RELATIVISTIC MAGNETOFLUIDS AND SYMMETRY MAPPINGS—II

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In this paper, it has been shown that the Greenberg's law of transport for the fluid velocity along the space-like congruence generated by the magnetic field lines can be deduced from the source-free part of Maxwell field equations. The heat-flux tube and the magnetic field tube are orthogonal when an imperfect magnetofluid is in 'steady state with rigid rotation'. Further, the FCRC quasi-symmetry properties admitted by the imperfect shear free magnetofluid space-times and the related conservation laws have been studied.

1. INTRODUCTION

The general relativistic treatment of a perfectly conducting fluid in a strong magnetic field is engaging more and more attention after the identification of pulsare as a rotating neutron star with a very high density. Banerji (1974) has predicted that the increase of magnetic intensity to a high value is responsible for the birth of a neutron star out of the collapsing star using the relativistic magnetohydrodynamics (RMHD) field equations given by Lichnerowicz (1967) and studied several consequences of astrophysical importance employing the conditions for the 'steady state with rigid rotation' formulated by Yodzis (1971). Several consequences of RMHD field equations have been deduced by Date (1976) constructing the stress-energy momentum tensor for thermally conducting, viscous, compressible fluid with infinite electrical conductivity and constant magnetic permeability. Author has also studied the RMHD field equations involving the kinematical parameters associated with the fluid flow lines, the electric field lines and the magnetic field lines (*see* Prasad and Ojha 1977, Prasad and Sinha 1978, Prasad 1978a).

The purpose of this paper is to investigate the compatibility of the theory of space-like congruence developed by Greenberg (1970) and is to point out the conservation expressions involving the symmetry methods developed by Davis *et al.* (1976), Oliver and Davis (1976), Norris *et al.* (1977) and Davis (1977).

2. KINEMATICAL PARAMETERS

Let x^i $(i = 0, 1, 2, 3)$ be an arbitrary coordinate system in a region of space-time of signature $(+, -, -, -)$ and the metric tensor g_{ij}. Then the covariant derivative of the unit 4-velocity vector u^i tangential to the world line is decomposed following Ehlers (1962)

$$u_{i;j} = \sigma_{ij} + w_{ij} + \theta\gamma_{ij} + Du_i u_j \qquad ...(2.1)$$

where the scalar $\theta = \frac{1}{3} u^i_{;i}$ is the expansion, $Du^i = u^i_{;j}\, u^j$ is the fluid acceleration vector, $\sigma_{ij} = u_{(i;j)} - Du_{(i}u_{j)} - \theta\gamma_{ij}$ is the shear tensor, $w_{ij} = u_{[i;j]} - Du_{[i}u_{j]}$ is the rotation tensor, $\gamma_{ij} = g_{ij} - u_i u_j$ is the projection tensor, the round bracket denotes symmetrization, the square bracket denotes skew-symmetrization and semi-colon stands for the covariant differentiation.

The space-like curves are defined by Greenberg (1970)

$$x^i = x^i(\overset{*}{\eta}{}^\alpha, s^*), \ (\alpha = 1, 2, 3) \qquad ...(2.2)$$

where $\overset{*}{\eta}{}^\alpha$ take the constant values for a particular curve and s^* is the parameter along this curve. The unit 4-vector tangential to this curve is given by

$$n^i = \frac{dx^i}{ds^*}, \ (\overset{*}{\eta}{}^\alpha \text{ fixed}) \qquad ...(2.3)$$

with $\quad n^i n_i = -1.$

The covariant derivative of the unit 4-vector n^i is decomposed as follows :

$$n_{i;j} = \overset{*}{\sigma}_{ij} + \overset{*}{w}_{ij} + \overset{*}{\theta}\overset{*}{\gamma}_{ij} - D^*n_i n_j + Dn_i u_j$$
$$- (Dn_k u^k)\, u_i u_j + (D^*n_k u^k)\, u_i n_j + n_{k;j}\, u^k u_i \qquad ...(2.4)$$

where $\overset{*}{\sigma}_{ij}$, $\overset{*}{w}_{ij}$ and $\overset{*}{\theta}$ denote the shear, rotation and expansion associated with the congruence of space-like curves respectively and are defined by (Greenberg 1970)

