SMALL LONGITUDINAL VIBRATIONS OF INITIALLY STRETCHED HOLLOW CYLINDRICAL POLARIZED DIELECTRIC

by A. N. CHAWLA *and* H. R. CHAUDHRY, *Department of Applied Sciences, Punjab Engineering College, Chandigarh*

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The theory of small deformation superposed on large deformation of an elastic dielectric (Verma and Chaudhry 1966) has been applied to determine the phase velocity of the small longitudinal vibrations of an initially stretched hollow cylindrical polarized dielectric. In the absence of polarization the first order approximation of the phase velocity of waves has been shown to reduce to the one already determined (Suhubi 1965, Nowinski 1969).

1. INTRODUCTION

In the past two to three decades, a number of problems on finite deformation of compressible and incompressible elastic materials have been discussed by various authors. The solutions of most of these problems are given in Green and Zerna (1954) and Green and Adkins (1960).

A theory suitable for an isotropic elastic dielectric subject to large deformation and polarization has been given by Eringen (1963). Following the lines of Green and Zerna (1954) and using the equations given by Eringen (1963), Verma and Chaudhry (1966) developed the theory valid for small deformation superposed on large deformation of an elastic dielectric.

The theory of small deformation superposed on large deformation given in Green and Zerna (1954) has been further employed by Suhubi (1965) to solve the problem of small longitudinal vibrations of an initially stretched circular cylinder for both compressible and incompressible materials. A similar problem of thermal waves in an elastic highly stretched cylindrical bar of incompressible material has been solved by Nowinsky (1969) employing the equations developed by Flavin (1962) and Flavin and Green (1961) which is a generalization to the thermoelastic case of the theory of superposition given in Green and Zerna (1954).

Employing the equations developed by Verma and Chaudhry (1966) and following the method of treatment given by Suhubi (1965) and Nowinski (1969), we make an attempt in the present paper, to determine the phase velocity of dielectric waves in an initially stretched hollow cylindrical polarized dielectric. For simplicity the material of the hollow cylinder has been taken as incompressible.

For this purpose, we reproduce the basic equations governing the finite deformation of hyperelastic dielectric in the presence of an electric field as given by Eringen (1963) in Section 2 and those for small deformation superposed on large deformation of an elastic dielectric as developed by Verma and Chaudhry (1966) in Section 3. In Section 4, the results of large uniform extension of a hollow cylindrical polarized dielectric as arrived at by Eringen (1963) are reproduced. Employing equations given in Sections 2, 3 and 4, we determine the first order approximation of the phase velocity of dielectric waves in Section 7 after the necessary calculations given in Sections 5 and 6. In the absence of polarization, the first order approximation of the phase velocity has been shown to reduce to the one already determined by Suhubi (1965) and Nowinski (1969).

2. Fundamental Equations of Hyperelastic Polarized Dielectric

(i) *Field Equations*

$$t^{k}_{l;k} + \rho f_l = 0 \qquad \dots(2.1)$$

$$_{L}E_k - \psi_{,k} = 0 \qquad \dots(2.2)$$

$$\epsilon \nabla^2 \psi - \text{div } P = - q_f \text{ in } V_d \qquad \dots(2.3)$$

where V_d is the volume that the dielectric occupies, semicolon and comma stand for covariant and ordinary partial differentiations respectively, t^k_l is the Cauchy stress, f_l the body force per unit mass, ρ the density, $_{L}E_k$ the local electric field, ψ the electrostatic potential, P the polarization vector, ϵ_0 the material constant and q_f the volume free charge.

(ii) *Boundary Conditions*

$$[[t^k_l]]\, n_k = 0 \qquad \dots(2.4)$$

$$[[\epsilon_0 \psi^k - P^k]]\, n_k + \omega_f = 0, \text{ on } S_d \qquad \dots(2.5)$$

where S_d is the surface enclosing the dielectric, Cauchy stress tensor t^k_l is defined by

$$t^k_l \equiv {}_{L}t^k_l - {}_{M}t^k_l \qquad \dots(2.6)$$

$$_{M}t^k_l \equiv \epsilon_0 \{\psi^k_{,}\, \psi_{,l} - \tfrac{1}{2} \psi^m_{,}\, \psi_{,m}\, \delta^k_l\} \qquad \dots(2.7)$$

and $_{L}t^k_l$ and $_{L}E^k$ are given by constitutive equations. In (2.5) n_k denotes the exterior normal to S_d, ω_f denotes the surface free charge and a double bracket stands for the discontinuity across the surface.

(iii) *Constitutive Equations*

$$
{}_{L}t^{l}_{k} = \frac{2\rho}{\rho_0}\Big[\frac{\partial\Sigma}{\partial I_1}\overset{-1}{C}{}^{l}_{k} + \Big(I_2\frac{\partial\Sigma}{\partial I_2} + I_3\frac{\partial\Sigma}{\partial I_3}\Big)\delta^{l}_{k}
- I_3\frac{\partial\Sigma}{\partial I_2}C^{l}_{k} + \Big(\frac{\partial\Sigma}{\partial I_4} + I_1\frac{\partial\Sigma}{\partial I_5}\Big)\overset{-1}{C}{}^{l}_{m}P^{m}.P_{k}
- I_2\frac{\partial\Sigma}{\partial I_5}P^{l}P_{k} + I_3\frac{\partial\Sigma}{\partial I_5}C^{l}_{m}P^{m}.P_{k} + \frac{\partial\Sigma}{\partial I_5}\overset{-1}{C}{}^{l}_{m}\overset{-1}{C}{}^{n}_{k}P_{n}.P^{m}\Big] \quad \text{...(2.8)}
$$

$$
{}_{L}E^{k} = \frac{2\rho}{\rho_0}\Big[\Big(\frac{\partial\Sigma}{\partial I_4} + I_1\frac{\partial\Sigma}{\partial I_5}\Big)\overset{-1}{C}{}^{k}_{l} + \Big(\frac{\partial\Sigma}{\partial I_6} - I_2\frac{\partial\Sigma}{\partial I_5}\Big)\delta^{k}_{l} + I_3\frac{\partial\Sigma}{\partial I_5}C^{k}_{l}\Big].P^{l} \quad \text{...(2.9)}
$$

where ρ_0 and ρ are densities in undeformed and deformed states respectively, $\Sigma = \Sigma(I_r)$, $(r = 1, 2, ..., 6)$ and I's are the invariants based on the strain measure of Finger $\overset{-1}{C}$ and the polarization vector P.

