma^{ij} R_{jk} u^k = \gamma^i_j \, (w^{jk}_{;k} - \sigma^{jk}_{;k} + \tfrac{2}{3}\theta^{,j}) + (w^i_k + \sigma^i_k) Du^k \quad ...(3.6)$$

we get

$$q^i = \gamma^i_j \,(w^{jk}_{;k} - \sigma^{jk}_{;k} + \tfrac{2}{3}\theta^{,j}) + (w^i_k + \sigma^i_k)\, Du^k \qquad ...(3.7)$$

which shows that the condition for the relativistic magnetofluid to be perfect is that $q^i = 0 \Rightarrow \theta = \sigma_{ij} = w_{ij} = 0$.

In view of (3.5) and (3.7) we conclude the statement.

Corollary 3.1 — The magnetic induction is divergence free iff the relativistic magnetofluid is free from the expansion, rotation and shear, and the magnetic field tubes lie in the surfaces of constant pressure.

PROOF : Contracting (3.1) with u_i, we get

$$B^j_{;j} + B^i Du_i = 0. \qquad ...(3.8)$$

Using Theorem 3.1 in (3.8) we prove the statement.

Contracting (3.4) with w_k and assuming $h_k w^k = 0$, we obtain

$$(\overset{*}{\rho} + p^*)\, w^k Du_k - p^*_{,j} w^j + (\overset{\vee}{\sigma}_{jk} + \overset{\vee}{\theta}\, \overset{\vee}{\gamma}_{jk})\, B^j h^k \mid w \mid$$

$$+ \nu \sigma^j_{k;j} w^k + w^k Dq_k + 4\theta w^k q_k + \sigma_{jk} q^j w^k = 0 \qquad ...(3.9)$$

where $\overset{\vee}{\sigma}_{jk}$, $\overset{\vee}{\theta}$ and $\overset{\vee}{\gamma}_{jk}$ are the shear, expansion, and the projection tensor associated with the vortex tubes.

The Ricci identity for the space-like vector n^i is

$$n^i_{;ij} - n^i_{;ji} = R_{ij} n^i. \qquad ...(3.10)$$

By means of (2.12) and (3.10), we have

$$2D^*\theta^* + D^*(Dn_k u^k) - 2\overset{*}{w}{}^2 + 2\overset{*}{\sigma}{}^2 + 2\overset{*}{\theta}{}^2 - 3\theta^2$$

$$- (D^* n^i)_{;i} - 2w_{ji} n^j Dn^i + D(D^* n_k u^k) + 3\theta(D^* n_k u^k)$$

$$+ n^k n^j Du_{k;j} + 3\theta \sigma_{kj} n^k n^j = R_{ij} n^i n^j \qquad ...(3.11)$$

Equations (2.17), (2.18) and (3.11) lead to

$$\tfrac{1}{2}(\rho - p - \mu \mid h \mid^2) = 2D^*\overset{*}{\theta} + D^*(Dn_k u^k) - 2w^{*2}$$

$$+ 2\overset{*}{\sigma}{}^2 + 2\overset{*}{\theta}{}^2 - 3\theta^2 - \nu\sigma_{ij} n^i n^j - (D^* n^i)_{;i} - 2w_{ji} n^j Dn^i$$

$$+ D(D^* n_k u^k) + 3\theta(D^* n_k u^k) + n^k n^j Du_{k;j} + 3\theta\sigma_{ij} n^i n^j \qquad ...(3.12)$$

where $h^i = \mid h \mid n^i$. (3.12) is a space-like counterpart of Raychaudhuri equation (1955) for the relativistic magnetofluids.

Again, using the Ricci identity

$$n_{i;jk} = n_{i;kj} + R_{lijk}n^l \qquad ...(3.13)$$

in (2.12), we get

$$D^*n_{i;j} = (D^*n^i)_{;j} - A_{ik}A^k_{\cdot j} + A_{ik}D^*n^kn_j$$

$$- A_{ik}Dn^ku_j + Dn_iu_{l;j}n^l + u_{l;k}n^l A^k_{\cdot j}u_i$$

$$- B_{lk}n^lD^*n^ku_in_j + B_{lk}n^lDn^ku_iu_j + R_{lijk}n^ln^k \qquad ...(3.14)$$

where

$$A_{ik} = \overset{*}{\sigma}_{ik} + \overset{*}{w}_{ik} + \overset{**}{\theta}\gamma_{ik} \qquad ...(3.15)$$

and

$$B_{lk} = \sigma_{lk} + w_{lk} + \theta\gamma_{lk}. \qquad ...(3.16)$$

Equation (3.14) is the propagation equation for the gradient of the space-like vector. This equation reveals that the change in the magnetic flux produces the deformation in the magnetic field tubes as well as in the fluid also. The magnetic flux vector will be defined by the expression $D^*n^i = n^i_{;j}n^j$.

