COUETTE FLOW IN HYDROMAGNETICS WITH TIME-DEPENDENT SUCTION

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The flow of a viscous, incompressible and electrically conducting fluid between two infinite parallel walls one of them moving with a uniform velocity, is considered when a magnetic field of constant strength acts perpendicular to the walls. The moving wall is subjected to a suction whose magnitude oscillates with respect to time over a constant mean. The effect of the magnetic field and suction frequency on fluctuating components of the skin friction and the current density at the walls is studied.

Introduction

The steady and unsteady Couette-type flows in hydrodynamics and hydromagnetics, subject to wall suction, have been discussed by several authors (Cramer 1959, Verma and Bansal 1966, Ramamoorty 1962, Rathy 1963, Shukla 1963). Katagiri (1962) has studied the Couette flow formation in MHD. Neglecting the induced field he has solved the momentum equation by the method of Laplace transform. Muhuri (1963) has considered the more general case wherein the velocity of the moving wall varies as $(\text{time})^n$ and the walls are subjected to uniform suction or injection. Balaram and Govindarajulu (1973) have obtained exact solutions for the same problem taking the induced field into account and for the magnetic Prandtl number $P_m = 1$. The present investigation is devoted to study the effect of suction on the skin friction and current density at the plate when the suction velocity varies periodically over a constant mean, and one of the walls moves with uniform velocity.

Basic Equations and Solutions

The basic equations of MHD neglecting the displacement currents and free charges are

$$\nabla . \bar{H} = 0 \qquad \text{...(1)}$$

$$\nabla \times \bar{H} = \bar{j} \qquad \text{...(2)}$$

$$\nabla \times \bar{E} = - \mu \frac{\partial \bar{H}}{\partial t} \qquad \text{...(3)}$$

Ohm's law

$$\bar{j} = \sigma(\bar{E} + \mu\bar{V} \times \bar{H}) \qquad ...(4)$$

the equation of continuity

$$\nabla \cdot \bar{V} = 0 \qquad ...(5)$$

and the momentum equation

$$\frac{\partial \bar{V}}{\partial t} + (\bar{V} \cdot \nabla)\bar{V} = -\rho^{-1}\nabla p + \nu\nabla^2\bar{V} + \mu\rho^{-1}\bar{j} \times \bar{H}. \qquad ...(6)$$

Here $\bar{V}$ is the velocity of the fluid, $\bar{H}$ the magnetic field, $\bar{E}$ the electric field and $\bar{j}$ the current density. ρ, μ, ν, σ and p denote respectively the density, magnetic permeability, kinematic viscosity, electric conductivity and fluid pressure.

Eliminating $\bar{E}$ and $\bar{j}$ from eqns. (2) - (4) we get

$$\frac{\partial \bar{H}}{\partial t} = \nabla \times (\bar{V} \times \bar{H}) + \eta\nabla^2\bar{H} \qquad ...(7)$$

where $\eta = (\mu\sigma)^{-1}$ is the magnetic diffusivity.

We consider the shear flow between two infinite parallel walls at a distance h apart. The lower wall moves with a uniform velocity U while the other is at rest. A suction whose magnitude oscillates in time over a constant mean is applied to the moving wall. A uniform magnetic field H_0 is imposed normal to the walls. The fluid is assumed to be viscous, incompressible and electrically conducting whereas the walls are taken to be non-conducting.

We take the x-axis along the lower wall and the y-axis normal to it. Since the plates are infinite in extent all quantities are functions of y and t only. We have $\bar{V} = (u, v, 0)$ and $\bar{H} = (H_x, H_y, 0)$. From eqn. (1) we get $H_y = H_0$, the applied field. Equation (5) gives

$$v(t) = -U(1 + \epsilon e^{i\omega t}),\ \epsilon \ll 1 \qquad ...(8)$$

which is the velocity of suction consisting of a basic steady value U with a weak time-varying component. Using (8) eqns. (6) and (7) may now be written in the nondimensional form,

$$\tfrac{1}{4}\frac{\partial u}{\partial t} - (1 + \epsilon e^{i\omega t})\frac{\partial u}{\partial y} = \frac{1}{R}\frac{\partial^2 u}{\partial y^2} + S^2\frac{\partial H}{\partial y} \qquad ...(9)$$

$$\tfrac{1}{4}\frac{\partial H}{\partial t} - (1 + \epsilon e^{i\omega t})\frac{\partial H}{\partial y} - \frac{\partial u}{\partial y} = \frac{1}{R_m}\frac{\partial^2 H}{\partial y^2} \qquad ...(10)$$

where $R = Uh/\nu$ and $R_m = Uh/\eta$ are the viscous and magnetic Reynolds numbers and $S = (\mu H_0^2/\rho U^2)^{1/2}$ is the magnetic pressure number.

