THE UNSTEADY FREE CONVECTION FLOW PAST A VERTICAL POROUS PLATE WITH VARIABLE SUCTION AND TRANSVERSE MAGNETIC FIELD

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(*Received* 13 *December* 1977)

The object of this paper is to study the unsteady flow of an incompressible, viscous, electrically conducting fluid past an infinite, vertical plate with variable suction. Approximate solution to the coupled non-linear equation governing the flow are derived using perturbation method with Eckert number $E\,(<< 1)$ as perturbation parameter. The detailed study has been made to know the effects of the suction parameter A on the amplitude and phase of the skin friction and the rate of heat transfer.

Notations

σ = electrical conductivity of the fluid

$|B|$ = amplitude of the skin friction

B_0 = applied magnetic field

K = thermal conductivity

M_r, M_i = fluctuating parts of the velocity profile

q' = rate of heat transfer

t' = time

t = dimensionless time

T' = temperature of fluid

T'_w = temperature of the plate

T'_∞ = temperature of the fluid in the free stream

T_r, T_i = fluctuating parts of the temperature profile

u', v' = velocity components in the x', y' directions

u = dimensionless velocity in the x-direction

U' = free stream velocity

U_0 = mean of $U'(t)$

U = dimensionless free stream velocity

g_x = acceleration due to gravity

$| Q |$ = amplitude of the rate of heat transfer

ϵU_0 = amplitude of free stream fluctuations

u_0 = mean velocity

u_1 = fluctuating part of the velocity

x', y' = co-ordinate system

y = dimensionless co-ordinate normal to the wall

ω' = frequency of free stream oscillations

ω = dimensionless frequency

τ' = skin friction

θ = dimensionless temperature

θ_0 = mean temperature

$\epsilon \theta_1$ = amplitude of the temperature fluctuations

α = phase angle of the skin friction

ρ' = density of the fluid in the boundary layer

ρ'_∞ = density of the fluid in the free stream.

1. Introduction

The oscillatory flow problem has attracted the attention of many research workers due to its important technological applications. In his pioneering work of the time-dependent oscillatory viscous flow, Lighthill (1954) considered the response of the laminar boundary layer of a fixed cylindrical body to unsteady fluctuations of free stream velocity. The idea was utilized by Stuart (1955) in studying the oscillatory flow of an incompressible, viscous fluid past an infinite horizontal, porous plate with constant suction. Reddy (1964) extended Stuart's problem by introducing a first-order velocity slip and temperature jump conditions at the plate. Soundalgekar (1977) completed a magnetohydrodynamic problem corresponding to that of Reddy. Messiha (1966) generalized Stuart's problem by introducing variable suction at the wall and found some interesting features. Soundalgekar (1969) solved Messiha's problem for electrically conducting fluid under the action of transverse magnetic field.

Recently Soundalgekar (1973a–b, 1975) has studied the effects of the oscillatory free stream and the free convection currents on the flow field past a vertical flat plate under a transverse magnetic field or without it and at the same time the temperature of the plate differs from the temperature of the free stream, causing the flow of a free convection currents in the boundary layer.

Here in this paper we have studied the free convection flow of a viscous liquid past an infinite porous plate with variable suction in the presence of the magnetic field. With the inclusion of viscous dissipation term in the energy equation, the problem becomes more general and mathematically difficult.

2. Mathematical Analysis

Here, x'-axis is taken along the plate in the direction of the flow whereas y'-axis is taken normal to the plate. The unsteady, two dimensional flow of an electrically conducting incompressible, viscous fluid past an infinite vertical porous plate with variable suction is assumed. A magnetic field of uniform strength is assumed to be applied transversely to the direction of the flow. The difference between the plate temperature and the main stream temperature is assumed to be so appreciable that the physical properties remain constant. Under these assumptions, the physical variables are functions of y' and t' only. Hence the unsteady free convection flow of an incompressible viscous liquid in the presence of a magnetic field is governed by the following equations :

Momentum equations

$$\rho'\left(\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'}\right) = -\frac{\partial p'}{\partial x'} - \rho' g_x + \mu\frac{\partial^2 u'}{\partial y'^2} - \sigma B_0^2 u' \qquad ...(1)$$

$$\frac{\partial v'}{\partial t'} = -\frac{1}{\rho'}\frac{\partial p'}{\partial y'}. \qquad ...(2)$$

Continuity equation

$$\frac{\partial v'}{\partial y'} = 0. \qquad ...(3)$$

Energy equation

$$\rho' C_p\left(\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'}\right) = K\frac{\partial^2 T'}{\partial y'^2} + \mu\left(\frac{\partial u'}{\partial y'}\right)^2. \qquad ...(4)$$

All the physical variables are defined in the notations. Also in (4), due to negligible induced magnetic field, the heat due to Joule dissipation is also assumed to be negligible. The boundary conditions are

$$\left.\begin{array}{l} u' = 0,\ T' = T'_w \text{ at } y' = 0 \\ u' \to U'(t'),\ \ T' \to T'_\infty \text{ as } y' \to \infty \end{array}\right\} \qquad ...(5)$$

where $U'(t') = U_0(1 + \epsilon e^{i\omega' t'})$.

From (1), we have for the free stream

$$\rho'\frac{\partial u'}{\partial t'} = -\frac{\partial p'}{\partial x'} - \rho'_\infty g_x - \sigma B_0^2\, U'. \qquad ...(6)$$

From (1) and (6), on eliminating $-\frac{\partial p'}{\partial x'}$, we have

$$\rho'\left(\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'}\right) = \rho'\frac{\partial U'}{\partial t'} + g_x(\rho'_\infty - \rho') + \mu\frac{\partial^2 u'}{\partial y'^2} + \sigma B_0^2 (U' - u'). \qquad ...(7)$$

Again from the equation of the state, we get

$$g_x(\rho'_\infty - \rho') = g_x\beta\rho'(T' - T'_\infty) \qquad ...(8)$$

where β is the coefficient of volume expansion. Substituting (8) in (7), we obtain

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \frac{\partial U'}{\partial t'} + g_x\beta(T' - T'_\infty) + \nu\frac{\partial^2 u'}{\partial y'^2} + \frac{\sigma B_0^2}{\rho'}(U' - u'). \qquad ...(9)$$

Equation (3) shows that v' is either a constant or a function of time and as we assume that the suction velocity oscillates in time about a constant mean, we have the following

$$v' = -v_0(1 + \epsilon A e^{i\omega' t'}) \qquad ...(10)$$

the negative sign indicates that the suction velocity is towards the plate. Also v_0 is the mean value of the suction at the plate and A is a suction parameter, substituting (10) in (9) and (4), we get

$$\frac{\partial u'}{\partial t'} - v_0(1 + \epsilon A e^{i\omega' t'})\frac{\partial u'}{\partial y'} = \frac{\partial U'}{\partial t'} + g_x\beta(T' - T'_\infty) + \nu\frac{\partial^2 u'}{\partial y'^2} + \frac{\sigma B_0^2}{\rho'} \times (U' - u') \qquad ...(11)$$

$$\frac{\partial T'}{\partial t'} - v_0(1 + \epsilon A e^{i\omega' t'})\frac{\partial T'}{\partial y'} = \frac{K}{\rho' C_p}\frac{\partial^2 T'}{\partial y'^2} + \frac{\nu}{C_p}\left(\frac{\partial u'}{\partial y'}\right)^2. \qquad ...(12)$$

Introducing the following non-dimensional quantities

$$y = y'v_0/\nu, \quad t = t'v_0^2/4\nu, \quad \omega = 4\nu\omega'/v_0^2,$$

$$u = u'/U_0, \quad U = U'/U_0, \quad \theta = \frac{T' - T'_\infty}{T'_w - T'_\infty}$$

