EXTENSIONS AND SPECIAL CASES OF TRANSPORTATION PROBLEM : A SURVEY

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(*Received* 2 *December* 1977)

The classical transportation problem is one of the many well-structured problems in the mathematical programming literature that has generated considerable interest. The purpose of this survey is to provide an up-to-date account of the theoretical and computational aspects of various special cases and extensions of the transportation problem.

1. INTRODUCTION

The classical transportation problem is one of the many well-structured problems in operations research that has been extensively studied in the literature. Other examples are the travelling salesman and shortest-route problems. As noted, the transportation problem is one of the subclasses of the linear programming problems for which simple and practical computational procedures have been developed that take advantage of the special structure of the problem. The transportation problem is probably the most important special linear programming problem—in terms of relative frequency with which it appears in the applications and also in the simplicity of the procedures developed for its solution. Among the many other important features of the problem, the following are considered to be most important. First, transportation problems were the earliest class of linear programs discovered to have totally unimodular matrices and integral extreme points resulting in considerable simplification of the simplex method. Second, study of the transportation problems laid the foundation for further theoretical and algorithmic development of the minimal cost network flow problems. Third, and most important, are the many applications in distribution scheduling, etc.

There are different types of transportation problems and the simplest of them that is now standard in the literature was first presented by Hitchcock (1941), along with a constructive solution and, later independently, by Koopman (1947). Koopman began to spearhead research on the potentialities of linear programs for the study of the problems in economics. His historic paper "Optimum Utilization of the Transportation Systems" was based on his war time experience. Because of this and the

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work done earlier by Hitchcock, the classical case is often referred as the Hitchcock-Koopman's transportation problem. Kantorovich (1942) published a paper on a continuous version of the problem and later with Gavurin, an applied study of the capacitated transportation problem (Kantorovich and Gavurin 1949).

The Hitchcock-Koopman's transportation problem may be expressed as the minimization of transport costs for moving a single commodity from m origins (sources) to n destinations (sinks) while operating within supply and demand constraints. The respective linear programming model is as follows:

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} = a_i, \quad i = 1, 2, \ldots, m \text{ (supply)} \qquad \ldots(1)$$

$$\sum_{i=1}^{m} x_{ij} = b_j, \quad j = 1, 2, \ldots, n \text{ (demand)} \qquad \ldots(2)$$

$$x_{ij} \geqslant 0 \text{ for all } i \text{ and} \qquad \ldots(3)$$

where

x_{ij} = the amount of goods moved from origin i to destination j

C_{ij} = the cost of moving a unit amount goods from origin i to destination j

a_i = the supply available at each origin i

b_j = the demand for goods at each destination j

m = total number of origins (sources)

n = total number of destinations (sinks).

For a feasible solution to exist it is necessary that total supply equals total demand ($\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$). If in reality, supply is greater than demand

$$\left(\sum_{i=1}^{m} a_i > \sum_{j=1}^{n} b_j \right),$$

then a fictitious destination may be used to create the desired equality. If demand exceeds supply ($\sum_{j=1}^{n} b_j > \sum_{j=1}^{m} a_i$), then a fictitious source may be introduced.

The purpose of this paper is to survey theoretical and computational aspects of various special cases and extensions of the transportation problem. However, no claim is made as to completeness of this survey.

The paper is organized as follows : In Section I, we survey a number of special cases and extensions of the problem. Section II is devoted to the multi-index transportation problem and Section III is devoted to a survey of time-cost trade-off in transportation problems.