3. Small Deformations Superposed on Large Deformations

As isotropic and homogeneous elastic dielectric with a strain energy function Σ is denoted by B_0, B and B' in its undeformed and unpolarized, deformed and polarized, and finally deformed and polarized states respectively. The displacement vector of a point is denoted by V in B and $V + \epsilon W$ in B', where ϵ is so small that ϵ^2, ϵ^3 etc., are neglected as compared to ϵ. We suppose that the base vectors are G_i, G^i in B_0; g_i, g^i in B and $g_i + \epsilon g'_i$, $g^i + \epsilon g'^i$ in B'. The metric tensors are G_{ij}, G^{ij} in B_0, g_{ij}, g^{ij} in B and $g_{ij} + \epsilon g'_{ij}$, $g^{ij} + \epsilon g'^{ij}$ in B'. Then

$$
g'_{ij} = W_i\|_j + W_j\|_i,\ g'^{ij} = -\,g^{ir}g^{js}g'_{rs} \quad \text{...(3.1)}
$$

where $W = W_m g^m = W^m g_m$ and a double line denotes covariant differentiation with respect to the coordinates of B state. The determinant of g_{ij} is g and that of $g_{ij} + \epsilon g'_{ij}$ is $g + \epsilon g'$ where $g' = g\, g^{ij} g'_{ij}$.

The invariants based on $\overset{-1}{C}{}^{k}_{l}$ and P in B' state are

$$
I_1 + \epsilon I'_1 = \delta^{k}_{l}(\overset{-1}{C}{}^{l}_{k} + \epsilon\,\overset{-1}{C}{}'^{l}_{k}),
$$

$$
I_2 + \epsilon I'_2 = \tfrac{1}{2}\,\delta^{km}_{ln}\,(\overset{-1}{C}{}^{l}_{k} + \epsilon\,\overset{-1}{C}{}'^{l}_{k})(\overset{-1}{C}{}^{n}_{m} + \epsilon\,\overset{-1}{C}{}'^{n}_{m}),
$$

$$
I_3 + \epsilon I'_3 = \tfrac{1}{6}\,\delta^{kmp}_{lnq}\,(\overset{-1}{C}{}^{l}_{k} + \epsilon\,\overset{-1}{C}{}'^{l}_{k})(\overset{-1}{C}{}^{n}_{m} + \epsilon\,\overset{-1}{C}{}'^{n}_{m})(\overset{-1}{C}{}^{q}_{p} + \epsilon\,\overset{-1}{C}{}'^{q}_{p}),
$$

(*continued on p.* 774)

$$I_4 + \epsilon I'_4 = (\overset{-1}{C}{}^k_l + \epsilon \overset{-1}{C}{}'^k_l)\, P^l . P_k,$$

$$I_5 + \epsilon I'_5 = (\overset{-2}{C}{}^k_l + \epsilon \overset{-2}{C}{}'^k_l)\, P^l . P_k,$$

$$I_6 + \epsilon I'_6 = P^2 \qquad \ldots(3.2)$$

where

$$\left. \begin{aligned} C'^l_k &= g'^{lm}\, G_{MK}\, X^M_{,m} X^K_{,k} \\ \overset{-1}{C}{}'^l_k &= g'_{km}\, G^{LM}\, x^l_{,L}\, x^m_{,M}. \end{aligned} \right\} \qquad \ldots(3.3)$$

For homogeneous isotropic elastic dielectric in B' state, the energy function Σ becomes $\Sigma + \epsilon\Sigma' = \Sigma(I_r + \epsilon I'_r)$ $(r = 1, 2, \ldots, 6)$. The scalar invariants $\Phi_r = \dfrac{\partial\Sigma}{\partial I_r}$ which for the body B are functions of I_r become functions of $I_r + \epsilon I'_r$ for the body B' and may be denoted by

$$\Phi_r + \epsilon\Phi'_r = \Phi_r(I_r + \epsilon I'_r) = \frac{\partial\Sigma}{\partial I_r} + \epsilon\frac{\partial\Sigma'}{\partial I_r}. \qquad \ldots(3.4)$$

By Taylor's expansion, we obtain

$$\Phi_r + \epsilon\Phi'_r = \Phi_r(I_r + \epsilon I'_r) = \Phi(I_r) + \epsilon \sum_{m=1,2,\ldots,6} I'_m \frac{\partial\Phi_r}{\partial I_m} \qquad \ldots(3.5)$$

to the first order in ϵ.

From (3.5), we get

$$\frac{\partial\Sigma'}{\partial I_r} = \sum_{m=1,2,\ldots,6} I'_m \frac{\partial^2\Sigma}{\partial I_m \partial I_r}. \qquad \ldots(3.6)$$

From (2.8), the local stress tensor in B' is given by

$$\begin{aligned} {}_L t^l_k + \epsilon\, {}_L t'^l_k = \frac{2(\rho + \epsilon\rho')}{\rho_0} \Bigg[& \left(\frac{\partial\Sigma}{\partial I_1} + \epsilon\frac{\partial\Sigma'}{\partial I_1}\right)(\overset{-1}{C}{}^l_k + \epsilon\overset{-1}{C}{}'^l_k) \\ & + \left\{ (I_3 + \epsilon I'_3)\left(\frac{\partial\Sigma}{\partial I_3} + \epsilon\frac{\partial\Sigma'}{\partial I_3}\right) \right. \\ & \left. + (I_2 + \epsilon I'_2)\left(\frac{\partial\Sigma}{\partial I_2} + \epsilon\frac{\partial\Sigma'}{\partial I_2}\right) \right\} \delta^l_k \\ & - (I_3 + \epsilon I'_3)\left(\frac{\partial\Sigma}{\partial I_2} + \epsilon\frac{\partial\Sigma'}{\partial I_2}\right)(C^l_k + \epsilon C'^l_k) + \end{aligned}$$