$G = \dfrac{\nu g_x\beta(T'_w - T'_\infty)}{U_0 v_0^2}$, the Grashof number

$P = \mu C_p/K$, the Prandtl number

$E = U_0^2/C_p(T'_w - T'_\infty)$, the Eckert number

$M = \sigma B_0^2 \nu/\rho' v_0^2$, the magnetic field parameter ...(13)

in (11) and (12), we get

$$\tfrac{1}{4}\frac{\partial u}{\partial t} - (1 + \epsilon A e^{i\omega t})\frac{\partial u}{\partial y} = \tfrac{1}{4}\frac{\partial U}{\partial t} + G\theta + \frac{\partial^2 u}{\partial y^2} + M(U - u) \qquad ...(14)$$

and

$$\frac{P}{4}\frac{\partial \theta}{\partial t} - (1 + \epsilon A e^{i\omega t}) P \frac{\partial \theta}{\partial y} = \frac{\partial^2 \theta}{\partial y^2} + PE\left(\frac{\partial u}{\partial y}\right)^2. \qquad ...(15)$$

The corresponding boundary conditions (5) in view of (13), reduce to

$$\left.\begin{array}{l} u = 0, \quad \theta = 1 \text{ at } y = 0 \\ u \to U(t),\ \theta = 0 \text{ as } y \to \infty. \end{array}\right\} \qquad ...(16)$$

To solve these coupled non-linear equations, we assume following Lighthill (1954), that the unsteady flow is superimposed on the mean steady flow. Mathematically this is represented in the neighbourhood of the plate as

$$u(y, t) = u_0(y) + \epsilon e^{i\omega t} u_1(y) \qquad ...(17)$$

$$\theta(y, t) = \theta_0(y) + \epsilon e^{i\omega t} \theta_1(y) \qquad ...(18)$$

and for free stream

$$U(t) = 1 + \epsilon e^{i\omega t}. \qquad ...(19)$$

Substituting (17) – (19) in (14), (15) and (16) equating the coefficients of the harmonic and nonharmonic terms, neglecting the coefficients of ϵ^2, we get

$$u_0'' + u_0' - M u_0 = - M - G\theta_0 \qquad ...(20)$$

$$u_1'' + u_1' - \left(M + \frac{i\omega}{4}\right) u_1 = \frac{- i\omega}{4} - M - G\theta_1 - A u_0' \qquad ...(21)$$

$$\theta_0'' + P\theta_0' = - PE u_0'^2 \qquad ...(22)$$

$$\theta_1'' + P\theta_1' - \frac{i\omega}{4} P\theta_1 = - AP\theta_0' - 2PE u_0' u_1' \qquad ...(23)$$

and

$$\left.\begin{array}{l} u_0 = 0,\ u_1 = 0,\ \theta_0 = 1,\ \theta_1 = 0 \text{ at } y = 0 \\ u_0 = 1,\ u_1 = 1,\ \theta_0 = 0,\ \theta_1 = 0 \text{ as } y \to \infty. \end{array}\right\} \qquad ...(24)$$

In (20) – (23), the primes denote the differentiation with respect to y. These equations are still coupled and non-linear and hence very difficult to solve. To solve them, u_0, u_1, θ_0, θ_1 are expanded in powers of E, the Eckert number, for the Eckert number for all incompressible fluids is always $\ll 1$ (Soundalgekar 1975). Hence we take

$$u_0(y) = u_{01}(y) + Eu_{02}(y) + O(E^2) \quad \text{...(25)}$$

$$u_1(y) = u_{11}(y) + Eu_{12}(y) + O(E^2) \quad \text{...(26)}$$

$$\theta_0(y) = \theta_{01}(y) + E\theta_{02}(y) + O(E^2) \quad \text{...(27)}$$

$$\theta_1(y) = \theta_{11}(y) + E\theta_{12}(y) + O(E^2). \quad \text{...(28)}$$

Substituting (25) - (28) in (20) - (24), equating the coefficients of different powers of E, we have the set of following coupled linear equations

$$u''_{01} + u'_{01} - Mu_{01} = -M - G\theta_{01} \quad \text{...(29)}$$

$$u''_{02} + u'_{02} - Mu_{02} = -G\theta_{02} \quad \text{...(30)}$$

$$u''_{11} + u'_{11} - \left(M + \frac{i\omega}{4}\right)u_{11} = -\left(M + \frac{i\omega}{4}\right) - G\theta_{11} - Au'_{01} \quad \text{...(31)}$$

$$u''_{12} + u'_{12} - \left(M + \frac{i\omega}{4}\right)u_{12} = -G\theta_{12} - Au'_{02} \quad \text{...(32)}$$

$$\theta''_{01} + P\theta'_{01} = 0 \quad \text{...(33)}$$

$$\theta''_{02} + P\theta'_{02} = -Pu'^2_{01} \quad \text{...(34)}$$

$$\theta''_{11} + P\theta'_{11} - \frac{i\omega P}{4}\theta_{11} = -AP\theta'_{01} \quad \text{...(35)}$$

$$\theta''_{12} + P\theta'_{12} - \frac{i\omega P}{4}\theta_{12} = -AP\theta'_{02} - 2Pu'_{01}u'_{11} \quad \text{...(36)}$$

and the boundary conditions are

$$\left.\begin{array}{l} u_{01} = 0,\ \theta_{01} = 1 \\ u_{02} = 0,\ \theta_{02} = 0 \\ u_{11} = 0,\ \theta_{11} = 0 \\ u_{12} = 0,\ \theta_{12} = 0 \end{array}\right\} \text{at } y = 0 \quad \text{...(37)}$$

and

$$\left.\begin{array}{l} u_{01} = 1,\ \theta_{01} = 0 \\ u_{02} = 0,\ \theta_{02} = 0 \\ u_{11} = 1,\ \theta_{11} = 0 \\ u_{12} = 0,\ \theta_{12} = 0 \end{array}\right\} \text{as } y \to \infty. \quad \text{...(38)}$$

As we are interested in the unsteady flow only, the solutions to u_1, θ_1, and those for the mean flow u_0 and θ_0 are given here. The solutions of (29) - (36) in virtue of the boundary conditions (37) and (38) are respectively given by

$$u_0(y) = 1 + \left(\frac{G}{P^2 - P - M} - 1\right) e^{-my} - \frac{Ge^{-Py}}{(P^2 - P - M)} + E[A'e^{-my} + B'e^{-Py} + C'e^{-2Py} + D'e^{-2my} + E'e^{-(P+m)y}] \quad \ldots(39)$$

$$u_1(y) = 1 - e^{-ly} + \frac{AGP}{\left(P^2 - P - M - \frac{i\omega}{4}\right)}\left(\frac{4i}{\omega} + \frac{1}{P^2 - P - M}\right)(e^{-ly} - e^{-Py})$$

$$+ \frac{Am}{\left(m^2 - m - M - \frac{i\omega}{4}\right)}\left(1 - \frac{G}{P^2 - P - M}\right)(e^{-ly} - e^{-my})$$

$$+ \frac{4GAPi}{\left(n^2 - n - M - \frac{i\omega}{4}\right)}(e^{-ny} - e^{-ly})$$

$$- EG\left[\frac{R}{(P + l)^2 - P(P + l) - \frac{i\omega P}{4}}\left\{\frac{e^{-(P+l)y} - e^{-ly}}{(P + l)^2 - (P + l) - M - \frac{i\omega}{4}} - \frac{e^{-ny} - e^{-ly}}{\left(n^2 - n - M - \frac{i\omega}{4}\right)}\right\}\right.$$