SECTION I : EXTENSIONS AND SPECIAL CASES

The time-minimizing, or bottleneck, transportation problem is one in which a time is associated with each shipping route. Rather than minimizing cost, the objective is to minimize the maximum time to transport all supply to the destinations. Hammer (1969) introduced the problem and an algorithm in the English literature. Szwarc (1971) noted that the problem was originally proposed ten years earlier by a Russian named Bar Sow and that several related papers were published in East European journals. He also corrects some errors in Hammer's original paper. Garfinkel and Rao (1971) present a primal approach and a threshold algorithm which generates a sequence of improving lower bounds on the maximum time. Hammer (1971) notes the work of Szwarc and Garfinkel and Rao in correcting his original paper. Recently Sharma and Swarup (1978) and Bhatia *et al.* (1974) have given iterative methods for the solution of time minimizing transportation problem.

Mathematically stated, suppose we are given to transport certain commodity from m origins to n destinations. Let a_i $(i = 1, 2, ..., m)$ be the quantity available at the ith origin and b_j $(j = 1, 2, ..., n)$ be the quantity required at the jth destination. Let t_{ij} be the time (in days, hours, or some convenient unit) required to transport a load however big or small from the ith origin to the jth destination. The problem is to find an optimum transportation scheme so as to minimize the total time required for delivery, i.e.

$$\text{Minimize } [\max_{(i,j)} t_{ij} : x_{ij} > 0]$$

subject to constraints (1) – (3), and $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$.

Bhatia *et al.* (1976a) present an algorithm for the solution of time-minimizing multi-index transportation problem having three indices which is known as solid transportation problem viz.

$$\text{Minimize } [\max_{(i,j,k)} t_{ijk} : x_{ijk} > 0]$$

subject to

$$\sum_{i=1}^{m} x_{ijk} = A_{jk} \quad \text{...(4)}$$

$$\sum_{j=1}^{n} x_{ijk} = B_{ki} \quad \text{...(5)}$$

$$\sum_{k=1}^{p} x_{ijk} = E_{ij} \quad \text{...(6)}$$

where

$$\sum_{=1}^{n} A_{jk} = \sum_{i=1}^{m} B_{ki}, \sum_{k=1}^{p} A_{jk} = \sum_{i=1}^{m} E_{ij}, \sum_{k=1}^{p} B_{ki} = \sum_{j=1}^{n} E_{ij} \quad \text{...(7)}$$

and

$$\sum_{j=1}^{n} \sum_{k=1}^{p} A_{jk} = \sum_{k=1}^{p} \sum_{i=1}^{m} B_{ki} = \sum_{i=1}^{m} \sum_{j=1}^{n} E_{ij} \quad \text{...(8)}$$

$$x_{ijk} \geqslant 0. \quad \text{...(9)}$$

Sharma and Swarup (1977a) present an iterative method for solving a time minimizing multidimensional transportation problem having three indices viz.

$$\text{Minimize } [\max_{(i,j,k)} t_{ijk} : x_{ijk} > 0]$$

subject to

$$\sum_{j=1}^{n} \sum_{k=1}^{p} x_{ijk} = a_i \quad \text{...(10)}$$

$$\sum_{i=1}^{m} \sum_{k=1}^{p} x_{ijk} = b_j \quad \text{...(11)}$$

$$\sum_{i=1}^{m} \sum_{j=1}^{n} x_{ijk} = c_k \quad \text{...(12)}$$

$$x_{ijk} \geqslant 0 \quad \text{...(13)}$$

and

$$\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j = \sum_{k=1}^{p} c_k. \quad \text{...(13a)}$$

Sharma and Swarup (1978a) present a transportation technique for time minimization in fractional functional programming problem with an objective function of the form

$$\text{Minimize } \frac{\sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij} x_{ij}}{[\max_{(i,j)} t_{ij} : x_{ij} > 0]} \quad \text{...(14)}$$

subject to constraints (1) – (3), and $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$.

Merrill and Tobin (1969) consider the problem in which total supply exceeds total demand and some C_{ij} are negative. In this case, it may be optional to overship, an algorithm is developed.