(*equation continued on p. 775*)

$$+\left\{\left(\frac{\partial\Sigma}{\partial I_4}+\epsilon\frac{\partial\Sigma'}{\partial I_4}\right)+(I_1+\epsilon I_1')\left(\frac{\partial\Sigma}{\partial I_5}+\epsilon\frac{\partial\Sigma'}{\partial I_5}\right)\right\}$$

$$\times\,(\overset{-1}{C}{}^l_m+\epsilon\,\overset{-1}{C}{}'^l_m)\,P^m.P_k-(I_2+\epsilon I_2')\left(\frac{\partial\Sigma}{\partial I_5}+\epsilon\frac{\partial\Sigma'}{\partial I_5}\right)P^l.P_k$$

$$+(I_3+\epsilon I_3')\left(\frac{\partial\Sigma}{\partial I_5}+\epsilon\frac{\partial\Sigma'}{\partial I_5}\right)(C^l_m+\epsilon C'^l_m)\,P^m.P_k$$

$$+\left(\frac{\partial\Sigma}{\partial I_5}+\epsilon\frac{\partial\Sigma'}{\partial I_5}\right)(\overset{-1}{C}{}^l_m+\epsilon\,\overset{-1}{C}{}'^l_m)(\overset{-1}{C}{}^n_k+\epsilon\,\overset{-1}{C}{}'^n_k)P_n.P^m\Big]\qquad\ldots(3.7)$$

where $\rho+\epsilon\rho'$ is the density in B'. We reduce (3.7) for the incompressible case by substituting $\rho=\rho_0$, $\rho'=0$, $I_3=1$ and compare the resulting equation with (2.8) to obtain

$$_Lt'^l_k=\left\{-p'+2\left(I_2\frac{\partial\Sigma'}{\partial I_2}+I_2'\frac{\partial\Sigma}{\partial I_2}\right)\right\}\delta^l_k$$

$$+2\Bigg[\left(\frac{\partial\Sigma}{\partial I_1}\overset{-1}{C}{}'^l_k+\frac{\partial\Sigma'}{\partial I_1}\overset{-1}{C}{}^l_k\right)-C^l_k\left\{\frac{\partial\Sigma'}{\partial I_2}+I_3'\frac{\partial\Sigma}{\partial I_2}\right\}$$

$$-\frac{\partial\Sigma}{\partial I_2}C'^l_k+\left\{\frac{\partial\Sigma'}{\partial I_4}\overset{-1}{C}{}^l_m+\frac{\partial\Sigma}{\partial I_4}\overset{-1}{C}{}'^l_m\right\}P^m.P_k$$

$$+\left\{I_1'\frac{\partial\Sigma}{\partial I_5}+I_1\frac{\partial\Sigma'}{\partial I_5}\right\}\overset{-1}{C}{}^l_m\,P^m.P_k$$

$$+I_1\frac{\partial\Sigma}{\partial I_5}\overset{-1}{C}{}'^l_m\,P^m.P_k-\left(I_2\frac{\partial\Sigma'}{\partial I_5}+I_2'\frac{\partial\Sigma}{\partial I_5}\right)P^l.P_k$$

$$+\left(\frac{\partial\Sigma'}{\partial I_5}+I_3'\frac{\partial\Sigma}{\partial I_5}\right)C^l_m\,P^m.P_k$$

$$+\frac{\partial\Sigma}{\partial I_5}C'^l_m\,P^m.P_k+\left(\frac{\partial\Sigma}{\partial I_5}\overset{-1}{C}{}'^l_m+\frac{\partial\Sigma'}{\partial I_5}\overset{-1}{C}{}^l_m\right)\overset{-1}{C}{}^n_k\,P_n.P^m$$

$$+\frac{\partial\Sigma}{\partial I_5}\overset{-1}{C}{}^l_m\,\overset{-1}{C}{}'^n_k\,P_n.P^m\Bigg]\qquad\ldots(3.8)$$

where p' the arbitrary hydrostatic stress is determined from the equations of equilibrium and boundary conditions. The local electric field and Maxwell stress tensor in B' are the same as those in B and are given by (2.9) and (2.7) respectively. It is more clear when we see the eqn. (2.7) in the electrostatic potential. Cauchy stress tensor in B' is given by

$$t^l_k+\epsilon t'^l_k={}_Lt^l_k+\epsilon\,{}_Lt'^l_k-{}_Mt^l_k\qquad\ldots(3.9)$$

The equations of motion in B in the absence of body forces are

$$t^{ij}\|_i = \rho f^j$$

and in B' are

$$t'^{ij}\|_i + \Gamma'^{j}_{ir} t^{ir} + \Gamma'^{r}_{ir} t^{ij} = \rho f'^j \qquad \ldots(3.10)$$

where

$$\Gamma'^{j}_{ir} = \tfrac{1}{2} g^{js} \{g'_{si,r} + g'_{sr,i} - g'_{ir,s}\} + \tfrac{1}{2} g'^{js} \{g_{si,r} + g_{sr,i} - g_{ir,s}\}.$$

When surface forces are prescribed at the surface of B', the boundary conditions are

$$(t^{ij} + \epsilon t'^{ij})(n_i + \epsilon n'_i) = \tau^j + \epsilon \tau'^j \qquad \ldots(3.11)$$

where $(n_i + \epsilon n'_i)$ are the covariant components of the exterior unit normal to the surface of B' referred to base vectors $g^i + \epsilon g'^i$ and $\tau^j + \epsilon \tau'^j$ are the contravariant components of the surface force vector. If the surface of B' is given in the form $F(\theta_1, \theta_2, \theta_3) = 0$ where $\theta_1, \theta_2, \theta_3$ are the curvilinear coordinates in the B state, the eqn. (3.11) can be written as

$$K(t^{ij} + \epsilon t'^{ij}) \frac{\partial F}{\partial \theta^i} = \tau^j + \epsilon \tau'^j$$

where

$$K \left\{ \frac{\partial F}{\partial \theta^r} \cdot \frac{\partial F}{\partial \theta^s} (g^{rs} + \epsilon g'^{rs}) \right\}^{1/2} = 1. \qquad \ldots(3.12)$$

