ON THE SLOW STEADY MOTION OF A NATURALLY PERMEABLE SPHERICAL SHELL PLACED AT THE AXIS OF A ROTATING VISCOUS INCOMPRESSIBLE FLUID

by P. D. VERMA *and* DILEEP SINGH CHAUHAN, *Department of Mathematics, University of Rajasthan, Jaipur*

(*Received* 22 *December* 1977; *after revision* 7 *April* 1978)

The present investigation concerns with the steady problem of a naturally permeable spherical shell placed at the axis of a rotating viscous incompressible fluid. It is assumed that the Reynolds number of the flow is small. We find that the drag coefficient decreases with permeability for $0 \leqslant K < 0.32$ as angular velocity of the fluid increases while it increases for $K > 0.32$ with the increase of the angular velocity.

1. INTRODUCTION

The motion of solid bodies in rotating, inviscid and incompressible fluids was investigated by Taylor (1921; 1922a, b; 1923), Grace (1922, 1923, 1924, 1926) and Morgan (1957) mostly in the early period of this century. Since then many workers made their contribution to this field. The slow motion of a sphere along the axis of a rotating inviscid fluid was considered by Stewartson (1952). In this paper, the experimental results that a column of the fluid cylinder of the same diameter as the sphere apparently pushed along in front of it, was mathematically verified. The motion of a sphere along the axis of a rotating inviscid fluid supposing that the fluid was undisturbed far-stream was again investigated by Stewartson (1958). He showed that if the above assumption holds good in unsteady motion, wave like solutions were excluded. Childress (1964) studied the slow motion of a sphere in a rotating viscous fluid. He obtained a modified Stokes' drag formula valid for small values of Reynolds and Taylor numbers, following the perturbation method due to Kaplun and Lagerstrom (1957). The slow steady rise of a solid sphere along the axis of a uniformly rotating viscous liquid was investigated by Davis (1965). He found, in general, by experiment, that the spheres which are less dense than the surrounding fluid, move to the axis of rotation and that the spheres which are more dense than the fluid, spiral out from the axis of rotation. He further obtained that the viscosity acts to destroy the two-dimensionality predicted by Taylor (1921; 1922a, b; 1923) for weak steady motion of an inviscid fluid under rotation, and that the effect of rotation is to reduce the speed of fall or rise of the sphere in Stokes' range. Maxworthy (1965) gave an experimental determination of the slow motion of a sphere in a rotating viscous fluid. The drag of the sphere was measured as it moved along the axis of a rotating viscous fluid. The results confirmed the theory of Childress (1964) for small values of Reynolds and Taylor numbers. Recently Verma and Gaur (1973) discussed the flow generated by the torsional oscillations of a sphere placed on the axis of a rotating viscous incompressible fluid.

The flow of fluids within porous media is of great practical importance and has been extensively studied. In the present investigation the effect of porosity on the drag of the sphere in a rotating viscous fluid is determined. The steady flow of a naturally permeable spherical shell placed at the axis of a rotating viscous incompressible fluid is considered. It is assumed that the Reynolds number of the flow is small. Thus, it is assumed that the Stokes' approximation holds good in the region beyond the outer and inner boundaries of the spherical shell and the well known Darcy's law holds in the porous region. The boundary conditions used as formulated by Jones (1973) are (i) the continuity of normal velocity, (ii) the continuity of pressure at the surfaces of shell and (iii) the slip condition for the tangential component of velocity at the outer surface of the shell, namely

$$e_{r\theta} = \beta(q_\theta - Q_\theta) \qquad \ldots(1)$$

where

$$e_{r\theta} = r\frac{\partial}{\partial r}\left(\frac{q_\theta}{r}\right) + \frac{1}{r}\frac{\partial q_\theta}{\partial \theta}, \quad \text{and } \beta = \frac{\alpha}{\sqrt{K}},$$

q_r, q_θ are radial and tangential velocity components in the free fluid region. Q_θ is the tangential velocity component in the porous region, α the constant depending upon the porous material and K the Darcy's constant usually known as permeability of the porous material.

It is found that the drag coefficient decreases with permeability for $0 \leqslant K < 0.32$ as angular velocity of the fluid increases while it increases for $K > 0.32$ with the increase of the angular velocity.

2. Equations of Motion and Boundary Conditions

We consider a homogeneous porous spherical shell placed at the axis of rotation of a viscous incompressible fluid. We impose a uniform velocity U at infinity parallel to the axis of rotation of the fluid. In spherical polar coordinate system (r, θ, ϕ), $\theta = 0$ is the axis of rotation of the fluid and $r = a$, $r = b$ are outer and inner radii of the spherical shell respectively. Let I, II and III be the outer most, porous and inner most region respectively. For slow motion, the Navier-Stokes equations of motion and the equation of continuity in a rotating frame of reference, are