The boundary conditions are

$$\left.\begin{aligned} y = 0 &: u = 1,\ H = 0 \\ y = 1 &: u = 0,\ H = 0. \end{aligned}\right\} \qquad \ldots(11)$$

We seek solutions of eqns. (9) and (10) for the particular case $R = R_m = 1$. For small values of ϵ, we write

$$\left.\begin{aligned} u(y, t) &= u_1(y) + \epsilon u_2(y)\, e^{i\omega t} \\ H(y, t) &= H_1(y) + \epsilon H_2(y)\, e^{i\omega t}. \end{aligned}\right\} \qquad \ldots(12)$$

The functions u_1, u_2, H_1 and H_2 then satisfy the set of equations

$$\left.\begin{aligned} & u_1'' + u_1' + S^2 H_1' = 0 \\ & H_1'' + H_1' + u_1' = 0 \\ & u_2'' + u_2' - \frac{i\omega}{4}\, u_2 + S^2 H_2' = -\, u_1' \\ & H_2'' + H_2' - \frac{i\omega}{4}\, H_2 + u_2' = -\, H_1' \end{aligned}\right\} \qquad \ldots(13)$$

with the boundary conditions

$$\left.\begin{aligned} y = 0 &: u_1 = 1,\ u_2 = 0,\ H_1 = H_2 = 0 \\ y = 1 &: u_1 = u_2 = 0,\ H_1 = H_2 = 0. \end{aligned}\right\} \qquad \ldots(14)$$

Here the prime denotes differentiation with respect to y. Equations (13) subject to (14) have been solved. Based on this we find that the skin friction τ and the current density j on the walls are given by

$$\frac{\tau_1}{\rho U^2} = \left(\frac{\partial u}{\partial y}\right)_{y=0} = -\,\{a(1 + S) + b(1 - S)\} + \epsilon \mid M_1 \mid e^{i(\omega t+\theta_1)} \qquad \ldots(15)$$

$$\frac{\tau_2}{\rho U^2} = \left(\frac{\partial u}{\partial y}\right)_{y=1} = -\,\{a(1 + S)\, e^{-(1+S)} + b(1 - S)\, e^{-(1-S)}\} + \epsilon \mid M_2 \mid e^{i(\omega t+\theta_2)} \qquad \ldots(16)$$

$$\frac{hj_1}{H_0} = -\left(\frac{\partial H}{\partial y}\right)_{y=0} = S^{-1}\,\{a(1 + S) - b(1 - S) - \epsilon \mid N_1 \mid e^{i(\omega t+\gamma_1)}\} \qquad \ldots(17)$$

$$\frac{hj_2}{H_0} = -\left(\frac{\partial H}{\partial y}\right)_{y=1} = S^{-1}\,\{a(1 + S)\, e^{-(1+S)} - b(1 - S)\, e^{-(1-S)} - \epsilon \mid N_2 \mid e^{i(\omega t+\gamma_2)}\} \qquad \ldots(18)$$

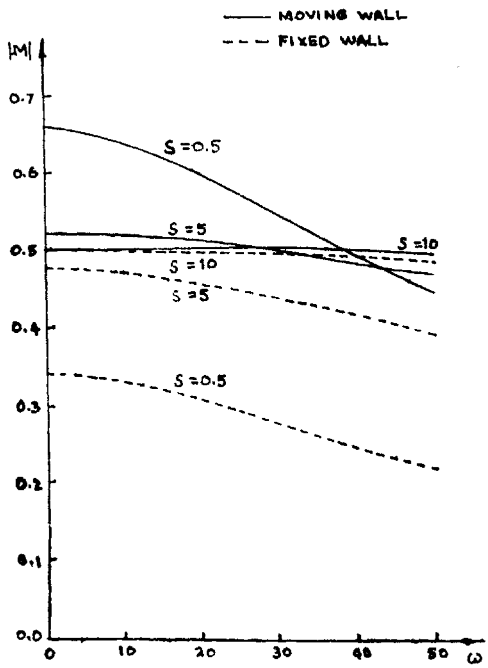

FIG. 1. Amplitude of skin friction.