4. Large Uniform Extension of a Hollow Cylindrical Polarized Dielectric

The problem of a thick, incompressible circular cylindrical shell carrying a uniform surface charge at the inner surface subjected to large uniform extension has been solved by Eringen (1963). To recapitulate his solution, cylindrical polar coordinates (R, Θ, Z) and (r, θ, z) have been taken to describe the position of a point in B_0 and B states respectively. The deformation field is described by

$$r = \mu R, \ \theta = \Theta, \ z = \lambda Z \qquad \ldots(4.1)$$

where λ and μ are constant extension ratios. The electric field and polarization components are given by $E = [E(r), 0, 0]$, $P = [P(r), 0, 0]$. The deformation tensor and strain invariants are

$$\| C^k_l \| = \begin{bmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda^{-2} \end{bmatrix}, \ \| \overset{-1}{C}{}^k_l \| = \begin{bmatrix} \frac{1}{\lambda} & 0 & 0 \\ 0 & \frac{1}{\lambda} & 0 \\ 0 & 0 & \lambda^2 \end{bmatrix} \qquad \ldots(4.2)$$

$$I_1 = \frac{2}{\lambda} + \lambda^2,\ I_2 = \frac{1}{\lambda^2} + 2\lambda,\ I_3 = 1,\ I_4 = \frac{1}{\lambda} P^2,\ I_5 = \frac{1}{\lambda^2} P^2,\ I_6 = P^2.$$

Maxwell stress tensor, the local stress tensor and electric field are obtained in the form

$$\left.\begin{aligned} {}_M t^{rr} &= {}_M t^r_r = -{}_M t^\theta_\theta = -r^2\,{}_M t^{\theta\theta} = -{}_M t^z_z = -{}_M t^{zz} \\ &= \frac{1}{2\epsilon_0}\left[\frac{bW_f}{r} - P(r)\right]^2 \\ {}_M t^r_\theta &= {}_M t^r_z = {}_M t^\theta_z = 0 \end{aligned}\right\} \qquad \ldots(4.3)$$

$$\left.\begin{aligned} {}_L t^r_r &= {}_L t^{rr} = \left[\left(\frac{2}{\lambda}\alpha_4 + \frac{4}{\lambda^2}\alpha_5\right)K_1^2 - \frac{(1+K_1)^2}{\epsilon_0}\right] \\ &\quad \times \frac{b^2W_f^2}{2}\left(\frac{1}{r^2} - \frac{1}{a^2}\right) + \frac{(1+K_1)^2 b^2 W_f^2}{2\epsilon_0 r^2} \\ {}_L t^r_r - {}_L t^\theta_\theta &= {}_L t^{rr} - r^2\,{}_L t^{\theta\theta} = 2\left(\frac{1}{\lambda}\alpha_4 + \frac{2}{\lambda^2}\alpha_5\right)\frac{K_1^2 b^2 W_f^2}{r^2} \\ {}_L t^z_z - {}_L t^\theta_\theta &= {}_L t^{zz} - r^2\,{}_L t^{\theta\theta} = -2\left(\frac{1}{\lambda} - \lambda^2\right)\left(\alpha_1 + \frac{1}{\lambda}\alpha_2\right) \\ {}_L t^r_\theta &= {}_L t^r_z = {}_L t^\theta_z = 0 \end{aligned}\right\} \qquad \ldots(4.4)$$

$${}_L E^r = -\left[\epsilon_0\left(\frac{2}{\lambda}\alpha_4 + \frac{2}{\lambda^2}\alpha_5 + 2\alpha_6\right) - 1\right]^{-1}\frac{bW_f}{r},\ {}_L E^\theta = {}_L E^z = 0 \qquad \ldots(4.5)$$

where

$$\left.\begin{aligned} K_1 &= \left[\epsilon_0\left(\frac{2}{\lambda}\alpha_4 + \frac{2}{\lambda^2}\alpha_5 + 2\alpha_6\right) - 1\right]^{-1} \\ P &= \frac{-K_1 b W_f}{r} \\ \Sigma &= \alpha_1(I_1 - 3) + \alpha_2(I_2 - 3) + \alpha_4 I_4 + \alpha_5 I_5 + \alpha_6 I_6 \end{aligned}\right\} \qquad \ldots(4.6)$$

and a and b are the external and internal radii of the shell.

5. Small Longitudinal Vibrations Superposed on Large Extensions

We now superpose an infinitesimal longitudinal motion characterized by the following displacement field on the deformed hollow cylindrical polarized dielectric

$$u_r = u(r, z, t),\ u_\theta = 0,\ u_z = w(r, z, t). \qquad \ldots(5.1)$$

From (3.1), (3.2), (3.6), (4.6) and (5.1) we have

$$g'_{ij} = \begin{bmatrix} 2\frac{\partial u}{\partial r} & 0 & \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \\ 0 & 2ru & 0 \\ \frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} & 0 & 2\frac{\partial w}{\partial z} \end{bmatrix},$$

$$g'^{ij} = \begin{bmatrix} -2\frac{\partial u}{\partial r} & 0 & -\frac{\partial u}{\partial z} - \frac{\partial w}{\partial r} \\ 0 & -\frac{2u}{r^3} & 0 \\ -\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} & 0 & -2\frac{\partial w}{\partial z} \end{bmatrix} \quad \ldots(5.2)$$

$$C'^{l}_{k} = \begin{bmatrix} -\frac{2}{\mu^2}\frac{\partial u}{\partial r} & 0 & \left(-\frac{\partial u}{\partial z} - \frac{\partial w}{\partial r}\right)\frac{1}{\lambda^2} \\ 0 & -\frac{2u}{\mu^2 r} & 0 \\ \left(-\frac{\partial u}{\partial z} - \frac{\partial w}{\partial r}\right)\frac{1}{\mu^2} & 0 & -\frac{2}{\lambda^2}\frac{\partial w}{\partial r} \end{bmatrix}$$

$$\overset{-1}{C}{}'^{l}_{k} = \begin{bmatrix} 2\mu^2\frac{\partial u}{\partial r} & 0 & \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right)\mu^2 \\ 0 & \frac{2u\mu^2}{r} & 0 \\ \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right)\lambda^2 & 0 & 2\lambda^2\frac{\partial w}{\partial z} \end{bmatrix} \quad \ldots(5.3)$$