4. Magnetofluid and Symmetry Mappings

In this section we deal with the magnetofluid space-times admitting certain symmetry properties and the related conservation expressions. In particular, we look at the symmetry properties in terms of $\underset{\xi}{\mathcal{L}} R_{ij} = 0$ for the time-like and space-like symmetry vectors ξ^i and explore some of the conditions that these Ricci collineations (RC) impose on the magnetofluid.

By virtue of (2.17) and (2.18), we have

$$R_{ij} = k\,[\mu_0u_iu_j - \mu_1\gamma_{ij} + \nu\sigma_{ij} - \mu h_ih_j + q_iu_j + q_ju_i] \qquad ...(4.1)$$

where

$$2\mu_0 = \rho + 3p + \mu\,|\,h\,|^2,\; 2\mu_1 = \rho - p + \mu\,|\,h\,|^2.$$

Let us consider time-like symmetry mapping vector of the form $\xi^i = \varphi u^i$, i.e. symmetry mapping in the direction of the fluid flow and observe the following result.

Theorem 4.1 — For the magnetofluid space-time admitting the symmetry property $\underset{\xi}{\mathcal{L}} R_{ij} = 0$ for $\xi^i = \varphi u^i$, the active gravitational density is constant along the flow lines when the magnetofluid of constant magnetic permeability is in 'steady state with rigid rotation' and the entropy is constant.

PROOF : It can be shown from (4.1) for the magnetofluid of constant magnetic permeability that $g^{ij} \underset{\xi}{\mathcal{L}} R_{ij} = 0$ is equivalent to

$$\tfrac{1}{2}\,\mu_{0,k}\xi^k - \mu h^i \underset{\xi}{\mathcal{L}} h_i + \mu_0 D\varphi + \varphi_{,i}q^i + \varphi q^i Du_i + \tfrac{1}{2}\nu\sigma^{ij} \underset{\xi}{\mathcal{L}} \sigma_{ij} = 0 \quad \text{...(4.2)}$$

The conservation law generator (Davis *et al.* 1976) $(R^i_j\, \xi_i)_{;i} = 0$ yields

$$(\mu_0\xi^i)_{;i} + \varphi_{,i}q^i + \varphi q^i_{;i} = 0. \quad \text{...(4.3)}$$

By virtue of (4.2), (4.3), the heat transfer equation (Date 1976)

$$T\rho_0 DS = 2\nu\sigma^2 + q_i Du^i - q^i_{;i} \quad \text{...(4.4a)}$$

where

$$(\rho_0 u^i)_{;i} = 0$$

and the equation due to Esposito and Glass (1977)

$$D(\tfrac{1}{2}\mu \mid h \mid^2) + 2\theta\mu \mid h \mid^2 + \mu\sigma_{ij}h^i h^j = 0 \quad \text{...(4.4b)}$$

we get

$$D\mu_0 + (6\mu_0 + 2\mu \mid h \mid^2)\,\theta + 4\mu\sigma_{ki}h^k h^i + 4\nu\sigma^2 - 2\rho_0 TDS - \nu g^{ij} \underset{\xi}{\mathcal{L}} \sigma_{ij} = 0 \quad \text{...(4.5)}$$

which proves the statement.

Now, we investigate few symmetry properties for space-like symmetry vectors imposing some conditions on the magnetofluid.

Theorem 4.2 — For a magnetofluid space-time with $\sigma_{ij}w^i w^j = 0$ admitting the symmetry property $\underset{\xi}{\mathcal{L}} R_{ij} = 0$ for $\xi^i = \varphi w^i$, the conservation law $(\alpha\mu_0 w^k)_{;k} = 0$, where $\alpha_{,k}w^k = 0$ holds iff

(i) $q^i = \lambda w^i$, $\lambda \neq 0$ and

(ii) $Dw^2 = 2\left\{D\ln\varphi - \dfrac{11\theta}{3}\right\}$

PROOF : It can be shown that $u^i u^j \underset{\xi}{\mathcal{L}} R_{ij} = 0$ is equivalent to

$$\mu_{0,k}\,\xi^k - 2\mu_0\,\xi^k Du_k + 2(q_k\,\xi^k_{;i}\,u^i - u_{i;k}q^i\xi^k) = 0. \quad \text{...(4.6)}$$

Using $\xi^k = \phi w^k$ and the identities $w^k_{;k} + 2w^k Du_k = 0$,

$$Dw^k = -w^j Du_j u^k + \tfrac{1}{2}\eta^{kijm}u_i Du_{j;m} - 2\theta w^k + \sigma^k_m w^m, \text{ and}$$

$Dw^2 = -\frac{4}{3}\theta w^2 - 2\sigma_{ij}w^iw^j + w^{ij}Du_{i;j}$, we observe that the last term of (4.6) vanishes iff $q^i = \lambda w^i$, $\lambda \neq 0$ and $Dw^2 = 2\left\{D\ln\phi - \frac{11\theta}{3}\right\}$. Consequently (4.6) yields

$$(\mu_0 w^k)_{;k} = 0 \qquad \ldots(4.7)$$

which is equivalent to

$$(\alpha\mu_0 w^k)_{;k} = 0, \qquad \alpha_{,k}w^k = 0 \qquad \ldots(4.8)$$

where α is a non-zero arbitrary function.