$$+ \frac{S}{(P + n)^2 - P(P + n) - \frac{i\omega P}{4}}\left\{\frac{e^{-(P+n)y} - e^{-ly}}{(P + n)^2 - (P + n) - M - \frac{i\omega}{4}} - \frac{e^{-ny} - e^{-ly}}{\left(n^2 - n - M - \frac{i\omega}{4}\right)}\right\}$$

$$- \frac{4X}{i\omega P}\left\{\frac{e^{-Py} - e^{-ly}}{P^2 - P - M - \frac{i\omega}{4}} - \frac{e^{-ny} - e^{-ly}}{\left(n^2 - n - M - \frac{i\omega}{4}\right)}\right\}$$

$$+ \frac{Y}{\left(2P^2 - \frac{i\omega P}{4}\right)}\left\{\frac{e^{-2Py} - e^{-ly}}{4P^2 - 2P - M - \frac{i\omega}{4}} - \frac{e^{-ny} - e^{-ly}}{\left(n^2 - n - M - \frac{i\omega}{4}\right)}\right\} + \frac{Z}{\left(4m^2 - 2mP - \frac{i\omega P}{4}\right)}$$

$$\times \left\{\frac{e^{-2my} - e^{-ly}}{4m^2 - 2m - M - \frac{i\omega}{4}} - \frac{e^{-ny} - e^{-ly}}{n^2 - n - M - \frac{i\omega}{4}}\right\} +$$

(*equation continued on p.* 1221)

$$+\frac{L}{(P+m)^2-P(P+m)-\frac{i\omega P}{4}}\left\{\frac{e^{-(P+m)y}-e^{-ly}}{(P+m)^2-(P+m)-M-\frac{i\omega}{4}}\right.$$

$$\left.-\frac{e^{-ny}-e^{-ly}}{n^2-n-M-\frac{i\omega}{4}}\right\}+\frac{T}{(m+l)^2-P(m+l)-\frac{i\omega P}{4}}$$

$$\times\left\{\frac{e^{-(m+l)y}-e^{-ly}}{(m+l)^2-(m+l)-M-\frac{i\omega}{4}}-\frac{e^{-ny}-e^{-ly}}{n^2-n-M-\frac{i\omega}{4}}\right\}$$

$$+\frac{U''}{(m+n)^2-P(m+n)-\frac{i\omega P}{4}}\left\{\frac{e^{-(m+n)y}-e^{-ly}}{(m+n)^2-(m+n)-M-\frac{i\omega}{4}}\right.$$

$$\left.\left.-\frac{e^{-ny}-e^{-ly}}{n^2-n-M-\frac{i\omega}{4}}\right\}\right]+AE\left[\frac{mA'(e^{-my}-e^{-ly})}{m^2-m-M-\frac{i\omega}{4}}\right.$$

$$+\frac{PB'(e^{-Py}-e^{-ly})}{P^2-P-M-\frac{i\omega}{4}}+\frac{2PC'(e^{-2Py}-e^{-ly})}{4P^2-2P-M-\frac{i\omega}{4}}$$

$$\left.+\frac{2mD'(e^{-2my}-e^{-ly})}{4m^2-2m-M-\frac{i\omega}{4}}+\frac{E'(P+m)(e^{-(P+m)y}-e^{-ly})}{(P+m)^2-(P+m)-M-\frac{i\omega}{4}}\right] \quad \ldots(40)$$

$$\theta_0(y)=e^{-Py}+E\left[\frac{G^2P}{2(P^2-P-M)^2}\{e^{-Py}-e^{-2Py}\}\right.$$

$$+\frac{mP(P^2-P-M-G)^2}{2(P^2-P-M)^2(2m-P)}(e^{-Py}-e^{-2my})$$

$$\left.+\frac{2P^2G(P^2-P-M-G)}{(P^2-P-M)^2(P+m)}(e^{-Py}-e^{-(P+m)y})\right] \quad \ldots(41)$$

$$\theta_1(y)=\frac{4APi}{\omega}(e^{-Py}-e^{-ny})+E\left[\frac{R}{(P+l)^2-P(P+l)-\frac{i\omega P}{4}}\{e^{-(P+l)y}-e^{-ny}\}\right.$$

$$+\frac{S}{(P+n)^2-P(P+n)-\frac{i\omega P}{4}}\{e^{-(P+n)y}-e^{-ny}\}-\frac{4X}{i\omega P}(e^{-Py}-e^{-ny})$$

$$+\frac{Y}{\left(2P^2-\frac{i\omega P}{4}\right)}(e^{-2Py}-e^{-ny})+\frac{Z}{4m^2-2mP-\frac{i\omega P}{4}}\{e^{-2my}-e^{-ny}\}+$$

(equation continued on p. 1222)

$$+ \frac{L}{(P+m)^2 - P(P+m) - \frac{i\omega P}{4}} \{e^{-(P+m)y} - e^{-ny}\}$$

$$+ \frac{T}{(m+l)^2 - P(m+l) - \frac{i\omega P}{4}}$$

$$\left. \times \{e^{-(m+l)y} - e^{-ny}\} + \frac{U''}{(m+n)^2 - P(m+n) - \frac{i\omega P}{4}} \{e^{-(m+n)y} - e^{-ny}\} \right] \quad ...(42)$$

where

$$m = \frac{1 + \sqrt{1+4M}}{2}, \; l = \frac{1 + \sqrt{1 + 4M + i\omega}}{2}, \; n = \frac{P + \sqrt{P^2 + i\omega P}}{2},$$

$$A' = \frac{G^3 P}{2(P^2 - P - M)^3} - \frac{G^3 P}{2(P^2 - P - M)^2 (4P^2 - 2P - M)}$$

$$+ \frac{mGP(P^2 - P - M - G)^2}{2(P^2 - P - M)^3 (2m - P)}$$

$$- \frac{PG(P^2 - P - M - G)^2 m}{2(P^2 - P - M)^2 (2m - P)(4m^2 - 2m - M)}$$

$$+ \frac{2P^2G^2(P^2 - P - M - G)}{(P^2 - P - M)^3 (P + m)}$$

$$- \frac{2P^2G^2(P^2 - P - M - G)}{(P^2 - P - M)^2 (P + m)\{(P+m)^2 - (P+m) - M\}}$$

$$B' = -\left[\frac{G^3 P}{2(P^2 - P - M)^3} + \frac{mGP(P^2 - P - M - G)^2}{2(P^2 - P - M)^3 (2m - P)} + \frac{2P^2G^2(P^2 - P - M - G)}{(P^2 - P - M)^3 (P + m)}\right]$$

$$C' = \frac{G^3 P}{2(P^2 - P - M)^2 (4P^2 - 2P - M)}$$

$$D' = \frac{mPG(P^2 - P - M - G)^2}{2(P^2 - P - M)^2 (2m - P)(4m^2 - 2m - M)}$$

$$E' = \frac{2P^2G^2(P^2 - P - M - G)}{(P^2 - P - M)^2 (P + m)\{(P+m)^2 - (P+m) - M\}}$$

$$R = \frac{-2P^2Gl}{(P^2 - P - M)} + \frac{2P^3G^2lA\left(\frac{4i}{\omega} + \frac{1}{P^2 - P - M}\right)}{(P^2 - P - M)\left(P^2 - P - M - \frac{i\omega}{4}\right)} +$$