$$-2\Omega \sin\theta q_{\phi i} = -\frac{1}{\rho}\frac{\partial p_i}{\partial r} + \nu\left[\frac{\partial^2 q_{ri}}{\partial r^2} + \frac{2}{r}\frac{\partial q_{ri}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 q_{ri}}{\partial \theta^2} + \frac{\cot\theta}{r^2}\frac{\partial q_{ri}}{\partial \theta} - \frac{2q_{ri}}{r^2} - \frac{2}{r^2}\frac{\partial q_{\theta i}}{\partial \theta} - \frac{2q_{\theta i}}{r^2}\cot\theta\right], \qquad \ldots(2.1)$$

$$-2\Omega\cos\theta\, q_{\phi i} = -\frac{1}{\rho_r}\frac{\partial p_i}{\partial \theta} + \nu\left[\frac{\partial^2 q_{\theta i}}{\partial r^2} + \frac{2}{r}\frac{\partial q_{\theta i}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 q_{\theta i}}{\partial \theta^2} + \frac{\cot\theta}{r^2}\frac{\partial q_{\theta i}}{\partial \theta} + \frac{2}{r^2}\frac{\partial q_{ri}}{\partial \theta} - \frac{q_{\theta i}}{r^2\sin^2\theta}\right], \quad \text{...(2.2)}$$

$$2\Omega(q_{ri}\sin\theta + q_{\theta i}\cos\theta) = \nu\left[\frac{\partial^2 q_{\phi i}}{\partial r^2} + \frac{2}{r}\frac{\partial q_{\phi i}}{\partial r} + \frac{1}{r^2}\frac{\partial^2 q_{\phi i}}{\partial \theta^2} + \frac{\cot\theta}{r^2}\frac{\partial q_{\phi i}}{\partial \theta} - \frac{q_{\phi i}}{r^2\sin^2\theta}\right], \quad \text{...(2.3)}$$

$$\frac{\partial q_{ri}}{\partial r} + \frac{2q_{ri}}{r} + \frac{1}{r}\frac{\partial q_{\theta i}}{\partial \theta} + \frac{q_{\theta i}}{r}\cot\theta = 0, \quad \text{...(2.4)}$$

where the suffix $i = 1$ and $i = 3$ will give the set of equations for the regions I and III respectively. The inertia terms are being neglected for slow motion and q's are independent of ϕ for axial symmetry. q_r, q_θ and q_ϕ are the velocity components in the increasing directions of r, θ and ϕ; ρ, the density; ν, the kinematic viscosity; Ω, the angular velocity of the fluid and p is the modified pressure which includes allowance for the effects of gravity and the centrifugal force arising from the rotation of the coordinate system.

The Darcy's law will govern the flow inside the porous medium-region II. In the absence of body forces, the Darcy's equations are

$$Q_r = -\frac{K}{\mu}\frac{\partial P}{\partial r}, \quad \text{...(2.5)}$$

$$Q_\theta = -\frac{K}{\mu}\frac{\partial P}{r\partial \theta}, \quad \text{...(2.6)}$$

and the equation of continuity is

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 Q_r) + \frac{1}{r\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, Q_\theta) = 0, \quad \text{...(2.7)}$$

where $Q = (Q_r, Q_\theta, 0)$ is the volumetric flow rate or filter velocity, K the constant called permeability of the medium, μ the viscosity of the fluid and P the mean pressure. The boundary conditions are :

(a) For $r = a$,

(i) continuity of pressure at the outer surface, $(p)_1 = (P)_2$,

(ii) continuity of normal velocity at the outer surface, $(q_r)_1 = (Q_r)_2$,

(iii) $e_{r\theta} = \beta[(q_\theta)_1 - (Q_\theta)_2]$, where

$$e_{r\theta} = r\frac{\partial}{\partial r}\left(\frac{q_\theta}{r}\right) + \frac{1}{r}\frac{\partial q_r}{\partial \theta},\ \beta = \frac{\alpha}{\sqrt{K}},$$

(iv) $(q_\phi)_1 = 0$.

(b) For $r = b$,

(v) continuity of pressure at the inner surface, $(p)_3 = (P)_2$,

(vi) continuity of normal velocity at the inner surface, $(q_r)_3 = (Q_r)_2$,

(vii) $e_{r\theta} = -\beta[(q_\theta)_3 - (Q_\theta)_2]$,

where the suffix denotes the corresponding region.

(viii) $(q_\phi)_3 = 0$.

Also, as $r \to \infty$,

$$\left.\begin{aligned} (q_r)_1 &\sim U\cos\theta \\ (q_\theta)_1 &\sim -U\sin\theta \\ (q_\phi)_1 &= 0. \end{aligned}\right\} \quad \ldots(2.8)$$