Shetty (1959) presents an iterative primal method for solving problems with a non-linear objective function of the form

$$\sum_{i=1}^{m} \sum_{j=1}^{n} C_{ij}x_{ij} + \sum_{i=1}^{m} P_i \left(\sum_{j=1}^{n} x_{ij} \right)$$

where P_i are continuous and non-decreasing functions. Beale (1959) develops a local search rule for problems with convex arc costs and proves that the local optima obtained are global.

AuCamp (1972) shows how to reduce a problem with a penalty for unsatisfied demand to an ordinary transportation problem.

Szwarc (1975) developed a non-iterative method for solving problems whose costs are of the form $C_{ij} = u_i + v_j$ with applications in shiploading and scheduling. Leu (1972) proposed a non-iterative method for a class of problems with tridiagonal admissible cell structure.

Klingman and Russell (1975) have developed an efficient procedure for solving transportation problems with additional linear constraints. Their method exploits the topological properties of basis trees within a generalized upper bounding framework.

DeMaio and Roveda (1971) and Srinivasan and Thompson (1973) proposed implicit enumeration and branch and bound algorithms for problems in which each destination is supplied by a single source and later Murthy (1976) considered the same problem and provided an efficient algorithm.

With a more practical flavour, Lee and Moore (1973) investigated goal programming procedures in problems with multiple objectives. Fong and Srinivasan (1974) developed an algorithm for determining all non-degenerate shadow princes in a transportation problem, i.e. those that are valid over a non-zero range and have a meaningful interpretation. Fong and Rao (1973) investigated aspects of parametric problems as related to logistics systems.

The fixed-charge transportation problem has been extensively studied. Since a survey devoted exclusively to this topic has recently appeared (Bair and Hefley 1973), no further consideration of this subject will be made here.

The generalized transportation problem (GTP) can be formulated as

$$\text{Minimize} \sum_{i=1}^{m} \sum_{=1}^{n} C_{ij}x_{ij}$$

subject to

$$\sum_{j=1}^{n} x_{ij} \leqslant a_i$$

$$\sum_{i=1}^{m} p_{ij} x_{ij} = b_j$$

$$x_{ij} \geqslant 0.$$

Although more difficult to solve than ordinary transportation problems, one can develop efficient algorithms for the GTP that exploit the topological structure of the basis. Laurie (1964) and Balas and Hammer (1964) discuss some theoretical aspects of the problem including basis structure and pivoting procedures. A dual method was developed by Balas (1966). Glover and Klingman (1973) discuss efficient procedures for computing dual prices and for pivoting.

Stochastic transportation problems, i.e., problems with stochastic demand and penalities for over supply and under supply, have been discussed by Williams (1963) and Szwarc (1964). The objective in such problems is to minimize total transportation costs plus expected penality costs. Recently, Wilson (1972, 1973, 1975) showed that a linear approximation can be used in order to solve the problem as an ordinary capacitated transportation problem.

Transportation problem with fractional functional objective function is studied by Swarup (1966) and later Corban (1973) extended the concept for multi-dimensional transportation problem.

Swarup (1970) developed a technique, similar to transportation technique in linear programming to minimize a locally indefinite quadratic function, subject to the constraints (1) - (3). Conditions for local optimality have been obtained. Sharma and Swarup (1977b) have developed the same concepts for multi-dimensional transportation problem.

Sharma (1976) has obtained optimality criteria for maximizing a particular convex programming problem with objective function of the form

$$\frac{\left(\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} C_{ijk} x_{ijk}\right)^2}{\left(\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} d_{ijk} x_{ijk}\right)}$$

subject to the constraints (10) - (13). Transportations problem with non-linear objective function is studied by Sharma (1973). Sharma (1978a) extended the concept for multidimensional transportation problem.