(*equation continued on p.* 1223)

$$+ \frac{2P^2GlAm\left(1 - \dfrac{G}{P^2 - P - M}\right)}{(P^2 - P - M)\left\{m^2 - m - M - \dfrac{i\omega}{4}\right\}}$$

$$- \frac{8P^3G^2lAi}{\omega\left\{n^2 - n - M - \dfrac{i\omega}{4}\right\}}$$

$$S = \frac{8P^3G^2Ani}{\omega\left\{n^2 - n - M - \dfrac{i\omega}{4}\right\}}$$

$$Y = \frac{-AP^3G^2}{(P^2 - P - M)^2} - \frac{2P^4G^2A\left(\dfrac{4i}{\omega} + \dfrac{1}{P^2 - P - M}\right)}{(P^2 - P - M)\left(P^2 - P - M - \dfrac{i\omega}{4}\right)}$$

$$X = \frac{AP^3G^2}{2(P^2 - P - M)^2} + \frac{AP^3m(P^2 - P - M - G)^2}{2(P^2 - P - M)^2(2m - P)} + \frac{2AP^4G(P^2 - P - M - G)}{(P^2 - P - M)^2(P + m)}$$

$$Z = \frac{-AP^2m^2(P^2 - P - M - G)^2}{(2m - P)(P^2 - P - M)^2} - 2P\left(1 - \frac{G}{P^2 - P - M}\right)^2 \times \frac{Am^3}{\left\{m^2 - m - M - \dfrac{i\omega}{4}\right\}}$$

$$L = \frac{-2AP^3G(P^2 - P - M - G)}{(P^2 - P - M)^2} - \frac{2P^2GAm^2(P^2 - P - M - G)}{(P^2 - P - M)^2\left\{m^2 - m - M - \dfrac{i\omega}{4}\right\}} - \frac{2P^3AGm(P^2 - P - M - G)\left(\dfrac{4i}{\omega} + \dfrac{1}{P^2 - P - M}\right)}{(P^2 - P - M)\left(P^2 - P - M - \dfrac{i\omega}{4}\right)}$$

$$T = \frac{-2Pm(P^2 - P - M - G)}{(P^2 - P - M)}\left[l - \frac{AGPl\left(\dfrac{4i}{\omega} + \dfrac{1}{P^2 - P - M}\right)}{\left(P^2 - P - M - \dfrac{i\omega}{4}\right)} - \right.$$

(equation continued on p. 1224)

$$- \frac{Aml(P^2 - P - M - G)}{(P^2 - P - M)\left\{m^2 - m - M - \frac{i\omega}{4}\right\}}$$

$$+ \frac{4AGPli}{\omega\left\{n^2 - n - M - \frac{i\omega}{4}\right\}}$$

$$U'' = \frac{8AGP^2nmi(P^2 - P - M - G)}{\omega\left\{n^2 - n - M - \frac{i\omega}{4}\right\}(P^2 - P - M)}.$$

Hence, the expressions for u and θ can be derived from (17) and (18) respectively. Knowing the velocity and temperature field, we can express them in terms of the fluctuating parts of the velocity and the temperature as

$$u(y, t) = u_0(y) + \epsilon(M_r \cos \omega t - M_i \sin \omega t)$$

$$\theta(y, t) = \theta_0(y) + \epsilon(T_r \cos \omega t - T_i \sin \omega t)$$

where $M_r + iM_i = u_1$

and

$$T_r + iT_i = \theta_1.$$

From the knowledge of the velocity, we now calculate the skin friction, which is given by

$$\tau' = \mu \left(\frac{\partial u'}{\partial y'}\right)_{y'=0} \qquad \text{...(43)}$$

and in virtue of (13), we have

$$\tau = \tau'/\rho' U_0 v_0 = \left(\frac{du}{dy}\right)_{y=0}$$

$$= \left(\frac{du_0}{dy} + \epsilon e^{i\omega t} \frac{du_1}{dy}\right)_{y=0}. \qquad \text{...(44)}$$

The first term in (44) viz. $\left(\frac{du_0}{dy}\right)_{y=0}$ is the mean skin friction τ_m. We now devote our attention to the unsteady part of it. So substituting for u_1 from (40) in (44), we have

$$\tau = \tau_m + \epsilon e^{i\omega t}\left[l + \frac{AGP\left(\frac{4i}{\omega} + \frac{1}{V}\right)(P - l)}{\left(V - \frac{i\omega}{4}\right)}\right.$$

$$+ \frac{Am(m - l)\left(1 - \frac{G}{V}\right)}{m^2 - m - M - \frac{i\omega}{4}} + \frac{4GAPi(l - n)}{\omega\left\{n^2 - n - M - \frac{i\omega}{4}\right\}} -$$

(*equation continued on p.* 1225)

$$- EG \left\{ \frac{-R}{(P+l)^2 - P(P+l) - \frac{i\omega P}{4}} \left\{ \frac{P}{(P+l)^2 - (P+l) - M - \frac{i\omega}{4}} \right. \right.$$

$$\left. + \frac{(l-n)}{n^2 - n - M - \frac{i\omega}{4}} \right\} + \frac{S}{(P+n)^2 - P(P+n) - \frac{i\omega P}{4}}$$

$$\times \left\{ \frac{(l-P-n)}{(P+n)^2 - (P+n) - M - \frac{i\omega}{4}} - \frac{(l-n)}{n^2 - n - M - \frac{i\omega}{4}} \right\}$$

$$- \frac{4X}{i\omega P} \left\{ \frac{(l-P)}{P^2 - P - M - \frac{i\omega}{4}} - \frac{(l-n)}{n^2 - n - M - \frac{i\omega}{4}} \right\}$$

$$+ \frac{Y}{\left(2P^2 - \frac{i\omega P}{4}\right)} \left\{ \frac{(l-2P)}{4P^2 - 2P - M - \frac{i\omega}{4}} - \frac{(l-n)}{n^2 - n - M - \frac{i\omega}{4}} \right\}$$

$$+ \frac{Z}{4m^2 - 2mP - \frac{i\omega P}{4}} \left\{ \frac{(l-2m)}{4m^2 - 2m - M - \frac{i\omega}{4}} - \frac{(l-n)}{n^2 - n - M - \frac{i\omega}{4}} \right\}$$

$$+ \frac{L}{(P+m)^2 - P(P+m) - \frac{i\omega P}{4}} \left\{ \frac{(l-P-m)}{(P+m)^2 - (P+m) - M - \frac{i\omega}{4}} \right.$$

$$\left. - \frac{(l-n)}{n^2 - n - M - \frac{i\omega}{4}} \right\}$$

$$+ \frac{T}{(m+l)^2 - P(m+l) - \frac{i\omega P}{4}} \left\{ \frac{-m}{(m+l)^2 - (l+m) - M - \frac{i\omega}{4}} \right.$$

$$\left. - \frac{(l-n)}{n^2 - n - M - \frac{i\omega}{4}} \right\}$$

$$+ \frac{U''}{(m+n)^2 - P(m+n) - \frac{i\omega P}{4}} \left\{ \frac{(l-m-n)}{(m+n)^2 - (m+n) - M - \frac{i\omega}{4}} \right.$$

$$\left. - \frac{(l-n)}{n^2 - n - M - \frac{i\omega}{4}} \right\} +$$

(*equation continued on p.* 1226)

$$+ AE\left(\frac{mA'(l-m)}{m^2-m-M-\frac{i\omega}{4}}+\frac{PB'(l-P)}{P^2-P-M-\frac{i\omega}{4}}\right.$$

$$+\frac{2PC'(l-2P)}{4P^2-2P-M-\frac{i\omega}{4}}$$

$$\left.\left.\left.+\frac{2mD'(l-2m)}{4m^2-2m-M-\frac{i\omega}{4}}+\frac{E'(P+m)(l-P-m)}{(P+m)^2-(P+m)-M-\frac{i\omega}{4}}\right)\right\}\right] \quad \ldots(45)$$

where $V = (P^2 - P - M)$.