Introducing the following non-dimensional quantities

$$R = \frac{r}{a},\ \sigma = \frac{b}{a},\ Re = \frac{Ua}{\nu},\ \bar{q}_r = \frac{q_r}{U},\ \bar{q}_\theta = \frac{q_\theta}{U},$$

$$\bar{q}_\phi = \frac{q_\phi}{U},\ w = \frac{a\Omega}{U},\ \bar{Q}_r = \frac{Q_r}{U},\ \bar{Q}_\theta = \frac{Q_\theta}{U},\ \bar{p} = \frac{a(p - p_\infty)}{\rho U \nu},$$

$$\bar{P} = \frac{a(P - p_\infty)}{\rho U \nu},$$

eqns. (2.1) to (2.7) on dropping the bars are reduced to

$$-2w\sin\theta\, q_{\phi i} = -\frac{1}{Re}\frac{\partial p_i}{\partial R} + \frac{1}{Re}\left[\frac{\partial^2 q_{ri}}{\partial R^2} + \frac{2}{R}\frac{\partial q_{ri}}{\partial R} + \frac{1}{R^2}\frac{\partial^2 q_{ri}}{\partial \theta^2} + \frac{\cot\theta}{R^2}\frac{\partial q_{ri}}{\partial\theta} - \frac{2}{R^2}q_{ri} - \frac{2}{R^2}\frac{\partial q_{\theta i}}{\partial\theta} - \frac{2q_{\theta i}}{R^2}\cot\theta\right], \quad \ldots(2.9)$$

$$-2w\cos\theta\, q_{\phi i} = -\frac{1}{Re}\frac{\partial p_i}{R\partial\theta} + \frac{1}{Re}\left[\frac{\partial^2 q_{\theta i}}{\partial R^2} + \frac{2}{R}\frac{\partial q_{\theta i}}{\partial R} + \frac{1}{R^2}\frac{\partial^2 q_{\theta i}}{\partial \theta^2} + \frac{\cot\theta}{R^2}\frac{\partial q_{\theta i}}{\partial\theta} + \frac{2}{R^2}\frac{\partial q_{ri}}{\partial\theta} - \frac{q_{\theta i}}{R^2\sin^2\theta}\right] \quad \ldots(2.10)$$

$$2w(q_{ri}\sin\theta + q_{\theta i}\cos\theta) = \frac{1}{Re}\left[\frac{\partial^2 q_{\phi i}}{\partial R^2} + \frac{2}{R}\frac{\partial q_{\phi i}}{\partial R} + \frac{1}{R^2}\frac{\partial^2 q_{\phi i}}{\partial \theta^2} + \frac{\cot\theta}{R^2}\frac{\partial q_{\phi i}}{\partial\theta} - \frac{q_{\phi i}}{R^2\sin^2\theta}\right], \quad \ldots(2.11)$$

$$\frac{\partial q_{ri}}{\partial R} + \frac{2q_{ri}}{R} + \frac{1}{R}\frac{\partial q_{\theta i}}{\partial \theta} + \frac{q_{\theta i}\cot\theta}{R} = 0 \qquad ...(2.12)$$

where

$Re = \frac{Ua}{\nu}$ is the Reynolds number.

$$Q_r = -K\frac{\partial P}{\partial R}, \qquad ...(2.13)$$

$$Q_\theta = -K\frac{\partial P}{R\partial \theta}, \qquad ...(2.14)$$

$$\frac{1}{R^2}\frac{\partial}{\partial R}(R^2 Q_r) + \frac{1}{R\sin\theta}\frac{\partial}{\partial \theta}(\sin\theta\, Q_\theta) = 0. \qquad ...(2.15)$$

And the boundary conditions (2.8) are reduced to

(a) For $R = 1$,

(i) $(p)_1 = (P)_2$,

(ii) $(q_r)_1 = (Q_r)_2$,

(iii) $e_{r\theta} = \beta[(q_\theta)_1 - (Q_\theta)_2]$,

(iv) $(q_\phi)_1 = 0$.

(b) For $R = \sigma$,

(v) $(p)_3 = (P)_2$,

(vi) $(q_r)_3 = (Q_r)_2$,

(vii) $e_{r\theta} = -\beta[(q_\theta)_3 - (Q_\theta)_2]$,

(viii) $(q_\phi)_3 = 0$.

And as $R \to \infty$,

$$\left.\begin{aligned} (q_r)_1 &\sim \cos\theta \\ (q_\theta)_1 &\sim -\sin\theta \\ (q_\phi)_1 &= 0. \end{aligned}\right\} \qquad ...(2.16)$$

3. Method of Solution

Eliminating pressure from (2.9) and (2.10), we get

$$\left[-2w\sin\theta\frac{\partial}{\partial\theta}(\sin\theta\, q_{\phi i}) + 2w\sin\theta\cos\theta\frac{\partial}{\partial R}(Rq_{\theta i})\right] = \frac{1}{Re}E^4\psi_i,$$
$$...(3.1)$$

where

$$E^2 = \frac{\partial^2}{\partial R^2} + \frac{1}{R^2}\frac{\partial^2}{\partial \theta^2} - \frac{\cot\theta}{R^2}\frac{\partial}{\partial\theta},$$

$$q_{ri} = \frac{1}{R^2 \sin\theta}\frac{\partial \psi_i}{\partial\theta},$$

$$q_{\theta i} = -\frac{1}{R\sin\theta}\frac{\partial\psi_i}{\partial R},$$

ψ_i is the Stokes' stream function.