$$\overset{*}{\sigma}_{ij} = \overset{*}{\gamma}{}^k_i\, \overset{*}{\gamma}{}^l_j\, n_{(k;l)} - \overset{*}{\theta}\,\overset{*}{\gamma}_{ij} \qquad ...(2.5)$$

$$\overset{*}{w}_{ij} = \overset{*}{\gamma}{}^k_i\, \overset{*}{\gamma}{}^l_j\, n_{[k;l]} \qquad ...(2.6)$$

and

$$\overset{*}{\theta} = \tfrac{1}{2}\,(n^i_{;i} - n_{j;k}u^j u^k) \qquad ...(2.7)$$

where the projection tensor $\overset{*}{\gamma}_{ij}$ is defined as

$$\overset{*}{\gamma}_{ij} = g_{ij} - u_i u_j + n_i n_j. \qquad ...(2.8)$$

The law of transport for the velocity vector along the space-like curve is

$$D^*u_k = Dn_k - (u^i Dn_i)\, u_k + (u^i D^*n_i)\, n_k \qquad ...(2.9)$$

where D^* denotes absolute differentiation along the magnetic field lines.

We can apply the above mentioned theory of the kinematical parameters associated with the congruence of space-like curves for the magnetic field lines (magnetic field tubes), the heat flux-tubes and the vortex tubes having some qualitative difference following the concluding remarks made by Greenberg (1970). We interpret the kinematical parameters $\overset{*}{\sigma}_{ij}$, $\overset{*}{w}_{ij}$, $\overset{*}{\theta}$ as the shear, rotation and expansion of the magnetic field tubes respectively.

3. Some Consequences of RMHD Field Equations

The field equations for the relativistic magnetofluid are (Lichnerowicz 1967)

$$R_{ij} - \tfrac{1}{2} R g_{ij} = k T_{ij} \qquad \ldots(3.1)$$

where k is a gravitational constant, R_{ij} is the usual Ricci tensor and T_{ij} is the stress-energy-momentum tensor defined by (Date 1976)

$$T_{ij} = (\overset{*}{\rho} + p^*)\, u_i u_j - p^* g_{ij} + \nu \sigma_{ij} + q_i u_j + q_j u_i - \mu h_i h_j \qquad \ldots(3.2)$$

which characterizes the thermally conducting, viscous, compressible fluid with infinite electrical conductivity and constant magnetic permeability. $\overset{*}{\rho}$, p^*, $\nu\ (\geqslant 0)$, q_i and h_i are the total energy density of the magnetofluid, the total pressure, the coefficient of shear viscosity, the heat energy-flux vector and the magnetic field vector respectively. ρ^* and p^* are defined by the expressions

$$\overset{*}{\rho} = \rho + \tfrac{1}{2}\mu \mid h \mid^2, \quad p^* = p + \tfrac{1}{2}\mu \mid h \mid^2 \qquad \ldots(3.3)$$

where ρ is the matter energy density of the fluid and μ is the magnetic permeability.

The heat flux-vector is (Eckart 1940)

$$q^i = K(T_{,j} - T D u_j)\, \gamma^{ij} \qquad \ldots(3.4)$$

where T is the rest temperature of the fluid and K is the heat conduction coefficient.

The Maxwell field equations are (Lichnerowicz 1967)

$$(u^i B^j - u^j B^i)_{;j} = 0 \qquad \ldots(3.5)$$

where B^i is the usual magnetic induction vector.

Transvecting (3.5) with $\overset{*}{\gamma}_{ik}$, we get

$$D^* u_k = D n_k - (u^i D n_i)\, u_k + (u^i D^* n_i) n_k \qquad \ldots(3.6)$$

where $B^i = \mid B \mid n^i$, $\mid B \mid > 0$. Equation (3.6) is exactly the Greenberg's law of transport for the fluid velocity vector along the space-like congruence generated by the magnetic field vector n^i.