From the physical point of view, it is convenient to express the skin friction in terms of the amplitude and the phase as

$$\tau = \tau_m + \epsilon \mid B \mid \cos(\omega t + \alpha) \quad \ldots(46)$$

where $B = B_r + iB_i =$ coefficient of $\epsilon e^{i\omega t}$ in (45) and

$$\tan \alpha = B_i/B_r. \quad \ldots(47)$$

Knowing the temperature field, it is important to know the rate of heat transfer between the fluid and the plate and is given by

$$q' = -K\left(\frac{\partial T'}{\partial y'}\right)_{y'=0}. \quad \ldots(48)$$

Hence, in non-dimensional form, it is given by

$$q = -\frac{q'\nu}{K v_0(T' - T'_\infty)} = \left(\frac{d\theta}{dy}\right)_{y=0}$$

$$= \left(\frac{d\theta_0}{dy} + \epsilon e^{i\omega t}\frac{d\theta_1}{dy}\right)_{y=0}. \quad \ldots(49)$$

The first term $q_m = \left(\frac{d\theta_0}{dy}\right)_{y=0}$ in (49) is the mean rate of heat transfer. Substituting for θ_1 from (42) in (49), we have

$$q = q_m + \epsilon e^{i\omega t}\left[\frac{4AP(n-P)i}{\omega} + E\left\{\frac{R(n-P-l)}{(P+l)^2-P(P+l)-\frac{i\omega P}{4}}\right.\right.$$

$$-\frac{SP}{(P+n)^2-P(P+n)-\frac{i\omega P}{4}}-\frac{4X(n-P)}{i\omega P}+\frac{Y(n-2P)}{2P^2-\frac{i\omega P}{4}}+$$

(*equation continued on p.* 1227)

$$+ \frac{Z(n-2m)}{4m^2 - 2mP - \frac{i\omega P}{4}} + \frac{L(n-P-m)}{(P+m)^2 - P(P+m) - \frac{i\omega P}{4}}$$

$$+ \frac{T(n-m-l)}{(m+l)^2 - P(m+l) - \frac{i\omega P}{4}}$$

$$\left. \left. - \frac{U''m}{(m+n)^2 - P(m+n) - \frac{i\omega P}{4}} \right\} \right]. \qquad \ldots(50)$$

We can also write the rate of heat transfer in terms of the amplitude and the phase as

$$q = q_m + \epsilon \mid Q \mid \cos(\omega t + \beta) \qquad \ldots(51)$$

where

$$Q = Q_r + iQ_i = \text{coefficient of } \epsilon e^{i\omega t} \text{ in (50)}$$

and

$$\tan \beta = Q_i/Q_r. \qquad \ldots(52)$$

3. Discussion of Results

Fig. 1 shows the effect of Grashoff number, magnetic field and the variable suction parameter 'A' on the amplitude of the skin friction, when $G > 0$ i.e. when cooling of the plate takes place. This figure shows that for small values of ω, the amplitude of skin friction is very large. When more cooling of the plate takes place the values of $| B |$ diminishes whether $A = 0$ or $A \neq 0$. An increase in the value of magnetic field strength augments the value of $| B |$; but the lower magnetic field strength in conjunction with higher Grashoff number enhances values of $| B |$ more. The effect of 'A', even with such small values as 0.2 and 0.4 is to increase $| B |$ to a great extent (cf. Soundalgekar 1975). Fig. 2 shows those effects when $G < 0$ i.e., when the heating of the plate takes place. When compared with Fig. 1, it shows that the larger values of $| B |$ in this case are much smaller. In this case the amplitude of the skin friction is not always large for small values of ω. When more heating of the plate takes place, the value of $| B |$ decreases for smaller values of ω, but for larger values, $| B |$ increases. But peculiarly it is seen that for higher values of 'A', $| B |$ decreases almost for all values of ω ($>$ 1.8 for mercury). Here also stronger the magnetic field, larger the value $| B |$. 'A' effect cannot be definitely concluded. G effect is quite noticable if we compare I and II of Fig. 2.

The phase of the skin friction $\tan \alpha$, is shown against ω in Figs. 3 and 4. In case of cooling of the plate by the free convection currents, it is shown in Fig. 3 and Tables I and II. Since the phase is always positive for all values of ω and M, we conclude that there is always a phase lead. Further it is noted that stronger

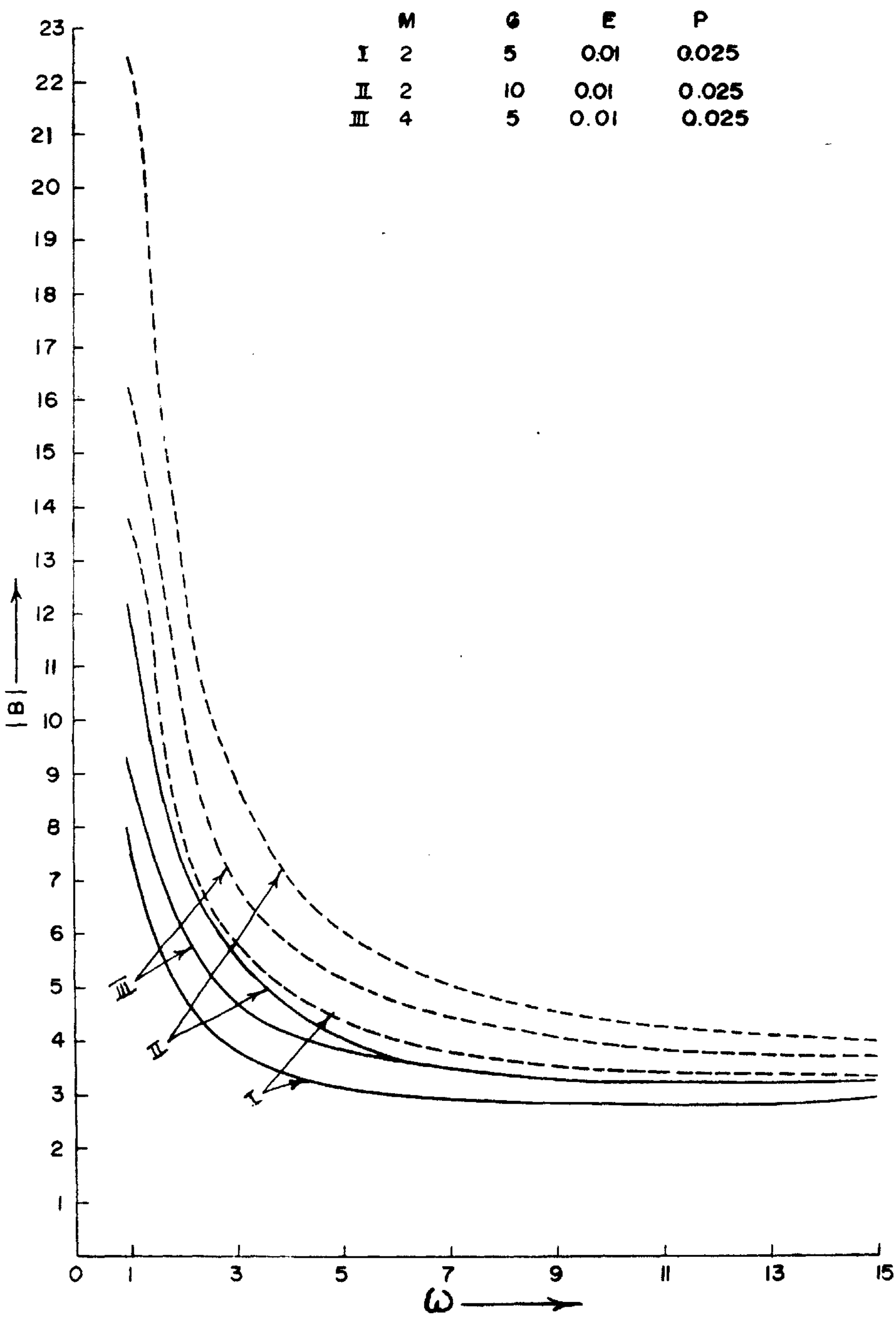

FIG. 1. Amplitude of skin friction $A = 0.2$ ——, $A = 0.4$ - - - - .