Expanding the physical quantities as power series in Re, we have

$$H_i(R, \theta) = H_{0i}(R, \theta) + Re\, H_{1i}(R, \theta) + Re^2 H_{2i}(R, \theta) + \ldots \qquad \ldots(3.2)$$

where H_i stands for ψ_i, q_{ri}, $q_{\theta i}$, $q_{\phi i}$, p_i, Q_{r2}, $Q_{\theta 2}$ and P_2.
Substituting (3.2) in (2.11), (2.13), (2.14) and (3.1) and equating the coefficients of equal powers of Re, we get the following set of equations.

Equations for Region I

$$E^4\psi_{01} = 0, \qquad \ldots(3.3)$$

$$E^4\psi_{11} = \left[-2w\sin\theta\frac{\partial}{\partial\theta}(\sin\theta\, q_{\phi 01}) + 2w\sin\theta\cos\theta\frac{\partial}{\partial R}(Rq_{\phi 01})\right], \ldots(3.4)$$

$$E^4\psi_{21} = \left[-2w\sin\theta\frac{\partial}{\partial\theta}(\sin\theta\, q_{\phi 11}) + 2w\sin\theta\cos\theta\frac{\partial}{\partial R}(Rq_{\phi 11})\right], \ldots(3.5)$$

$$\frac{\partial^2 q_{\phi 01}}{\partial R^2} + \frac{2}{R}\frac{\partial q_{\phi 01}}{\partial R} + \frac{1}{R^2}\frac{\partial^2 q_{\phi 01}}{\partial\theta^2} + \frac{\cot\theta}{R^2}\frac{\partial q_{\phi 01}}{\partial\theta} - \frac{q_{\phi 01}}{R^2\sin^2\theta} = 0, \quad \ldots(3.6)$$

$$\frac{\partial^2 q_{\phi 11}}{\partial R^2} + \frac{2}{R}\frac{\partial q_{\phi 11}}{\partial R} + \frac{1}{R^2}\frac{\partial^2 q_{\phi 11}}{\partial\theta^2} + \frac{\cot\theta}{R^2}\frac{\partial q_{\phi 11}}{\partial\theta} - \frac{q_{\phi 11}}{R^2\sin^2\theta}$$
$$= 2w\sin\theta\, q_{r01} + 2w\cos\theta\, q_{\phi 01}. \qquad \ldots(3.7)$$

Equations for Region II

$$Q_{rj2} = -K\frac{\partial P_{j2}}{\partial R}, \qquad \ldots(3.8)$$

$$Q_{\theta j2} = -K\frac{1}{R}\frac{\partial P_{j2}}{\partial\theta}. \qquad \ldots(3.9)$$

Equations for Region III

$$E^4\psi_{03} = 0, \qquad \ldots(3.10)$$

$$E^4\psi_{13} = \left[-2w\sin\theta\frac{\partial}{\partial\theta}(\sin\theta\, q_{\phi 03}) + 2w\sin\theta\cos\theta\frac{\partial}{\partial R}(Rq_{\phi 03})\right]$$
$$\ldots(3.11)$$

$$E^4\psi_{23} = \left[-2w \sin\theta \frac{\partial}{\partial\theta} (\sin\theta\, q_{\phi 13}) + 2w \sin\theta \cos\theta \frac{\partial}{\partial R} (R q_{\phi 13}) \right] \quad \ldots(3.12)$$

The boundary conditions (2.16) are reduced to :

(a) For $R = 1$,

(i) $p_{j1} = P_{j2}$,

(ii) $q_{rj1} = Q_{rj2}$,

(iii) $e_{r\theta j1} = \beta[q_{\theta j1} - Q_{\theta j2}]$,

(iv) $q_{\theta j1} = 0$.

(b) For $R = \sigma$,

(v) $p_{j3} = P_{j2}$,

(vi) $q_{rj3} = Q_{rj2}$,

(vii) $e_{r\theta j3} = -\beta[q_{\theta j3} - Q_{\theta j2}]$,

(viii) $q_{\phi j3} = 0$.

(c) For $R \to \infty$,

$$\left.\begin{aligned} q_{r01} &\sim \cos\theta, \\ q_{\theta 01} &\sim -\sin\theta, \\ q_{\phi j1} &= 0. \end{aligned}\right\} \quad \ldots(3.13)$$

Here the first suffix $j = 0, 1$ and 2 will give the corresponding equations for the zeroth, first and second order solutions respectively, while the second suffix denotes the corresponding region.

4. Solutions

The solutions are obtained by using well known method of separating the variables.