$$\left.\begin{aligned} I'_1 &= 2\mu^2\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) + 2\lambda^2\frac{\partial w}{\partial z} \\ I'_2 &= 2\mu^2\left[(\lambda^2+\mu^2)\left(\frac{\partial u}{\partial r} + \frac{u}{r}\right) + 2\lambda^2\frac{\partial w}{\partial z}\right] \\ I'_3 &= 2\lambda^2\mu^4\left(\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z}\right) \\ I'_4 &= \frac{2}{\lambda}\frac{\partial u}{\partial r}P^2,\ I'_5 = \frac{4}{\lambda^2}\left(\frac{\partial u}{\partial r}\right)^2 P^2,\ I'_6 = 0 \end{aligned}\right\} \quad \ldots(5.4)$$

$$\frac{\partial \Sigma'}{\partial I_1} = \frac{\partial \Sigma'}{\partial I_2} = \frac{\partial \Sigma'}{\partial I_3} = \frac{\partial \Sigma'}{\partial I_5} = \frac{\partial \Sigma'}{\partial I_6} = \frac{\partial \Sigma'}{\partial I_4} = 0. \quad \ldots(5.5)$$

Employing (3.2), (3.3), (3.6), (3.8) in (4.6), the components of the additional local stress tensor are obtained as follows :

$$\left.\begin{aligned}
{}_Lt_r^{'r} &= -p' - 4\mu^2(\alpha_1 + \alpha_2\mu^2)\frac{dw}{dz} - 4\mu^2\left(\alpha_1 + \frac{\alpha_2}{\mu^4}\right)\frac{u}{r} \\
&\quad + P^2 . 4\frac{\partial u}{\partial r}\left\{\mu^4\alpha_4 + \alpha_5\left(4\mu^4 + \frac{1}{\mu^2}\right)\right\} \\
{}_Lt_\theta^{'\theta} &= -p' - 4\mu^2\left[\left(\alpha_1 + \frac{\alpha_2}{\mu^4}\right)\frac{\partial u}{\partial r} + (\alpha_1 + \alpha_2\mu^2)\frac{\partial w}{\partial z}\right] \\
{}_Lt_z^{'z} &= -p' + \frac{4}{\mu^4}(\alpha_1 + \alpha_2\mu^2)\frac{\partial w}{\partial z} \\
{}_Lt_z^{'r} &= 2\mu^2(\alpha_1 + \alpha_2\mu^2)\left(\frac{\partial u}{\partial r} + \frac{\partial w}{\partial z}\right) + 2\alpha_5\mu^4\left(\frac{\partial u}{\partial r} + \frac{\partial w}{\partial z}\right)P^2 \\
{}_Lt_r^{'z} &= 2\left(\frac{\partial u}{\partial r} + \frac{\partial w}{\partial z}\right)\left(\frac{\alpha_1}{\mu^4} + \frac{\alpha_2}{\mu^2}\right) + P^2\left(\frac{\partial u}{\partial r} + \frac{\partial w}{\partial z}\right) \\
&\quad \times\left(\frac{\alpha_4}{\mu^4} + I_1\frac{\alpha_5}{\mu^4} + \frac{\alpha_5}{\mu^2}\right) \\
{}_Lt_\theta^{'r} &= {}_Lt_r^{'\theta} = {}_Lt_z^{'\theta} = {}_Lt_\theta^{'z} = 0.
\end{aligned}\right\} \quad \ldots(5.6)$$

The components of the Maxwell stress tensor and local electric field in B' state are the same as those in B and are given by (4.3) and (4.5)

Using (2.6), (4.3) and (5.6), the additional Cauchy stresses become

$$\left.\begin{aligned}
t_r^{'r} &= -p' - 4\mu^2(\alpha_1 + \alpha_2\mu^2)\frac{\partial w}{\partial z} - 4\mu^2\left(\alpha_1 + \frac{\alpha_2}{\mu^4}\right)\frac{u}{r} \\
&\quad + 4\left(\frac{-K_1bW_f}{r}\right)^2\left\{\mu^2\alpha_4 + \alpha_5\left(4\mu^4 + \frac{1}{\mu^2}\right)\right\}\frac{\partial u}{\partial r} \\
t_\theta^{'\theta} &= -p' - 4\mu^2\left\{\left(\alpha_1 + \frac{\alpha_2}{\mu^4}\right)\frac{\partial u}{\partial r} + (\alpha_1 + \alpha_2\mu^2)\frac{\partial w}{\partial z}\right\} \\
t_z^{'z} &= -p' + \frac{4}{\mu^4}(\alpha_1 + \alpha_2\mu^2)\frac{\partial w}{\partial z} \\
t_z^{'r} &= 2\mu^2(\alpha_1 + \alpha_2\mu^2)\left(\frac{\partial w}{\partial z} + \frac{\partial u}{\partial r}\right) + 2\alpha_5\mu^4\left(\frac{\partial u}{\partial r} + \frac{\partial w}{\partial z}\right)P^2 \\
t_z^{'\theta} &= t_\theta^{'r} = 0.
\end{aligned}\right\} \quad \ldots(5.7)$$

Through (2.6), (4.4) and (4.5), the components of Cauchy stress tensor in B state can be written in the form

$$\left.\begin{aligned} t^r_r &= P^2\mu^2(\alpha_4 + 2\mu^2\alpha_5) - \frac{1}{2\epsilon_0}\left(\frac{bW_f}{r} - P\right)^2 \\ t^\theta_\theta &= \frac{1}{2\epsilon_0}\left(\frac{bW_f}{r} - P\right)^2 \\ t^z_z &= -p + 2\left(\frac{1}{\mu^4}\alpha_1 + \frac{2}{\mu^2}\alpha_2\right) + \frac{1}{2\epsilon_0}\left(\frac{bW_f}{r} - P\right)^2 \\ t^r_\theta &= t^r_z = t^\theta_z = 0. \end{aligned}\right\} \quad \ldots(5.8)$$