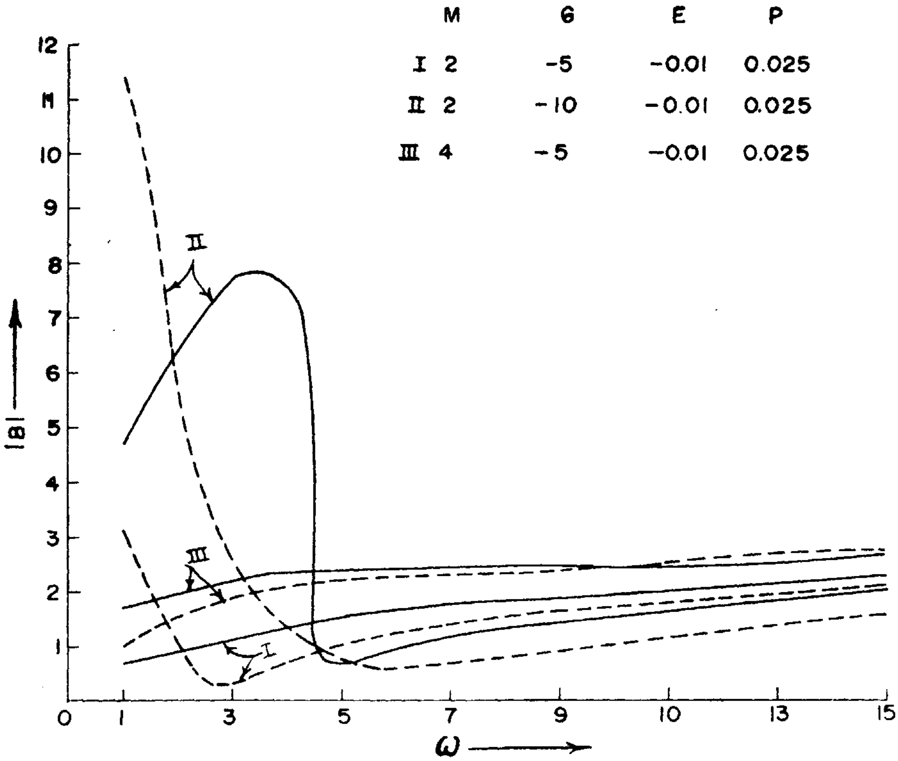

FIG. 2. Amplitude of skin friction $A = 0.2$ ——, $A = 0.4$ - - - - .

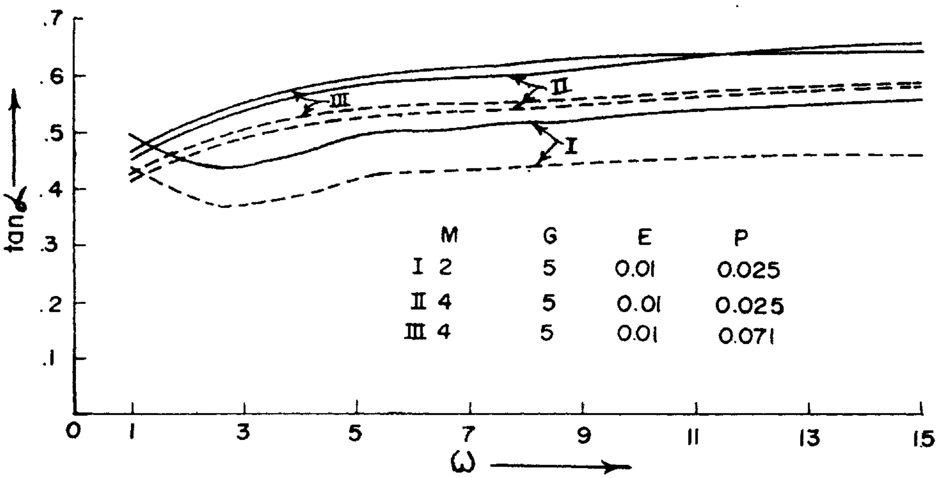

FIG. 3. Phase of skin friction $A = 0.2$ ——, $A = 0.4$ - - - - .

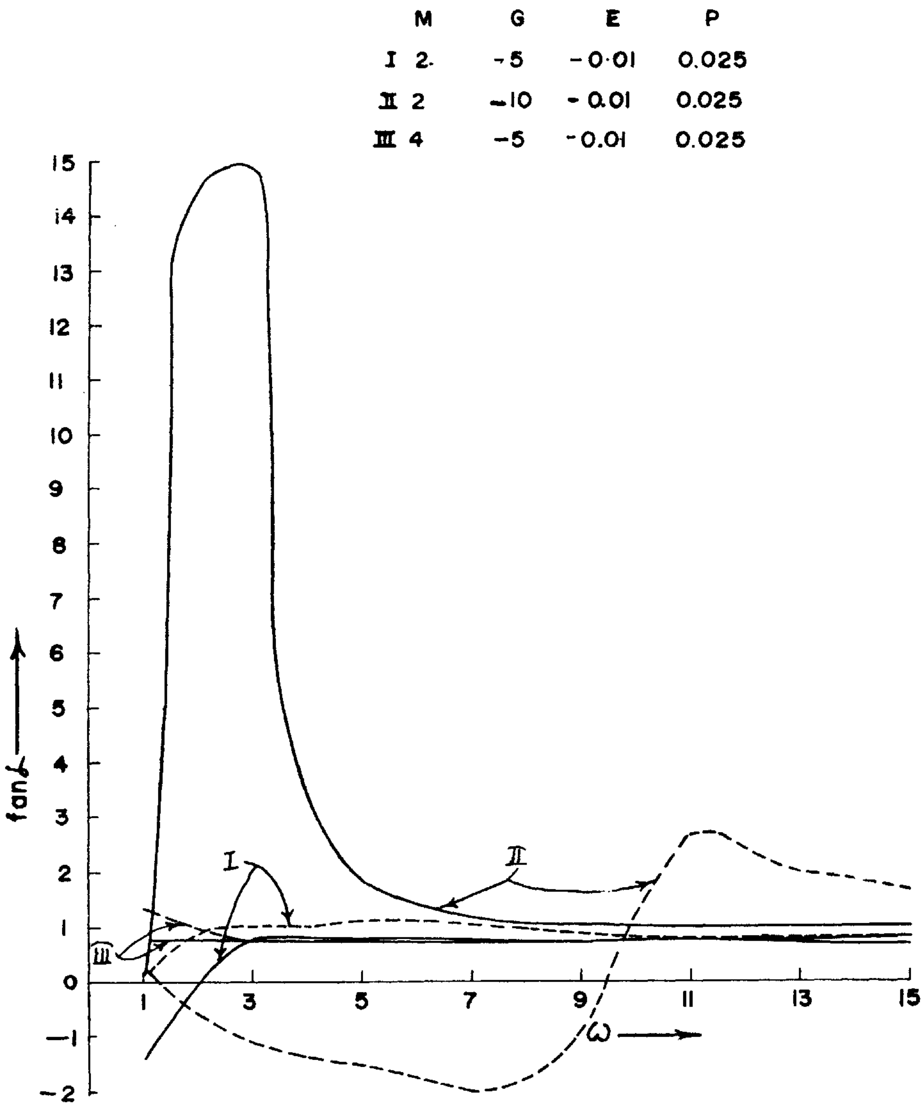

FIG. 4. Phase of skin friction $A = 0.2$ ——, $A = 0.4$ - - - - .

magnetic field diminishes skin friction phase. It is also evident that 'A' effect is to annul ω effect on the phase of skin friction and the graphs both for $A = 0.2$ and 0.4 are almost parallel to ω-axis. Fig. 4 is shown where the plate is being heated by the free convection current. Here also as in the previous case there is phase-lead almost everywhere. When the plate is less heated due to convection, the fluctuation of the curves is quite large. But where greater heating of the plate takes place, 'A' effect is to annul the ω effect of the skin friction and the higher value of the magnetic field strength is not much effective to bring down the value further.