The conservation equations $T^{ij}_{;j} = 0$ yield the local "energy-balance" equations (Asgekar and Date 1977) in the form

$$D(\overset{*}{\rho} + p^*)\, u^i + (\overset{*}{\rho} + p^*)\, Du^i + 3\theta(\overset{*}{\rho} + p^*)\, u^i - p^*_{,j}\, g^{ij} + \nu\sigma^{ij}_{;j}$$
$$+ Dq^i + 3\theta q^i + q^j_{;j}\, u^i + q^j u^i_{;j} - \mu h^i_{;j}\, h^j - \mu h^i\, h^j_{;j} = 0. \quad ...(3.7)$$

Using (2.1), (2.4) for the space-like vector $a^i = q^i / \mid q \mid (\mid q \mid > 0)$ and (3.4) in the equation obtained by the contraction of (3.7) with q_i, we get

$$(\overset{*}{\rho} + p^*) \mid q \mid^2 / KT - p^*_{,j}\, q^j + \nu q_i\, \sigma^{ij}_{;j} - (\tfrac{1}{2} \mid q \mid^2 u^i)_{;i}$$
$$- \frac{5\theta}{2} \mid q \mid^2 + \sigma_{ij} q^i q^j - \mu (q_i h^i)^2 / KT - \mu (q_i h^i)_{;j} h^j$$
$$+ \mu \mid q \mid_{,j} h^j (a_i h^i) + \mu \mid q \mid (\overset{\vee}{\sigma}_{ij} + \overset{\vee}{\theta} \overset{\vee}{\gamma}_{ij} - \overset{\vee}{D} a_i a_j)\, h^i h^j = 0 \quad ...(3.8)$$

where $\overset{\vee}{\sigma}_{ij}$, $\overset{\vee}{\theta}$ denote the shear and expansion of the heat-flux tubes respectively. Equation (3.8) suggests that the heat flux tube is always oblique at the boundary of a perfectly conducting star.

The 'steady state with rigid rotation' is defined by the conditions (Yodzis 1971)

$$\theta = \sigma_{ij} = 0. \quad ...(3.9)$$

Using (3.9), Banerji (1974) proved that

$$h^i w_{ij} = 0, \;\; w_{ij} \neq 0. \quad ...(3.10)$$

The q^i is given by (Prasad 1978b).

$$q^i = \gamma^i_j\, (w^{jk}_{;k} - \sigma^{jk}_{;k} + \tfrac{2}{3} \theta^{,j}) + (w^i_k + \sigma^i_k)\, Du^k \quad ...(3.11)$$

By virtue of (3.9), (3.10), (3.11) and (2.4), we get

$$q^i h_i = - \mid h \mid \overset{*}{w}_{jk} w^{jk} = - 2 \mid h \mid \overset{*}{w}^i w_i, \;\; \overset{*}{w}^i = \tfrac{1}{2} \eta^{ijkm} u_j w_{km} \quad ...(3.12)$$

which gives the following theorem :

Theorem 3.1 — The heat-flux tube and the magnetic field tube are orthogonal when an imperfect magnetofluid is in 'steady state with rigid rotation'.

Corollary 3.1 — The fluid acceleration and the magnetic field are orthogonal when an imperfect magnetofluid of constant rest temperature is in 'steady state with rigid rotation'.

The proof follows from (3.4) and Theorem 3.1.

When an imperfect magnetofluid is in 'steady state with rigid rotation', eqn. (3.8) reduces to

$$(\overset{*}{\rho} + p^*) \mid q \mid^2/KT - p^*_{,j}\, q^j - (\tfrac{1}{2} \mid q \mid^2 u^i)_{;i}$$
$$+ \mu \mid q \mid (\overset{\vee}{\sigma}_{ij} + \overset{\vee\vee}{\theta} \gamma_{ij})\, h^i h^j = 0 \qquad ...(3.13)$$

which suggests that there is a consẻrvation law for the thermal heat energy along the world line, i.e. $(\frac{1}{2} \mid q \mid^2 u^i)_{;i} = 0$.

4. Imperfect Shear-free Magnetofluid and Symmetry Mappings

In this section we will discuss the family of contracted Ricci collineations (FCRC) admitted by the imperfect shear-free magnetofluid space-times. Norris *et al.* (1977) have defined the FCRC admitted by the four dimensional Riemannian manifold V_4 as follows :

$$\underset{\xi}{\mathcal{L}} R_{ij} = H_{ij},\ g^{ij} H_{ij} = 0 \qquad ...(4.1)$$

where H_{ij} is any trace-free symmetric tensor that is not identically equal to $\underset{\xi}{\mathcal{L}} R_{ij}$. Here $\underset{\xi}{\mathcal{L}}$ denotes the operation of Lie differentiation with respect to the vector ξ^i. When, for a particular H_{ij} in a given space-time, a vector ξ^i satisfying (4.1) can be determined to within a multiplicative constant, then we say that the given space-time admits FCRC symmetry property. When a particular choice of H_{ij} in a given space-time does not permit the determination of the vector ξ^i (e.g., when ξ^i is only determined to within an arbitrary multiplicative function f satisfying $\xi^i f_{,i} = 0$) then we say that given space-time admits an FCRC quasi-symmetry property. It can be shown that the conservation expression (Oliver and Davis 1976)

$$(\sqrt{-g}\, R^i_j\, \xi^j)_{;i} = 0 \qquad ...(4.2)$$

holds for both FCRC symmetry and quasi-symmetry properties.