To interpret this theorem we use the relation (Greenberg 1971)

$$\frac{1}{A}\check{D}A = -\frac{1}{|w|}\check{D}|w| + \frac{w^kDu_k}{|w|} \qquad \ldots(4.9)$$

where $\check{D}$ denotes absolute differentiation along the vortex tube and A is the proper area subtended by the vortex lines as they pass through the screen is the 2-surface dual to the surface formed by u_i and w_i.

Combining (4.7), (4.9) and the identity $w^k_{;k} + 2w^kDu_k = 0$, we get

$$\check{D}(\mu_0^{1/2} A\,|w|) = 0 \qquad \ldots(4.10)$$

which shows that $(\rho + 3p + \mu|h|^2)^{1/2} A\,|w|$ is constant along the vortex tube. This result may be regarded as a generalization of the Kelvin-Helmholtz theorem in case of the relativistic magnetofluid. If the magnetic field vanishes, we get the result obtained by Oliver and Davis (1976).

Theorem 4.3 — For a magnetofluid space-time admitting the symmetry property $\underset{\xi}{\mathcal{L}} R_{ij} = 0$, for $\xi^i = \varphi B^i$, the conservation law $(\mu_0^{1/2}B^k)_{;k} = 0$ holds iff the magnetofluid is expansion free.

PROOF : The symmetry condition $u^iu^j \underset{\xi}{\mathcal{L}} R_{ij} = 0$ is equivalent to

$$\mu_{0,k}B^k - 2\mu_0 B^kDu_k + 2\,[q_kDB^k - u_{i;k}q^iB^k] = 0. \qquad \ldots(4.11)$$

Contracting (3.1) with q_i, we get

$$q_iDB^i + 3\theta q_iB^i - u_{i;j}q^iB^j = 0. \qquad \ldots(4.12)$$

By virtue of (4.11) and (4.12), we have

$$\mu_{0,k}B^k - 2\mu_0 B^kDu_k - 6\theta q_iB^i = 0, \quad q_iB^i \neq 0. \qquad \ldots(4.13)$$

Using the assumption that the magnetofluid is expansion free and (3.9), we obtain

$$(\mu_0^{1/2} B^k)_{;k} = 0 \qquad \ldots(4.14)$$

which proves the statement.

Further development is given in another paper.

References

Asgekar, G. G., and Date, T. H. (1977). Collineations and motions in self-gravitating magnetofluids. *J. Math. Phys*,, **18**, 738.

Banerji, S. (1974). The perfectly conducting medium in general relativity. *Nuovo Cim.*, **23B**, 345.

Date, T. H. (1976). On the relativistic magnetofluids. *Ann. Inst. Henri Poincare*, **24**, 417.

Davis, W. R. (1974). Conservation laws in Einstein's general theory of relativity, in *Lanczos Festschrift*, edited by B. K. P. Scaife (London and N.Y.), p. 29 ; Symmetry properties and conservation laws in relativistic continuum mechanics, in *Proc. Symp. on Symmetry, Similarity, and Group Theoretic Methods in Mechanics*, edited by P. G. Glockner and M. C. Singh (Calgary, Alta), p. 119.

Davis, W. R., Green, L. H., and Norris, L. K. (1976). Relativistic matter fields admitting Ricci collineations and related conservation laws. *Nuovo Cim.*, **34B**, 256.

Eckart, C. (1940). The hydrodynamics of irreversible process II, Relativistic theory of simple fluid. *Phys. Rev.*, **58**, 919.

Ehlers, J. (1962). Recent Development in General Relativity. Pergamon Press, London.

Esposito, F. P., and Glass, E. N. (1977). General relativistic magnetofluid. *J. Math. Phys.*, **18**, 705.

Glass, E. N. (1975). Weyl tensor and shear-free perfect fluids. *J. Math. Phys.*, **16**, 2361.

Greenberg, P. J. (1971). The general theory of space-like congruences with an application to vorticity in relativistic hydrodynamics. *J. Math. Analysis Applic.*, **30**, 128.

Lichnerowicz, A. (1967). Relativistic Hydrodynamics and Magnetohydrodynamics. Benjamin, New York.

Oliver, D. R., and Davis, W. R. (1976). Perfect fluids and symmetry mappings leading to conservation laws. *J. Math. Phys.*, **17**, 1790.

Prasad, G. (1978). On the geometry of relativistic electromagnetic fluid flows with geometrical symmetries. *Indian J. pure appl. Math.*, **9**, 457.

Raychaudhuri, A. K. (1955). Relativistic cosmology—I. *Phys. Rev.*, **98**, 1123.

Shaha, R. R. (1974). *Ann. Inst. Henri Poincare*, **20**, 189.

Yodzis, P. (1971). Some general relations in relativistic magnetohydrodynamics. *Phys. Rev.*, D3, 2941.