Zeroth Order Solutions

For Region I

$$\psi_{01} = \left(\frac{A}{R} + BR - \tfrac{1}{2} R^2 \right) \sin^2\theta,$$

$$q_{r01} = \left(\frac{2A}{R^3} + \frac{2B}{R} - 1 \right) \cos\theta,$$

(*equation continued on p.* 804)

$$q_{\theta 01} = \left(\frac{A}{R^3} - \frac{B}{R} + 1\right)\sin\theta,$$

$$q_{\phi 01} = 0,$$

$$p_{01} = \frac{2B\cos\theta}{R^2}. \qquad \ldots(4.1)$$

For Region II

$$Q_{r02} = K\left(\frac{2E}{R^3} - F\right)\cos\theta,$$

$$Q_{\theta 02} = K\left(\frac{E}{R^3} + F\right)\sin\theta,$$

$$Q_{\phi 02} = 0,$$

$$P_{02} = \left(\frac{E}{R^2} + FR\right)\cos\theta. \qquad \ldots(4.2)$$

For Region III

$$\psi_{03} = (CR^2 + DR^4)\sin^2\theta,$$

$$q_{r03} = (2C + 2DR^2)\cos\theta,$$

$$q_{\theta 03} = -(2C + 4DR^2)\sin\theta,$$

$$q_{\phi 03} = 0,$$

$$p_{03} = 20DR\cos\theta. \qquad \ldots(4.3)$$

On using the boundary conditions (3.13), we have

$$A = \beta[(\tfrac{1}{4} - \tfrac{3}{2}K)(3\sigma^4 + \beta\sigma^5) - \tfrac{1}{4}(30K\beta + 3\sigma + \beta\sigma^2)]/S,$$

$$B = \tfrac{3}{4}(\beta + 2)(30K\beta + 3\sigma + \beta\sigma^2 - 3\sigma^4 - \beta\sigma^5)/S,$$

$$C = -\frac{9K}{2}(5K\beta + \tfrac{3}{2}\sigma + \beta\sigma^2)(\beta + 2)/S,$$

$$D = \tfrac{9}{4}K\beta(\beta + 2)/S,$$

$$E = -\tfrac{3}{2}(\beta + 2)(3\sigma^4 + \beta\sigma^5)/S,$$

$$F = \tfrac{3}{2}(\beta + 2)(30K\beta + 3\sigma + \beta\sigma^2)/S, \qquad \ldots(4.4)$$

where

$$S = [(3 + \beta + 3K + \tfrac{3}{2}K\beta)(3\sigma + \beta\sigma^2 + 30K\beta) - (3 + \beta - 6K)(3\sigma^4 + \beta\sigma^5)].$$

The solutions of zeroth order given by (4.1) to (4.4) are in agreement with Jones (1973).

First Order Solutions

For Region I

$$\psi_{11} = w^2 \sin^2\theta \cos\theta \left(-\tfrac{1}{2} C_1 R^{-2} + C_2\right),$$

$$q_{r11} = w^2(3\cos^2\theta - 1)\left(-\tfrac{1}{2} C_1 R^{-4} + C_2 R^{-2}\right),$$

$$q_{\theta 11} = w^2 \sin\theta \cos\theta(-C_1 R^{-4}),$$

$$q_{\phi 11} = w \sin\theta \cos\theta \left[\left(A + \frac{B}{2}\right) R^{-3} - AR^{-1} - \frac{B}{2} R\right],$$

$$p_{11} = w^2(3\cos^2\theta - 1)(2C_2 R^{-3}). \qquad \ldots(4.5)$$

For Region II

$$Q_{r12} = -Kw^2(3\cos^2\theta - 1)(2L_1 R - 3L_2 R^{-4}),$$

$$Q_{\theta 12} = 6Kw^2 \sin\theta \cos\theta(L_1 R + L_2 R^{-4}),$$

$$Q_{\phi 12} = 0,$$

$$P_{12} = w^2(3\cos^2\theta - 1)(L_1 R^2 + L_2 R^{-3}). \qquad \ldots(4.6)$$

For Region III

$$\psi_{13} = w^2 \sin^2\theta \cos\theta\left(\tfrac{1}{3} C_3 R^3 + \tfrac{1}{5} C_4 R^5\right),$$

$$q_{r13} = w^2(3\cos^2\theta - 1)\left(\tfrac{1}{3} C_3 R + \tfrac{1}{5} C_4 R^3\right),$$

$$q_{\theta 13} = w^2 \sin\theta \cos\theta(-C_3 R - C_4 R^3),$$

$$q_{\phi 13} = 0,$$

$$p_{13} = w^2(3\cos^2\theta - 1)\left(\tfrac{7}{5} C_4 R^2\right) \qquad \ldots(4.7)$$

where C_1, C_2, L_1, L_2, C_3 and C_4 are the constants of integration.

Second Order Solutions

For Region I

$$\psi_{21} = \left\{w^2 \sin^2\theta\left[X_1 R^{-1} + X_2 R + \frac{A}{6} R^3 - \frac{28}{35}\alpha_2 R^{-3} - \frac{B}{36} R^5\right] + w^2 \sin^4\theta\left[\alpha_1 R^{-1} + \alpha_2 R^{-3} + \frac{A}{12}(R - R^3) + \frac{B}{24}\left(R + \frac{R^5}{2}\right)\right]\right\}.$$