By virtue of (3.1) and (3.2), we have

$$R_{ij} = k\, \{\mu_0 u_i u_j - \mu_1 \gamma_{ij} + \nu \sigma_{ij} - \mu h_i h_j + q_i u_j + q_j u_i\}$$

where

$$2\mu_0 = \rho + 3p + \mu \mid h \mid^2,\ 2\mu_1 = \rho - p + \mu \mid h \mid^2. \qquad ...(4.3)$$

Let us consider any space-like symmetry mapping vector ξ^i which satisfies the property $\xi^i u_i = 0$. Then (4.3) assumes the form

$$\underset{\xi}{\mathcal{L}} R_{ij} = A(u_i u_j - \tfrac{1}{4} g_{ij}) + 2(\mu_0 + \mu_1)(u_i w_{kj} + u_j w_{ki})\xi^k$$
$$+ 2(q_i w_{jk} + q_j w_{ik})\xi^k - (Du_k\xi^k)(q_i u_j + q_j u_i)$$
$$- \mu_1(L\xi)_{ij} + \nu \underset{\xi}{\mathcal{L}} \sigma_{ij} - (q_k\xi^k)(2\sigma_{ij} + 2\theta\gamma_{ij}$$
$$+ Du_i u_j + Du_j u_i) + 2\{u_{(i} q_{j);k} - q_{k;(i} u_{j)}\}\xi^k$$
$$+ 2\mu\{h_{k;(i} h_{j)} - h_{(i} h_{j);k}\}\xi^k + (h_k\xi^k)(h_{i;j}$$
$$+ h_{j;i}) - \tfrac{1}{2} g_{ij}(\mu_1\xi^k)_{;k} + \tfrac{1}{2} g_{ij}(q_k\xi^k u^l)_{;l}$$
$$- \tfrac{1}{2} g_{ij}(B_k\xi^k h^l)_{;l} \qquad \ldots(4.4)$$

where

$$A = (\mu_0 + \mu_1)_{,k}\,\xi^k - (\mu_0 + \mu_1)\,\xi^k Du_k$$

and

$$(L\xi)_{ij} = \xi_{j;i} + \xi_{i;j} - \tfrac{1}{2} g_{ij}\xi^k_{\ ;k}\,.$$

Here we assume that the imperfect magnetofluid is shear-free. If ξ^i is an FCRC mapping vector then the last three terms on the right-hand side of (4.4) vanish by (4.2). Further, (4.4) will satisfy the property $g^{ij} \underset{\xi}{\mathcal{L}} R_{ij} = 0$ only when the following conditions

$$(w^i_{\ k} + \theta\gamma^i_{\ k})\, q_i\xi^k + \xi^k Dq_k = 0 \qquad \ldots(4.5)$$

$$(\tfrac{1}{2}\mu \mid h \mid^2)_{,k}\,\xi^k + \mu \mid h \mid \xi^k D^* h_k = 0 \qquad \ldots(4.6)$$

and

$$h^i_{\ ;i} = 0 \qquad \ldots(4.7)$$

are satisfied. It is clear from the above analysis that the form of H_{ij} in (4.4) can determine ξ^i under special cases. To determine ξ^i we suppose that $\xi^k(u_i w_{kj} + u_j w_{ki}) = 0$ which suggests that $\xi^k = \varphi w^k$, where φ is an arbitrary function and w^k is the vorticity vector. The determination of ξ^i is also possible in other directions but we are interested to explore few conditions on the imperfect shear-free magnetofluids involving FCRC quasi symmetry property for the symmetry vector $\xi^i = \varphi w^i$ and to focus attention on the skeleton structures formed by this quasi-symmetry property because the role of skeleton structures is important for the study of actual symmetry properties.