The contravariant and mixed tensors in B and B' state are related by

$$t^r_m = t^r_s\, g^{ms} \quad \ldots(5.9)$$

and

$$(t^{rm} + \epsilon t'^{rm}) = (t^r_s + \epsilon t'^r_s)(g^{ms} + \epsilon g'^{ms}) \quad \ldots(5.10)$$

so that

$$t'^{rm} = t^r_s\, g'^{ms} + t'^r_s\, g^{ms}. \quad \ldots(5.11)$$

Using (5.7), (5.8) and (5.11), we obtain the non-vanishing stress component in B' state in the form

$$\left.\begin{aligned} t'^{rr} &= \left(\alpha_1 + \frac{\Psi}{\lambda^2}\right)\frac{2u}{r} + (\alpha_1' - p)\,2\frac{\partial u}{\partial r} + 2\alpha_2'\frac{\partial w}{\partial z} \\ &\quad + p' + \frac{1}{r^2}\frac{\partial u}{\partial r}\alpha_6' \\ r^2 t'^{\theta\theta} &= (\alpha_1' - p)\frac{2u}{r} + \left(\alpha_1' + \frac{\Psi}{\lambda^2}\right)2\frac{\partial u}{\partial r} \\ &\quad + 2\alpha_2'\frac{\partial w}{\partial z} + p' + \alpha_5'\frac{u}{r^3} \\ t'^{zz} &= 2\alpha_2'\frac{u}{r} + 2\alpha_2'\frac{\partial u}{\partial r} + 2\alpha_3'\frac{\partial w}{\partial z} + p' + \alpha_5' \cdot \frac{1}{r^2}\frac{\partial w}{\partial z} \\ t'^{rz} &= -(\lambda\Psi + p)\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right) + \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r}\right)\frac{1}{r^2}\alpha_7' \end{aligned}\right\} \quad \ldots(5.12)$$

where

$$\alpha_1' = \frac{A}{\lambda^2} + \left(\lambda + \frac{1}{\lambda^2}\right)^2 B + 2\left(1 + \frac{1}{\lambda^3}\right)F,$$

$$\alpha_2' = \lambda A + 2\left(\frac{1}{\lambda} + \lambda^2\right)B + (3 + \lambda^2)F + \lambda\Psi,$$

$$\alpha_3' = \lambda^4 A + 4\lambda^3 F + 4\lambda^2 B - p,$$

$$\alpha_5' = - b^2 W_f^2 (1 + K_1^2)/\epsilon_0,$$

$$\alpha_6' = b^2 W_f^2 \left[K_1^2 \left(2\mu^2\alpha_4 + 12\mu^4\alpha_5 + \frac{4\alpha_5}{\mu^2} \right) \right] + \frac{1}{\epsilon_0} (1 + K_1^2) \Big],$$

$$\alpha_7' = - b^2 W_f^2 \left[\mu^2\alpha_4 K_1^2 + \frac{1}{2\epsilon_0} (1 + K_1^2) \right],$$

$$p = - \frac{1}{\lambda} \left[\Phi + \left(\frac{1}{\lambda} + \lambda^2 \right) \Psi \right],$$

$$A = 2 \frac{\partial^2 \Sigma}{\partial I_1^2},$$

$$B = 2 \frac{\partial^2 \Sigma}{\partial I_2^2},$$

$$F = 2 \frac{\partial^2 \Sigma}{\partial I_1 \partial I_2}.$$

6. Equations of Motion and Boundary Conditions

Equations of motion (3.10) for u and w reduce to

$$\frac{\partial}{\partial r} t'^{11} + \frac{\partial}{\partial z} t'^{13} + \frac{1}{r} t'^{11} - r t'^{22} + \frac{\partial^2 u}{\partial z^2} t^{33} = \rho \frac{\partial^2 u}{\partial t^2} \qquad \text{...(6.1)}$$

$$\frac{\partial}{\partial r} t'^{13} + \frac{\partial}{\partial z} t'^{33} + \frac{1}{r} t'^{13} + \left(\frac{\partial^2 u}{\partial r \partial z} + \frac{1}{r} \frac{\partial u}{\partial z} + 2 \frac{\partial^2 w}{\partial z^2} \right) t^{33} = \rho \frac{\partial^2 w}{\partial t^2} \qquad \text{...(6.2)}$$

whereas second equation of motion for $u_\theta = v$ is identically satisfied.

Substituting the values from (5.12) and using the incompressibility condition,

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0 \qquad \text{...(6.3)}$$

the eqns. (6.1) and (6.2) reduce to

$$\beta_1 \frac{\partial^2 u}{\partial z^2} + \beta_2 \frac{\partial^2 w}{\partial r \partial z} + \frac{\partial p'}{\partial r} - 2 \frac{\alpha_6'}{r^3} \frac{\partial u}{\partial r} + \frac{\alpha_5'}{r^3} \cdot \frac{u}{r} + \frac{\alpha_6'}{r^2} \frac{\partial^2 u}{\partial r^2} + \alpha_7' \left(\frac{\partial^2 w}{\partial r \partial z} + \frac{\partial^2 u}{\partial z^2} \right) = \rho \frac{\partial^2 u}{\partial t^2} \qquad \text{...(6.4)}$$

$$\beta_3 \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) + \beta_4 \frac{\partial^2 w}{\partial z^2} + \frac{\partial p'}{\partial z} + \frac{\alpha_7'}{r^2} \left(\frac{\partial^2 u}{\partial r \partial z} + \frac{\partial^2 w}{\partial r^2} - \frac{1}{r} \frac{\partial u}{\partial z} - \frac{1}{r} \frac{\partial w}{\partial r} \right) = \rho \frac{\partial^2 w}{\partial t^2} \qquad \text{...(6.5)}$$

where

$$\beta_1 = \lambda^2 \left(\Phi + \frac{\Psi}{\lambda}\right),\ \beta_2 = 2\alpha_2' - 2(\alpha_1' - p) + \frac{1}{\lambda}\left(\Phi + \frac{\Psi}{\lambda}\right),$$

$$\beta_3 = \frac{1}{\lambda}\left(\Phi + \frac{\Psi}{\lambda}\right),\ \beta_4 = 2\alpha_3' - 2\alpha_2' + \left(\lambda^2 - \frac{2}{\lambda}\right)\left(\Phi + \frac{\Psi}{\lambda}\right). \qquad \text{...(6.5a)}$$