(*equation continued on p.* 806)

$$q_{r21} = \Big\{ w^2 \cos\theta \Big[2X_1R^{-3} + 2X_2R^{-1} + \frac{A}{3} R - \frac{56}{35}\alpha_2 R^{-5} - \frac{B}{18} R^3 \Big]$$
$$+ w^2 \sin^2\theta \cos\theta \Big[4\alpha_1 R^{-3} + 4\alpha_2 R^{-5} + \frac{A}{3}(R^{-1} - R)$$
$$+ \frac{B}{6}\Big(R^{-1} + \frac{R^3}{2}\Big)\Big]\Big\},$$

$$q_{\theta 21} = \Big\{ w^2 \sin\theta \Big[X_1R^{-3} - X_2R^{-1} - \frac{A}{2} R - \frac{84}{35}\alpha_2 R^{-5} + \frac{5}{36} BR^3 \Big]$$
$$+ w^2 \sin^3\theta \Big[\alpha_1 R^{-3} + 3\alpha_2 R^{-5} + \frac{A}{4}(-\tfrac{1}{3}R^{-1} + R)$$
$$- \frac{B}{24}(R^{-1} + \tfrac{5}{2}R^3)\Big]\Big\},$$

$$p_{21} = \Big\{ w^2 \cos\theta \Big[2X_2R^{-2} - 4\alpha_1 R^{-4} + A(-\tfrac{2}{3}R^{-2} + 1) - \frac{B}{3}(R^{-2} + R^2) \Big]$$
$$+ w^2 \sin^2\theta \cos\theta \Big[10\alpha_1 R^{-4} + A(R^{-2} - \tfrac{1}{2}) + \frac{B}{2} R^{-2} - \tfrac{1}{3}R^2)\Big]\Big\}. \quad \ldots(4.8)$$

For Region II

$$P_{22} = \{w^2 \cos\theta(N_3R + N_4R^{-2} - \tfrac{2}{5}N_1R^3 - \tfrac{2}{5}N_2R^{-4})$$
$$+ w^2 \sin^2\theta \cos\theta(N_1R^3 + N_2R^{-4})\},$$
$$Q_{r22} = \{- Kw^2 \cos\theta(N_3 - 2N_4R^{-3} - \tfrac{6}{5}N_1R^2 + \tfrac{8}{5}N_2R^{-5})$$
$$- Kw^2 \sin^2\theta \cos\theta(3N_1R^2 - 4N_2R^{-5})\},$$
$$Q_{\theta 22} = \Big\{ Kw^2 \sin\theta \Big(N_3 + N_4R^{-3} - \frac{12}{5} N_1R^2 - \frac{12}{5} N_2R^{-5}\Big)$$
$$+ Kw^2 \sin^3\theta(3N_1R^2 + 3N_2R^{-5})\Big\}. \quad \ldots(4.9)$$

For Region III

$$\psi_{23} = \{w^2 \sin^2\theta(M_1R^2 + M_2R^4) + w^2 \sin^4\theta(M_3R^4 + M_4R^6)\},$$
$$q_{r23} = \{w^2 \cos\theta(2M_1 + 2M_2R^2) + w^2 \sin^2\theta \cos\theta(4M_3R^2 + 4M_4R^4)\},$$
$$q_{\theta 23} = \{w^2 \sin\theta(- 2M_1 - 4M_2R^2) + w^2 \sin^3\theta(- 4M_3R^2 - 6M_4R^4)\},$$
$$p_{23} = \Big\{ w^2 \cos\theta \Big(20M_2R + 16M_3R - \frac{336}{35} M_4R^3 \Big)$$
$$+ w^2 \sin^2\theta \cos\theta(24M_4R^3)\Big\}. \quad \ldots(4.10)$$

The constants involved in (4.8) to (4.10) are evaluated by using the boundary conditions (3.13).

5. Drag on the Shell

The drag on the shell is given by

$$\text{Drag} = -\int_0^\pi 2\pi a^2 \sin\theta[\tau_{RR}\cos\theta - \tau_{R\theta}\sin\theta]_{R=1}\,d\theta, \qquad \ldots(5.1)$$

where, τ_{RR} and $\tau_{R\theta}$ are the components of the stress tensor for Newtonian fluids. Thus the drag coefficient is given by

$$C_D = \frac{1}{Re}\int_0^\pi [2p\sin\theta\cos\theta]_{R=1}\,d\theta - \int_0^\pi \frac{2\sin\theta}{Re}\Bigg[2\cos\theta\frac{\partial q_r}{\partial R} - \sin\theta\frac{\partial q_\theta}{\partial R} + \sin\theta\frac{q_\theta}{R} - \frac{\sin\theta}{R}\frac{\partial q_r}{\partial\theta}\Bigg]_{R=1} d\theta. \qquad \ldots(5.2)$$

We take the values of p, q_r and q_θ upto second order and neglect the higher terms. By substituting these values in eqn. (5.2), we obtain

$$C_D = \frac{8B}{Re} + 4w^2\,Re\left(2X_2 + \frac{46}{15}\alpha_1 - \frac{7172}{315}\alpha_2 + \frac{16}{15}A + \frac{119}{180}B\right), \qquad \ldots(5.3)$$

where A, B, α_1, α_2 and X_2 are the constants.

6. Particular Cases

(i) When $w \to 0$, $C_D = \dfrac{8B}{Re}$, ...(6.1)

which is the drag coefficient of porous shell in non-rotating case and agrees with Jones (1973).

(ii) When $w \to 0$, $K \to 0$, we have $B = \frac{3}{4}$.