SECTION II : MULTI-INDEX TRANSPORTATION PROBLEMS

The Hitchcock-Koopman's transportation problem is usually conceived of as a two-dimensional problem. Schell (1955) and Galler and Dwyer (1957) considered the multi-index transportation problem which can be thought of as an n-dimensional problem, where $n \geqslant 3$. This problem is closely related to the multi-commodity transportation problem (MCTP) as it is known today was introduced by Haley (1962, 1963, 1965) called the solid (or multi-index) transportation problem. In such problems, the central variables are of the form $x_{ij \ldots k}$, rather than x_{ij} as in the Hitchcock transportation problem. In 1955 Schell has considered the three-dimensional transportation problem in the context of a soap manufacturer with m factories (index i), p types of soap (index k) non-overlapping sales areas (index j) and considers unit costs, C_{ijk}, which might include such items as raw materials costs (if different from plant to plant), variable manufacturing costs, shipping costs to the jth area, and local advertising costs.

Schell considers four cases of the three-dimensional problem, as represented by the nature of the given restrictions namely :

1. Three planar sums
2. Two planar sums
3. One planar and one axial sum
4. Three axial sums.

For the case of three planar sums, the problem may be stated as follows:

Determine $x_{ijk} \geqslant 0$ for all i, j, k which minimize

$$Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} C_{ijk} x_{ijk} \qquad \ldots(15)$$

subject to the constraints (4) – (8).

Schell shows that the above stated three-dimensional transportation problem will have at most $mnp - (m-1)(n-1)(p-1)$ independent relations and, accordingly, the minimum number of zero entries will be given by the symmetric functions with alternating signs :

$$mnp - (mn + np + mp) + (m + n + p - 1)$$

and states that these results can be generalized to higher dimensional problems. Schell further shows that, unlike the two-dimensional case, feasible solutions do not always exist, but develops a procedure for constructing an initial basic feasible solution, if one exists. Finally for a particular example Schell shows how to proceed from a basic feasible solution to the optimum feasible solution, using a slight generalization of the Dantzig (1951) transportation technique.

Haley (1962) proposed an extension of the MODI method for its solution. He gave a comparison between solid problem and transportation problem and developed a procedure for constructing an initial basic feasible solution, if one exists by taking a small numerical example. He also dealt with the case of degeneracy in the problem.

The multi-commodity transportation problem is usually expressed as follows :

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} C_{ijk} x_{ijk}$$

subject to

$$\sum_{i=1}^{m} x_{ijk} = A_{jk}$$

$$\sum_{j=1}^{n} x_{ijk} = B_{ki}$$

$$\sum_{k=1}^{p} x_{ijk} \leqslant E_{ij}$$

$$x_{ijk} \geqslant 0$$

where

$$\sum_{j=1}^{n} A_{jk} = \sum_{i=1}^{m} B_{ki}$$

one can easily show that an $(m, n, p+1)$ MITP is equivalent to an (m, n, p) MCTP. Some special theoretical properties have been derived concerning the MCTP. Rebman (1974) showed that two commodity problem has a totally unimodular constraint matrix if and only if the set of arcs with finite capacities E_{ij} forms a tree in the graph. Evans *et al.* (1975) showed that for an m-source, 2-sink, r-commodity problem, $(m, r \geqslant 2)$, the MCTP can be realized as an equivalent single commodity transportation problem by the following constraints :

$$-\sum_{i=1}^{m} x_{i1k} = -A_{1k}$$

$$\sum_{j=1}^{n} x_{ijk} = B_{ki}$$

$$-\sum_{k=1}^{p} x_{i2k} - S_{i2} = -E_{i2}$$

$$S_{i1} + S_{i2} = E_{i1} + E_{i2} - B_{i1} - B_{i2}$$

$$-\sum_{i=1}^{m} S_{i1} = -\sum_{j=1}^{n} E_{j1} + \sum_{k=1}^{p} A_{1k}$$

where S_{ij} are slack variables. Truemper (1976) proved that the MCTP is unimodular if and only if $m = 2$ or $n = 2$, correcting the claim of total unimodularity by Evans *et al.* (1975). Evans (1976) further showed that an $(m, 2, r)$ MCTP is equivalent to an ordinary capacitated transportation problem.