The solution to the boundary value problem described by three field eqns. (6.3), (6.4) and (6.5) taken in the form of simple harmonic waves is characterized by

$$\left.\begin{aligned} u(r, z, t) &= \phi_1(r)\, e^{i(qz+\omega t)} \\ w(r, z, t) &= \phi_2(r)\, e^{i(qz+\omega t)} \\ p'(r, z, t) &= \phi_3(r)\, e^{i(qz+\omega t)} \end{aligned}\right\} \qquad \text{...(6.6)}$$

where ϕ_i, $i = 1, 2, 3$ are functions depending on the radial coordinate r only, $\omega/2\pi$ is the frequency of waves and $2\pi/q$ their wave length so that the phase velocity is given by ω/q. By substituting (6.6) into the field eqns. (6.3), (6.4) and (6.5), the following system of three differential equations with three unknown functions

ϕ_1, ϕ_2 and ϕ_3 is obtained.

$$\phi_1' + \frac{1}{r}\phi_1 + iq\phi_2 = 0 \qquad \text{...(6.7)}$$

$$\delta_1\phi_1 + \beta_2 iq\phi_2' + \phi_3' + \frac{\alpha_6'}{r^2}\left(\phi_1'' - \frac{2}{r}\phi_1'\right) + \frac{\alpha_5'}{r^2}\cdot\frac{\phi_1}{r^2}$$

$$+ \frac{\alpha_7'}{r^2}(\phi_2' iq - q^2\phi_1) = 0 \qquad \text{...(6.8)}$$

$$\beta_3\left(\phi_2'' + \frac{1}{r}\phi_2'\right) + \delta_2\phi_2 + iq\phi_3 + \frac{\alpha_7'}{r^2}\left(\phi_1' iq + \phi_2''\right.$$

$$\left. - \frac{1}{r}\phi_1 iq - \frac{1}{r}\phi_2'\right) = 0 \qquad \text{...(6.9)}$$

where the prime denotes a differentiation with regard to radial coordinate and

$$\delta_1 = \rho\omega^2 - \beta_1 q^2,\ \delta_2 = \rho\omega^2 - \beta_4 q^2.$$

For solving these linear differential equations, we eliminate ϕ_2, ϕ_3, ϕ_2', ϕ_3' and ϕ_2'' from (6.8) with the help of (6.7) and (6.9) and obtain a linear differential equation of the fourth order, namely

$$\phi_1''''\left(\frac{-\beta_3}{q^2}-\frac{\alpha_7'}{r^2q^2}\right)+\phi_1'''\left(\frac{2\alpha_7'}{r^3q^2}-\frac{2\beta_2}{rq^2}\right)+\phi_1''\left\{\frac{1}{r^2}\left(\frac{3\beta_3}{q^2}-2\alpha_7'+\alpha_6'\right)+\frac{3\alpha_7'}{r^4q^2}-\left(\beta_2+\frac{\delta_2}{q^2}\right)\right\}$$
$$+\phi_1'\left\{\frac{-15\alpha_7'}{r^5q^2}+\frac{1}{r^3}\left(2\alpha_7'-\frac{3\beta_3}{q^2}-2\alpha_6'\right)+\frac{1}{r}\left(\frac{-\delta_2}{q^2}-\beta_2\right)\right\}$$
$$+\phi_1\left\{\frac{15\alpha_7'}{r^6q^2}+\frac{1}{r^4}\left(\frac{3\beta_3}{q^2}-2\alpha_7'+\alpha_5'\right)+\frac{1}{r^2}\left(\frac{\delta_2}{q^2}+\beta_2-\alpha_7'q^2\right)+\delta_1\right\}=0. \qquad ...(6.10)$$

Following Frobenius method of solving differential equations, we assume

$$\phi_1(r)=\sum_0^\infty A_n r^{m+n}$$

to be the solution of (6.10), where $A_0, A_1, A_2 \ldots A_n$ are arbitrary constants to be determined.

We thus obtain

$$\phi_1(r)=A[r^5-S_0r^7-(R_2-S_0S_2)r^9+\{P_4+R_4S_0+S_4(R_2-S_0S_2)\}r^{11}+\ldots]$$
$$+B[r^3-R_0r^7+(P_2+R_0S_2)r^9+\{R_4R_0-S_4(P_2+S_0S_2)\}r^{11}+\ldots]$$
$$+C[r+P_0r^7-P_0S_2r^9+(-P_0R_4+P_0S_2S_4)r^{11}+\ldots]+Dr^{-1} \qquad ...(6.11)$$

where A, B, C, D are new independent constants given by

$$A=A_{10}+B,$$
$$B=A_8+C,$$
$$C=A_6+D,$$
$$D=A_4.$$

From (6.7) and (6.9), we get

$$\phi_2(r)=-\frac{1}{iq}[A\{6r^4-8S_0r^6-10(R_2-S_0S_2)r^8+12(P_4+R_4S_0+S_4.\overline{R_2-S_0S_2})r^{10}+\ldots\}$$
$$+B\{4r^2-8R_0r^6+10(P_2+R_0S_2)r^8+12(R_0R_4-S_4.\overline{P_4+S_0S_2})r^{10}+\ldots\}$$
$$+C\{2+8P_0r^6-10P_0S_2r^8+12(-P_0R_4+P_0S_2S_4)r^{10}+\ldots\}] \qquad ...(6.12)$$

$$\phi_3(r) = A\Bigg[\frac{-48\alpha'_7}{q^2} + \left(\frac{-96\beta_3}{q^2} + \frac{192S_0\alpha'_7}{q^2} - 4\alpha'_7\right) r^2$$

$$+ \left\{\frac{288\beta_3 S_0}{q^2} + 6S_0\alpha'_7 - 6\,\frac{\delta_2}{q^2} + \frac{\alpha'_7}{q^2}(R_2 - S_0S_2)\,480\right\} r^4$$

$$+ \left\{\frac{640(R_2 - S_0S_2)}{q^2}\,\beta_3 + 8(R_2 - S_0S_2)\,\alpha'_7 + \frac{8S_0\delta_2}{q^2}\right.$$