TABLE I

Values of $|B|$ *and* $\tan \alpha$ *when* $E = 0.01$, $P = 0.071$, $A = 0.3$, $G = 5$

	Values of $|B|$		Values of $\tan \alpha$	
ω \ M	2	4	2	4
1	10.7962	12.6786	0.468283	0.44359
3	4.8216	5.88915	0.431371	0.524896
5	3.78527	4.55391	0.468182	0.558902
7	3.38975	4.01709	0.481640	0.581393
9	3.21672	3.74604	0.491355	0.595481
11	3.14510	3.59906	0.501102	0.605737
13	3.12688	3.52042	0.511586	0.614122
15	3.13935	3.48353	0.522679	0.621604

TABLE II

Values of $\tan \alpha$ *when* $E = 0.01$, $P = 0.071$, $A = 0.3$, $G = 10$

ω \ M	2	4
1	0.453107	1.11899
3	2.10727	0.521249
5	0.384795	0.549085
7	0.413900	0.573724
9	0.420936	0.583500
11	0.425325	0.588071
13	0.430707	0.590734
15	0.437647	0.593015

TABLE III

Values of amplitude of rate of heat transfer $(= |Q|)$ *for* $E = -0.01$, $P = 0.025$

ω \ A	0.2	0.4	0.2	0.4	0.2	0.4
	$M = 2, G = -5$		$M = 2, G = -10$		$M = 4, G = -5$	
1	0.111629	0.109794	0.109831	0.105433	0.111697	0.110259
3	0.0678421	0.0670442	0.0681405	0.0668148	0.0676449	0.0667611
5	0.0534481	0.0527087	0.0536170	0.0523932	0.0532152	0.0525539
7	0.0455769	0.0449396	0.0457988	0.0447139	0 0453247	0.0447506
9	0.0404424	0.0398727	0.0407073	0.0397155	0.0401787	0.0396675
11	0.0367528	0.0362319	0.0370540	0.036129	0.0364785	0.0360133
13	0.0339360	0.0334524	0.0342696	0.0333955	0.0336517	0.0332218
15	0.0316943	0.0312405	0.0320573	0.0312241	0.0314004	0.0309987

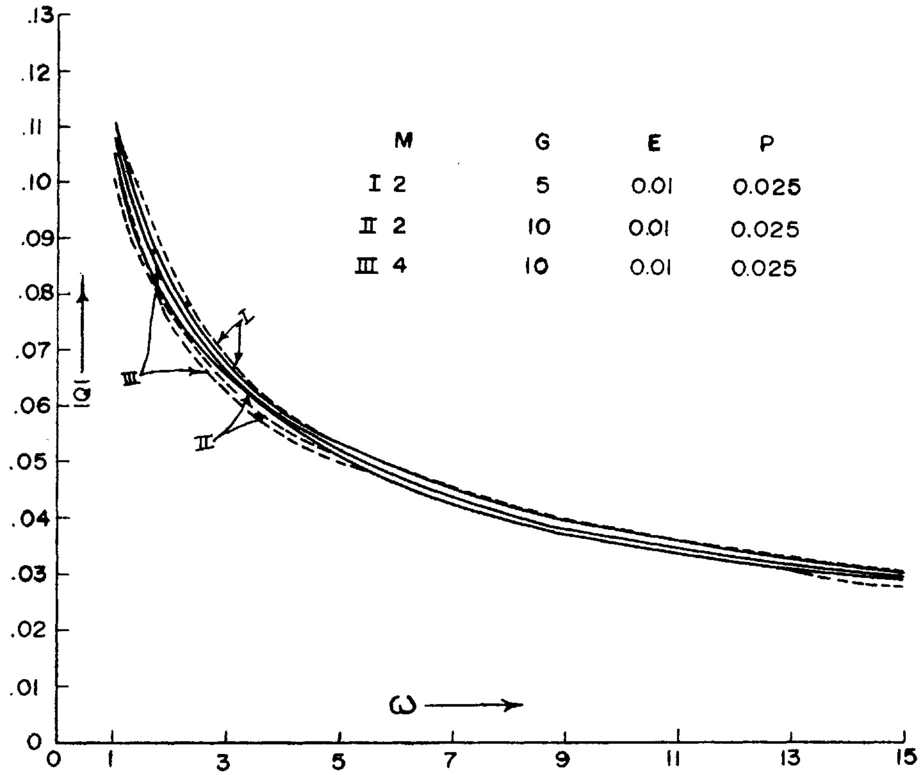

FIG. 5. Amplitude of rate of heat transfer $A = 0.2$ ——, $A = 0.4$ - - - - .

The amplitude of the rate of heat transfer $|Q|$ is shown in Fig. 5 for the cooling of the plate and in Table III in case of heating of the plate by the free convection current. Figure 5 and Table III show that the rate of heat transfer is appreciably affected by the frequency ω during cooling but during heating, rate of heat transfer is affected for smaller values of ω and not for larger values of ω (vide Table III). Greater heating of the plate leads to an increase in $|Q|$, but greater cooling of the plate does not affect much of $|Q|$ value (excepting for some small values of ω).

The phase $\tan \beta$ of the rate of heat transfer is shown in Figs. 6 and 7. $\tan \beta$ is observed to be negative for all values of ω both for $G > 0$ and $G < 0$. The graphs (when $G \gtrless 0$, $A = 0$), which are almost convex in nature now become concave in the lower half of the co-ordinate plane. Hence numerically $\tan \beta$ increases for both $G > 0$ and $G < 0$.