Thus $C_D = \dfrac{6}{Re}$ ...(6.2)

which is the Stokes' drag when an impermeable sphere is in motion through viscous, incompressible fluid.

(iii) When $w \to 0$, $\sigma \to 0$, we have

$$B = \frac{(\beta + 2)}{(4K + 2\beta K + \frac{4}{3}\beta + 4)},$$

and the drag coefficient is given by

$$C_D = \frac{6(\beta + 2)}{Re(3K + \frac{3}{2}K\beta + \beta + 3)}, \quad \text{...(6.3)}$$

which is the drag coefficient of a porous sphere in non-rotating case and this result is in full agreement with that obtained by Jones (1973)

(iv) When $\sigma \to 0$ and $w \neq 0$, the drag coefficient of a moving porous sphere placed at the axis of a rotating viscous, incompressible fluid is given by

$$C_D = \frac{8B}{Re} + 4w^2 Re\left(2X_2 + \frac{46}{15}\alpha_1 - \frac{7172}{315}\alpha_2 + \frac{16}{15}A + \frac{119}{180}B\right) \quad \text{...(6.4)}$$

where

$$A = \frac{-\beta}{(12K + 6K\beta + 4\beta + 12)},$$

$$B = \frac{3(\beta + 2)}{(12K + 6K\beta + 4\beta + 12)},$$

$$\alpha_1 = -\left[\frac{\left(\frac{23}{6} + \frac{4}{3}\beta + 30K + 7K\beta\right)B + \left(15K + \frac{21}{2}K\beta - \frac{2\beta}{3} - \frac{4}{3}\right)A}{(900K + 210K\beta + 8\beta + 56)}\right],$$

$$\alpha_2 = -\left[\left(\frac{15}{2}K + 1\right)\alpha_1 + \left(\frac{1}{16} + \frac{K}{4}\right)B + \frac{3}{8}KA\right],$$

$$N_1 = -\left[\frac{\left(\frac{59}{3} + \frac{19}{6}\beta\right)B + \frac{15}{2}\beta - \left(\frac{124}{3} + 300K + \frac{32}{3}\beta\right)A}{(900K + 210K\beta + 8\beta + 56)}\right],$$

$$N_3 = \frac{-\left[N_1\left(\frac{2}{5} + \frac{6}{5}K - \frac{2}{3}\beta - \frac{24}{5}K\beta\right) + \alpha_1(-4 + 4\beta + 16) + \alpha_2\left(\frac{1736}{35} + \frac{168}{35}\beta\right) + A\left(-3 + \frac{\beta}{3}\right) + B\left(\frac{103}{18} + \frac{19}{18}\beta\right)\right]}{(6 + 2\beta + 6K + 3K\beta)},$$

$$X_2 = \left[-\frac{A}{6} + \frac{B}{3} + \frac{N_3}{2} - \frac{N_1}{5} + 2\alpha_1\right]. \quad \text{...(6.5)}$$

For this case, the drag coefficient C_D is plotted against the permeability K for various values of w and $Re = 0.1$, $\beta = 300$ and $\sigma = 0$ (Fig. 1). It is interesting to note that the drag coefficient decreases with permeability for $0 \leqslant K < 0.32$ as w increases, while it increases for $K > 0.32$ with the increase of w. This may be due to that as the angular velocity of the fluid increases, a greater force is required to keep the sphere on the axis of rotation which causes the increase in drag with the increase of