Equations (4.5) – (4.7) reduce to

$$KTw^i D^2 u_i = (q_k w^k) \qquad \ldots(4.8)$$

$$n^i \underset{w}{\mathcal{L}} h_i = -\tfrac{1}{2} D^* \epsilon \qquad \ldots(4.9)$$

where ϵ is the charge density.

$$h^i_{;i} = 0 \qquad \ldots(4.10)$$

for the symmetry vector $\xi^i = \varphi w^i$. The last three terms on the right-hand side of (4.4) are the conservation expressions given by

$$(\varphi \mu_1 w^k)_{;k} = 0 \qquad \ldots(4.11a)$$

$$(\varphi q_k w^k u^i)_{;i} = 0 \qquad \ldots(4.11b)$$

$$(\mu \varphi h_k w^k h^i)_{;i} = 0. \qquad \ldots(4.11c)$$

Equation (4.11*a*) reduces to the well-known conservation expression obtained by Norris *et al.* (1977) when the magnetic field vanishes. Using (4.10) and $h_k w^k = -\epsilon/2$ in (4.11*c*), we get

$$D^*(\varphi \mu \epsilon) = 0 \qquad \ldots(4.11d)$$

which is a conservation of charge together with the magnetic permeability along the magnetic field tubes. (4.11*b*) gives the conservation law for the thermal heat energy in case of rotating fluid.

Now we write the different type of choices for the tensor H_{ij} in the manner of Norris *et al.* (1977) as follows :

$$\overset{1}{H}_{ij} = A(u_i u_j - \tfrac{1}{4} g_{ij}) - (\xi^k D u_k)(q_i u_j + q_j u_i) - \mu_1 (L\xi)_{ij} \qquad \ldots(4.12)$$

$$\overset{2}{H}_{ij} = A(u_i u_j - \tfrac{1}{4} g_{ij}) - (\xi^k D u_k)(q_i u_j + q_j u_i) - (q_k \xi^k)(2\theta \gamma_{ij} + D u_i u_j + D u_j u_i) \qquad \ldots(4.13)$$

$$\overset{3}{H}_{ij} = -(\xi^k D u_k)(q_i u_j + q_j u_i) - \mu_1 (L\xi)_{ij} - (q_k \xi^k)(2\theta \gamma_{ij} + D u_i u_j + D u_j u_i) \qquad \ldots(4.14)$$

$$\overset{4}{H}_{ij} = A(u_i u_j - \tfrac{1}{4} g_{ij}) - (\xi^k D u_k)(q_i u_j + q_j u_i) \qquad \ldots(4.15)$$

$$\overset{5}{H}_{ij} = A(u_i u_j - \tfrac{1}{4} g_{ij}) - \mu_1 (L\xi)_{ij} \qquad \ldots(4.16)$$

$$\overset{6}{H}_{ij} = A(u_i u_j - \tfrac{1}{4} g_{ij}) - (q_k \xi^k)(2\theta \gamma_{ij} + D u_i u_j + D u_j u_i) \qquad \ldots(4.17)$$

$$\overset{7}{H}_{ij} = -(\xi^k D u_k)(q_i u_j + q_j u_i) - \mu_1 (L\xi)_{ij} \qquad \ldots(4.18)$$

$$\overset{8}{H}_{ij} = -\mu_1 (L\xi)_{ij} - (q_k \xi^k)(2\theta \gamma_{ij} + D u_i u_j + D u_j u_i) \qquad \ldots(4.19)$$

$$\overset{9}{H}_{ij} = -(\xi^k Du_k)(q_i u_j + q_j u_i) - (q_k \xi^k)(2\theta \gamma_{ij} + Du_j u_i + Du_i u_j). \quad ...(4.20)$$

Similarly other types of choices [e.g., $\overset{10}{H}_{ij} = -\mu_1(L\xi)_{ij}$] are also possible but such choices will impose stronger conditions on the imperfect shear-free magnetofluids. We discuss these quasi-symmetry structures given by (4.12) - (4.20) one by one.

