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DISTURBANCE OF SH TYPE DUE TO BODY FORCES AND DUE TO SHEARING STRESS-DISCONTINUITY IN A PRE-STRESSED SEMI-INFINITE VISCOELASTIC MEDIUM

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Effect of existing pre-stresses inside the earth on the disturbance of SH-type due to body forces and due to shearing stress-discontinuity in a semi-infinite viscoelastic medium has been shown in this paper. Biot's incremental deformation theory has been used to solve the problem. The expressions for the surface displacements are obtained in terms of modified Bessel functions.

1. INTRODUCTION

The problem of evaluating the disturbances produced in an isotropic semi-infinite medium due to sources has been studied by several authors. Mitra[10] used the method of Cagniard[3] to determine the surface displacement due to different types of body forces. Nag[11,12] has investigated the SH-type of motion due to different types of body forces and obtained the results in closed form.

The disturbance produced due to shearing stress discontinuity in a semi-infinite medium or layered media has been studied by several authors. Spencer[19] was the first to discuss sources consisting of stress discontinuity suddenly created inside an infinite medium. Noted contributions in this field have also been given in Das Gupta[4], Nag[12,13], Nag and Pal[14–16].

Wave propagation in a viscoelastic medium has been discussed by Jeffreys[9] and also by Hanin[7], Jeffreys took up the case of the step-pulse disturbance and his result consist of certain asymptotic formulae and a real integral representation. Hanin[7] discussed the wave generated by a Dirac-pulse disturbance is a visco-elastic medium. Bhattacharya[1] discussed on the strong SH-motion in clay (viscoelastic) overlying hard elastic material due to explosions. Sarkar[18] investigated the surface displacement of the SH-type motion due to time dependent body forces and due to sudden introduction of a discontinuity in the shearing stress in a semi-infinite, homogeneous, isotropic

viscoelastic medium. The displacement due to a point force and due to a stress discontinuity over a fixed region have been calculated numerically for different degrees of visco-elasticity and at different distances along the surface. Nag and Pal[17] studied the problem on the SH-type waves due to shearing stress discontinuity in a viscoelastic layered half space.

Earth is a pre-stressed or initially-stressed elastic medium. So, it will be more appropriate to consider the effect of initial stresses while studying problems on earth model. This view point has led to consider the present problem to show the effect of initial stresses on the SH-type motion generated due to body forces and shearing stress discontinuity in a viscoelastic medium of the standard linear type. Here the stress decreases exponentially with time. In section I the, effect of initial stresses on the surface displacements of the SH-type of motion generated due to different types of body forces in an initially-stressed semi-infinite viscoelastic medium have been shown and in section II same has been studied due to shearing stress discontinuity in the interface of an initially stressed viscoelastic layer and viscoelastic half space. The expressions for the surface displacements in terms of modified Bessel functions in both cases have been derived in the closed form. The results depict the effect of initial stresses and the viscoelastic damping characteristics explicitly. The problem has been solved using Biot's incremental deformation theory with the help of Laplace and Fourier transforms.

As the earth is a pre-stressed medium and certain seismic observations, e.g. damping of earthquake waves (Jaeger[8]). may be explained on the supposition that the material of the earth is viscoelastic of the standard linear type and as the formation of crack is usually characterised by a discontinuity in the stress, such problems may be of interest in seismology and in connection with the formation of crack.

2. Mathematical Derivation

Section I

Consider a viscoelastic halfspace $z \geqslant 0$, and x-axis along the free surface (Fig. 1). It is assumed that the whole medium is under the constant initial compressive stress P along x-direction. Only the SH-type motion is being considered here with the assumption that the displacements and the body forces are independent of y.