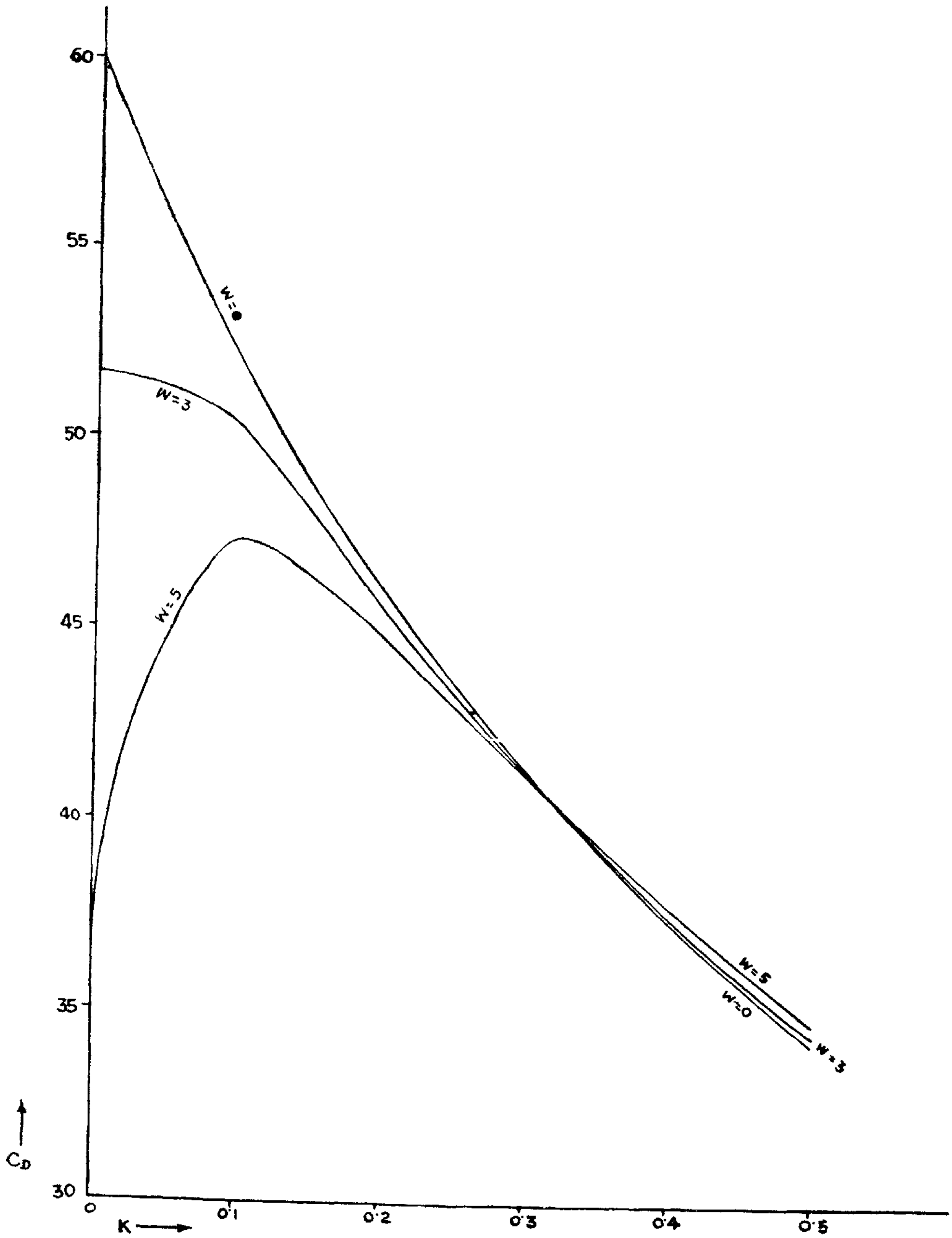

FIG. 1. Drag coefficient C_D versus permeability K for various values of w and $Re = 0.1, \beta = 300$ and $\sigma = 0$.

w for $K > 0.32$. Thus we conclude that the effect of permeability and the angular velocity of the fluid is to reduce the drag. But for $w = 5$, the drag coefficient increases as K increases from 0 to 0.1 and then decreases with K.

ACKNOWLEDGEMENT

One of us (DSC) is thankful to the Council of Scientific and Industrial Research for awarding a junior research fellowship.

REFERENCES

Childress, S. (1964). The slow motion of a sphere in a rotating viscous fluid. *J. Fluid Mech.*, **20**, 305.

Davis, Philip K. (1965). Motion of a sphere in a rotating fluid at small Reynolds numbers. *Phys. Fluids*, **8**, 560.

Grace, S. F. (1922). Free motion of a sphere in a rotating liquid parallel to the axis of rotation. *Proc. R. Soc.*, A **102**, 89.

——— (1923). Free motion of a sphere in a rotating liquid at right angles to the axis of rotation. *Proc. R. Soc.*, A **104**, 278.

——— (1924). A spherical source in a rotating fluid. *Proc. R. Soc.*, A **105**, 532.

——— (1926). On the motion of a sphere in a rotating liquid. *Proc. R. Soc.*, A **113**, 46.

Jones, I. P. (1973). Low Reynolds number flow past a porous spherical shell. *Proc. Camb. phil. Soc.*, **73**, 231.

Kaplun, S., and Lagerstrom, P. A. (1957). Asymptotic expansions of Navier-Stokes solutions for small Reynolds numbers. *J. Math. Mech.*, **6**, 585.

Maxworthy, T. (1965). An experimental determination of the slow motion of a sphere in a rotating viscous fluid. *J. Fluid Mech.*, **23**, 373.

Morgan, G. W. (1957). A study of motions in a rotating liquid. *Proc. R. Soc.*, A **206**, 108.

Stewartson, K. (1952). On the slow motion of a sphere along the axis of a rotating fluid. *Proc. Camb. phil. Soc.*, **48**, 168.

——— (1958). On the motion of a sphere along the axis of a rotating fluid. *Q. Jl Mech. appl. Math.*, **11**, 39.

Taylor, G. I. (1921). Experiments with the rotating fluids. *Proc. Camb. phil. Soc.*, **20**, 326.

——— (1922a). Experiments with the rotating fluids. *Proc. R. Soc.*, A **100**, 114.

——— (1922b). The motion of a sphere in a rotating liquid. *Proc. R. Soc.*, A **102**, 180.

——— (1923). Experiments on the motion of solid bodies in rotating fluids. *Proc. R. Soc.*, A **104**, 213.

Verma, P. D., and Gaur, Y. N. (1973). On the torsional oscillations of a sphere placed at the axis of a rotating viscous incompressible fluid. *J. Mécanique*, **12**, 183.