CONCLUSION

The main object of this paper is to study the effect of 'A' (variable suction parameter) on the skin friction and the rate of heat transfer in conjunction with

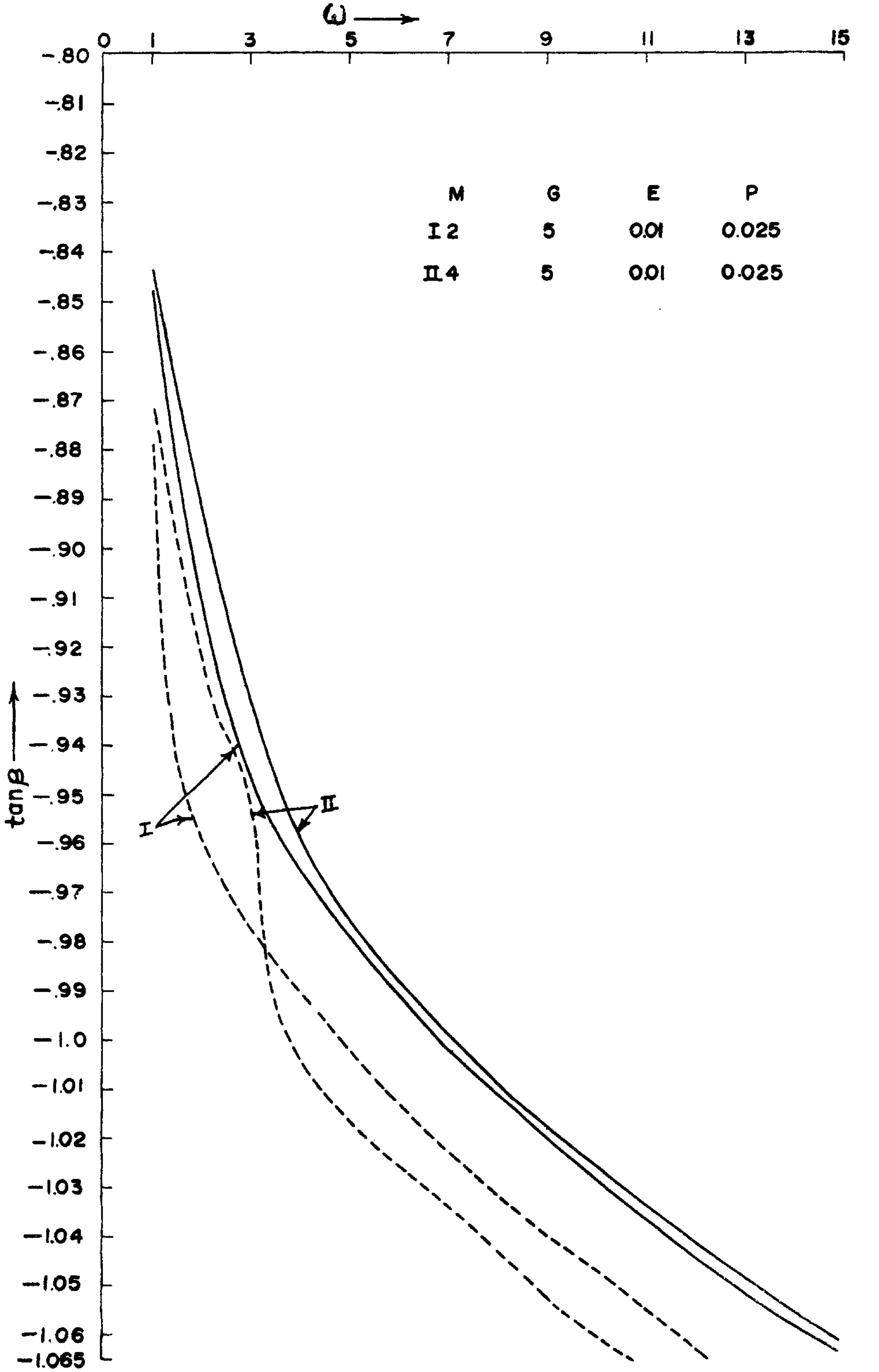

FIG. 6. Phase of rate of heat transfer $A = 0.2$ ——, $A = 0.4$ - - - - .

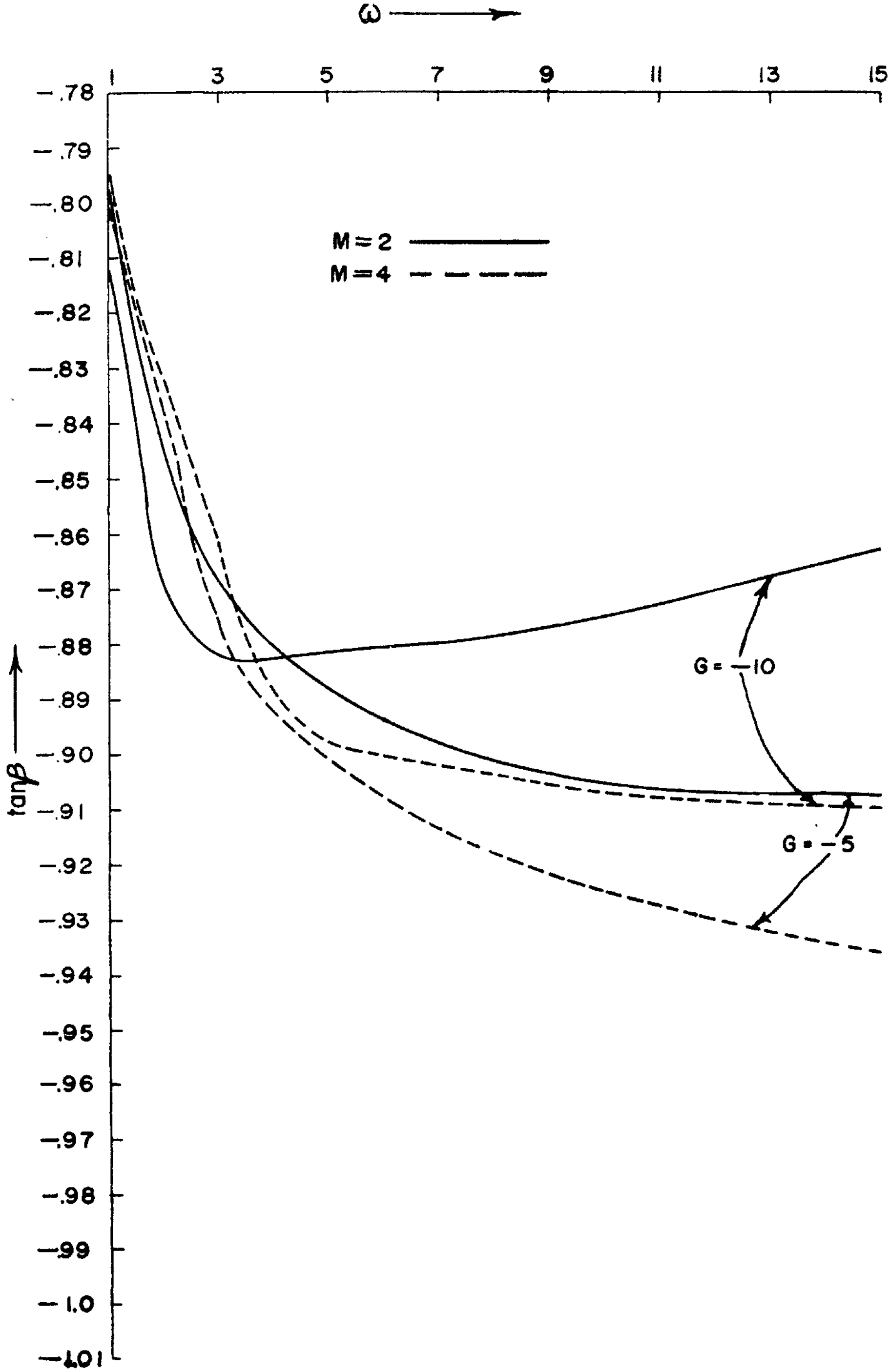

FIG. 7. Phase of rate of heat transfer $E = -0.01$, $P = 0.025$, $A = 0.2$.

other parameters governing hydromagnetic flow. Some important results we summarise as follows :

(1) amplitude $|B|$ of the skin friction increases even for small increment of the parameter A, when $G > 0$;

(2) $|B|$ decreases for all values of ω ($>$ 1.8 for mercury) when 'A' has higher values;

(3) 'A' effect is to annul ω effect on the phase of the skin friction when $G > 0$. For $G < 0$, the same conclusion holds, when the plate is more heated;

(4) when $G > 0$ or $G < 0$, rate of heat transfer is affected by ω, but for $A = 0$, it is not affected.

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