$$\left. - \frac{960\alpha'_7}{q^2}(P_4 + R_4S_0 + S_4\,.\,\overline{R_2 - S_0S_2})\right\} r^6 + \ldots\Bigg]$$

$$+ B\Bigg[\left(\frac{-16\beta_3}{q^2} - 2\alpha'_7\right) + \left(\frac{-4\delta_2}{q^2} + \frac{192R_0\alpha'_7}{q^2}\right) r^2$$

$$+ \left\{\frac{288R_0\beta_3}{q^2} + 6\alpha'_7\,R_0 - \frac{480\alpha'_7}{q^2}(P_2 + R_0S_2)\right\} r^4$$

$$+ \left\{-(P_2 + R_0S_2)\left(\frac{640\beta_3}{q^2} + 8\alpha'_7\right) - \frac{960\alpha'_7}{q^2}\right.$$

$$\left. \times (R_0R_4 - S_4\,.\,\overline{P_2 + S_0S_2}) + \frac{8R_0\delta_2}{q^2}\right\} r^6 + \ldots\Bigg]$$

$$+ C\Bigg[\frac{-2\delta_2}{q^2} + \frac{-192P_0\alpha'_7}{q^2}\,r^2 + \left(\frac{-288P_0\beta_3}{q^2}\right.$$

$$\left. - 6P_0\alpha'_7 + \frac{480P_2S_2\alpha'_7}{q^2}\right) r^4$$

$$+ \left\{\frac{640P_0S_2\beta_3}{q^2} + 8P_0S_2\alpha'_7 - \frac{8P_0\delta_2}{q^2}\right.$$

$$\left. - \frac{960\alpha'_7}{q^2}(-P_0R_4 + P_0S_2S_4)\right\} r^6 + \ldots\Bigg]$$

$$+ D\left[\frac{2\alpha'_7}{r^4}\right] \qquad \ldots(6.13)$$

where

$$P_n = \frac{q^2\delta_1}{\alpha'_7(n+2)(n+4)(n+6)(n+8)},$$

$$R_n = \frac{P_n}{\delta_1}\left[\left(\beta_2 + \frac{\delta_2}{q^2}\right)(n+2)(n+4) + \alpha'_5\,q^2\right],$$

$$S_n = \frac{P_n}{\delta_1}\left[\frac{\beta_3}{q^2}(n+6)(n+4)^2(n+2) + 2\alpha'_7(n+4)^2\right.$$

$$\left. - \alpha'_6(n+2)(n+5) - \alpha'_5\right]$$

valid for $n = 0, 1, 2, \ldots \infty$.

In eqns. (6.11), (6.12) and (6.13), the only unknown constants are A, B, C and D which are to be determined from the boundary conditions

$$\left.\begin{aligned} \tau'^{11} &= 0 \text{ at } r = a \text{ and } r = b \\ \tau'^{13} &= 0 \text{ at } r = a \text{ and } r = b. \end{aligned}\right\} \qquad \ldots(6.14)$$

The explicit form of these boundary conditions (6.14) become

$$\left.\begin{aligned} &\frac{2}{a}\left(\alpha_1' + \frac{\psi}{\lambda^2}\right) u(a) + (\alpha_1' - p)\, 2u,_r(a) + 2\alpha_2' \, w,_z(a) \\ &\qquad + p'(a) + \frac{1}{a^2} u,_r(a)\, \alpha_6' = 0 \\ &\frac{2}{b}\left(\alpha_1' + \frac{\psi}{\lambda^2}\right) u(b) + (\alpha_1' - p)\, 2u,_r(b) + 2\alpha_2' \, w,_z(b) \\ &\qquad + p'(b) + \frac{1}{b^2} u,_r(b)\, \alpha_6' = 0 \\ &\left\{-(\lambda\psi + p) + \frac{\alpha_7'}{a^2}\right\} \{u,_z(a) + w,_r(a)\} = 0 \\ &\left\{-(\lambda\psi + p) + \frac{\alpha_7'}{b^2}\right\} \{u,_z(b) + w,_r(b)\} = 0. \end{aligned}\right\} \qquad \ldots(6.15)$$

Substituting the series obtained in (6.11), (6.12) and (6.13) into the boundary conditions (6.15) and using (6.6), we obtain four linear algebraic equations for determining the four unknown constants A, B, C and D. Elimination of these four unknown constants gives rise to a determinantal equation which is the desired frequency equation from which the wave velocity can be found.

7. Phase Velocity of Dielectric Waves

We assume that internal and external radii b and a of the cylindrical tube are small compared to their second and higher powers. This assumption is justified since we consider the cylindrical tube as very long so that no reflection of the waves may occur in the time interval considered.

Further, for explicit results, we take the strain energy function for the dielectric in the standard form

$$\Sigma = \alpha_1(I_1 - 3) + \alpha_2(I_2 - 3) + \alpha_4 I_4 + \alpha_5 I_5 + \alpha_6 I_6. \qquad \ldots(7.1)$$

Using (5.13) and (6.5a), we obtain

$$\alpha_1' = \alpha_2' = \Psi = 0,$$

$$\beta_1 = \beta_4 = \lambda^2\Phi, \ \beta_2 = -\beta_3 = -\frac{\Phi}{\lambda} = p. \qquad \ldots(7.2)$$

Substituting these values into the boundary conditions (6.15) and eliminating A, B, C and D after neglecting higher powers of a and b, the frequency equation of the first order approximation becomes,

$$\frac{\omega}{q}=\left[\frac{1}{\rho}\left\{2\alpha_7'+\frac{2\Phi}{\lambda}+\lambda^2\Phi\right\}\right]^{1/2} \qquad \ldots(7.3)$$

This shows that the velocity of the waves (ω/q) depends upon the polarization factor α_7' only.

In the absence of polarization, the eqn. (7.3) reduces to

$$\frac{\omega}{q}=\left[\frac{\Phi}{\rho}\left(\frac{2}{\lambda}+\lambda^2\right)\right]^{1/2} \qquad \ldots(7.4)$$

which is in agreement with the result already obtained by Suhubi (1965) and Nowinski (1969) for isothermal case.

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