Kennington (1975) specialized the generalized upper bounding procedure to the MCTP. The largest problem solved was $m = n = r = 13$. He also showed that any multi-commodity minimal cost flow problem can be transformed to an equivalent MCTP, a result similar to Ford and Fulkerson for single commodity problems. A subsequent paper with Shalaby (Kennington and Shalaby 1975) developed a resource-directed heuristic using sub-gradient optimization.

The problem with three-axial sums is to :

Determine $x_{ijk} \geqslant 0$ for all i, j, k which minimize

$$\text{Minimize } Z = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{p} C_{ijk} x_{ijk} \qquad \ldots(16)$$

subject to the constraints (10) - (12).

An economic interpretation of this problem given by Corban (1964) is the following : C_{ijk} is the cost of transporting a unit of product from origin i to destination j by means of transportation k; x_{ijk} is the amount of product transferred from origin i to destination j by means of transportation k; a_i is the supply available at the origin i; b_j is the amount of product required at the destination j and c_k is the possible amount to be transferred with the kth means of transportation.

The problem contains a particular case of ordinary transportation problems and other similar problems. The economic interpretation of these problems, occurring in the practice of planning, derives from the economic interpretation of the problem.

Schell (1955) shows how one can proceed to an optimum solution of this problem by a procedure quite similar to that used in two-dimensional problems. Corban (1964) deals with the mathematical foundation of this problem and method of solving but does not deal with possible adaptations.

For the problem with two planar sums, Schell shows how it reduces to m separate two-dimensional problems which are, of course, easily solved. Similarly, Schell shows that the problem with one planar and one axial sum reduces to one two-dimensional problem of dimensions $p \times mn$.

Galler and Dwyer (1957) proposed a procedure for solving the special class of k-dimensional multi-index problems ($k \geqslant 2$) which, for $k = 3$, yields the above three-dimensional axial-sum transportation problem, further they observe that an algebraic solution of the given constraints might yield negative values. They then show how one can eliminate negative solutions and arrive at a non-negative—but not necessarily integral solution of the given problem.

SECTION III : TIME-COST TRADE-OFF IN TRANSPORTATION PROBLEMS

Bhatia, *et al.* (1976c) present an enumerative technique to obtain successive time-cost-commodity in pipe-line trade-off relationships in a transportation problem. The time-cost-minimum commodity in the pipe-line trade-off relationships are also obtained in the process. The procedure leads ultimately to the minimum time (with minimum commodity in the pipe-line at this time) of transportation of goods from sources to destinations. The algorithm also gives the minimum time at a given cost for which the transportation schedule is known. In this procedure, extreme point solutions of the ordinary cost transportation problem are enumerated until the minimum time of transportation (with minimum pipe-line) is achieved. Sharma and Swarup (1976) also developed same enumerative technique to obtain time-cost trade-off in a multidimensional transportation problem.

Sharma with Swarup (1978b) and Agrawal (1976b) present a time-enumerative technique for obtaining successive time-cost trade-off relationships in a given transportation problem (two and multidimensional). The technique is based on the concept of second best solution, third best solution etc., in a transportation problem. The algorithm developed starts with a solution giving minimum time of transportation and defining the concept of second best moves towards the optimality with respect to cost and in the process generates time-cost trade-off relationships. Bhatia *et al.* (1976b) also developed a time-enumerative technique to obtain time-cost trade-off in a solid transportation problem.

CONCLUSION

In retrospect, one can observe part of the history and growth of operational research through just the transportation problem : its origins in applications, the theoretical development and an increasing degree of sophistication, a parallel development with advances in Computer technology, and generalizations to more complex problems.

ACKNOWLEDGEMENT

The author is thankful to Professor Kanti Swarup for continued help and inspirations and to Council of Scientific and Industrial Research for the financial assistance.

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