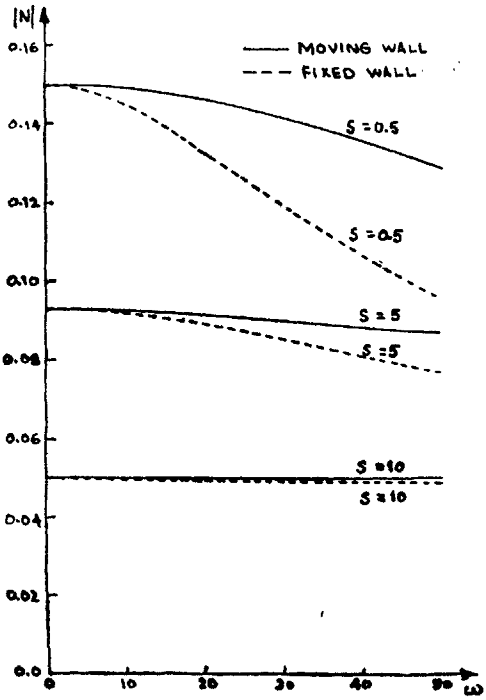

FIG. 2. Amplitude of current density.

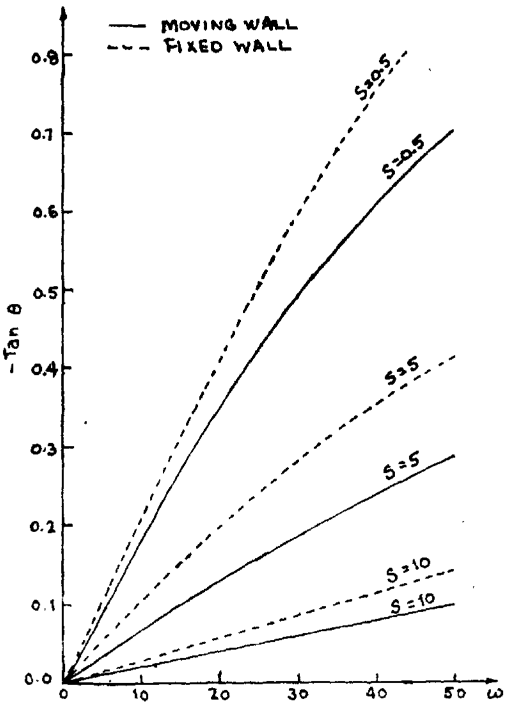

FIG. 3. Phase of skin friction.

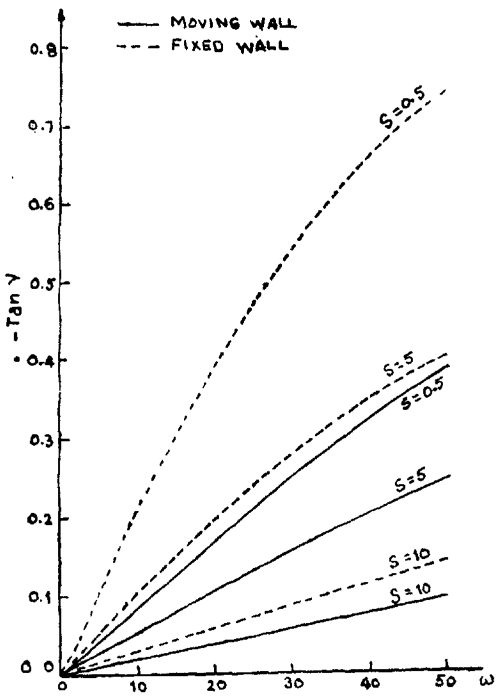

FIG. 4. Phase of current density.

where

$$2a = \{1 - e^{-(1+S)}\}^{-1/2}, \; 2b = \{1 - e^{-(1-S)}\}^{-1/2}$$

and M_i, N_i, θ_i and γ_i are functions of S and ω.

Results

The parameters which affect the flow characteristics are S and ω. To study the behaviour of the fluctuating components of the skin friction and the current density at the walls, their amplitudes and phases are plotted against the frequency in Figs. 1–4 for $S = 0.5, 5, 10$. We find that the amplitudes of the skin friction and the current density at the fixed wall is always less than at the moving one for all frequencies excepting that for higher values of S they nearly become equal and uniform. As S increases the amplitude of the skin friction at the wall in motion decreases for small frequencies but increases for higher frequencies and at the other wall it increases steadily. On the other hand the amplitude of the current density at both the walls decreases as S increases. As the frequency of the suction velocity increases the amplitude decreases, the decrease being more rapid for smaller values of S. The phase lags of the skin friction and the current density increase with frequency but decrease as S increases and are greater at the stationary wall.

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