Comparing (4.12) with (4.4) and using (4.8) - (4.11*c*), we get

$$(2\theta\gamma_{ij} + Du_i u_j + Du_j u_i) = 0 \text{ as } (q_k \xi^k) \neq 0 \quad ...(4.21)$$

which holds only when $\theta = Du_i = 0$. Thus we observe the following theorem :

Theorem 4.1 — An imperfect shear-free magnetofluid space-time admits an FCRC quasi-symmetry propetry (4.12) with

$$\xi^i = \varphi w^i \text{ iff (i) } (\varphi \mu_1 w^k)_{;k} = (\varphi q_k w^k u^i)_{;i} = (\mu \varphi h_k w^k h^i)_{;i} = 0$$

and (ii) the fluid flows are non-expanding geodesics.

Comparing (4.13) with (4.4) and using (4.8) - (4.11*c*), we have

$$\xi_{i;j} + \xi_{j;i} = \tfrac{1}{2} \xi^k_{;k} g_{ij} \quad ...(4.22)$$

which shows that an FCRC degenerates into conformal motion. Thus we have the following theorem :

Theorem 4.2 — An imperfect shear-free magnetofluid space-time admits an FCRC quasi-symmetry property (4.13) with

$$\xi^i = \varphi w^i \text{ iff (i) } (\varphi \mu_1 w^k)_{;k} = (\varphi q_k w^k u^i)_{;i} = (\mu \varphi h_k w^k h^i)_{;i} = 0$$

and (ii) FCRC degenerates into conformal motion.

Comparing (4.14) with (4.4) and using (4.8) - (4.11*c*), we get

$$(\mu_0 + \mu_1)_{,k} \xi^k - (\mu_0 + \mu_1) \xi^k Du_k = 0 \quad ...(4.23)$$

since $(u_i u_j - \frac{1}{4} g_{ij}) \neq 0$. Using the identity

$w^k_{;k} + 2w^k Du_k = 0$ in (4.23), we obtain

$$\{(\mu_0 + \mu_1)^2 w^k\}_{;k} = 0 \quad ...(4.24)$$

which gives the following theorem :

Theorem 4.3 — An imperfect shear-free magnetofluid space-time admits an FCRC quasi-symmetry property (4.14) with

$$\xi^i = \varphi w^i \text{ iff (i) } (\varphi \mu_1 w^k)_{;k} = (\varphi q_k w^k u^i)_{;i} = (\mu \varphi h_k w^k h^i)_{;i} = 0$$

and (ii) $\{(\mu_0 + \mu_1)^2 w^k\}_{;k} = 0$

Similarly we have the following theorems :

Theorem 4.4 — An imperfect shear-free magnetofluid space-time admits an FCRC quasi-symmetry property (4.15) with

$$\xi^i = \varphi w^i \text{ iff (i) } (\varphi\mu_1 w^k)_{;k} = (\varphi q_k w^k u^i)_{;i} = (\mu\varphi h_k w^k h^i)_{;i} = 0,$$

(ii) the fluid flows are non-expanding geodesics and (iii) FCRC degenerates into conformal motion.

Theorem 4.5 — An imperfect shear-free magnetofluid space-time admits an FCRC quasi-symmetry property (4.16) with

$$\xi^i = \varphi w^i \text{ iff (i) } (\varphi\mu_1 w^k)_{;k} = (\varphi q_k w^k u^i)_{;i} = (\mu\varphi h_k w^k h^i)_{;i} = 0,$$

(ii) the fluid acceleration and the vorticity are orthogonal and (iii) the fluid flows are non-expanding geodesics.

Theorem 4.6 — An imperfect shear-free magnetofluid space-time admits an FCRC quasi-symmetry property (4.17) with

$$\xi^i = \varphi w^i \text{ iff (i) } (\varphi\mu_1 w^k)_{;k} = (\varphi q_k w^k u^i)_{;i} = (\mu\varphi h_k w^k h^i)_{;i} = 0,$$

(ii) the fluid acceleration and vorticity are orthogonal and (iii) FCRC degenerates into conformal motion.

Theorem 4.7 — An imperfect shear-free magnetofluid space-time admits an FCRC quasi-symmetry property (4.19) with

$$\xi^i = \varphi w^i \text{ iff } (\varphi\mu_1 w^k)_{;k} = (\varphi q_k w^k u^i)_{;i} = (\mu\varphi h_k w^k h^i)_{;i} = 0,$$

(ii) $\{\mu_0 + \mu_1)^2 w^k\}_{;k} = 0$ and (iii) the fluid acceleration and vorticity are orthogonal.

Further development will be communicated soon.

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