For SH-type of motion $u = w = 0$, so that two equations of motion are identically satisfied and the third equation of motion (under initial stress $P = -S_{11}$) reduces to[2].

$$\frac{\partial s_{21}}{\partial x} + \frac{\partial s_{23}}{\partial z} - P\frac{\partial \omega_z}{\partial x} + \rho Y = \rho \frac{\partial^2 v}{\partial t^2} \qquad \ldots(2.1)$$

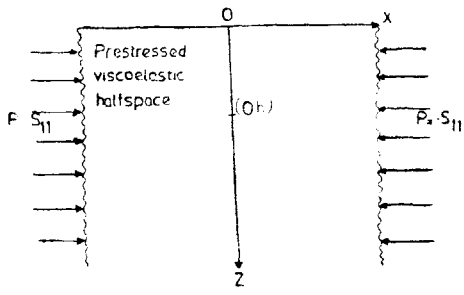

FIG. 1. Geometry of the problem.

where s_{ij} are incremental stresses, v is the displacement. ρ is the density, Y is the body force and ω_z the rotational component given by

$$\omega_z = \frac{1}{2}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial v}\right). \qquad \ldots(2.2)$$

The incremental stress components s_{21} and s_{23} are related to the strain components (e_{ij}) by[2].

$$s_{21} = 2\,Q_3\,e_{xy},\ s_{23} = 2\,Q_1\,e_{yz} \qquad \ldots(2.3)$$

where Q_1 and Q_3 are incremental shear moduli in x and z-direction respectively.

The equation of motion (2.1) with the help of (2.2) and (2.3) takes the form

$$\left(Q_3 - \frac{P}{2}\right)\frac{\partial^2 v}{\partial x^2} + Q_1\frac{\partial^2 v}{\partial z^2} + \rho\,Y = \rho\,\frac{\partial^2 v}{\partial t^2}. \qquad \ldots(2.4)$$

If the medium under initial stress remains isotropic for incremental plane strain, then

$$Q_1 = Q_3 = \mu. \qquad \ldots(2.5)$$

For standard linear viscoelastic solid (Voigt type) the governing equation will be obtained replacing $\mu\,(= Q_1 = Q_3)$ by $\mu + \mu'\,\partial/\partial t$ (*cf.* Ewing *et al.*)[6] in eqn. (2.4) which is

$$\left(\mu + \mu'\frac{\partial}{\partial t} - \frac{P}{2}\right)\frac{\partial^2 v}{\partial x^2} + \left(\mu + \mu'\frac{\partial}{\partial t}\right)\frac{\partial^2 v}{\partial z^2} + \rho\,Y = \rho\,\frac{\partial^2 v}{\partial t^2}. \qquad \ldots(2.6)$$

It is assumed that,

$$v = \varphi\,(x, z)\,e^{-pt} \text{ and } Y = F\,(x, z)\,e^{-pt}. \qquad \ldots(2.7)$$

Substitution of (2.7) in (2.6) gives

$$\frac{\mu - \mu' p}{\rho}(1 - I)\frac{\partial^2 \varphi}{\partial x^2} + \frac{\mu - \mu' p}{\rho}\frac{\partial^2 \varphi}{\partial z^2} + F = p^2 \varphi \qquad \ldots(2.8)$$

where

$$I = \frac{P}{2(\mu - \mu' p)} .$$

The above equation may be written as

$$p^2 \varphi = F + \sigma^2 \left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{1}{k^2}\frac{\partial^2 \varphi}{\partial z^2} \right) \qquad \ldots(2.9)$$

where

$$k^2 = (1 - I),\ \sigma^2 = \frac{(\mu - \mu' p)}{\rho} k^2$$

and it is assumed that $\sigma^2 > 0$. Let $f_1(\eta, z)$ be the Fourier transform of the function $f(x, z)$ and $f_2(\eta, q)$ be the Laplace transform of $f_1(\eta, z)$, defined by

$$f_1(\eta, z) = \int_{-\infty}^{\infty} e^{-i\eta x} f(x, z)\, dx \qquad \ldots(2.10)$$

and

$$f_2(\eta, q) = \int_0^{\infty} e^{-qz} f_1(\eta, z)\, dz. \qquad \ldots(2.11)$$

Applying the Fourier transform (2.10) in (2.9), one obtains

$$p^2 \varphi_1 = F_1 + \sigma^2 \left[-\eta^2 \varphi_1(\eta, z) + \frac{1}{k^2}\frac{\partial^2 \varphi_1}{\partial z^2} \right]$$

again applying Laplace transform (2.11) to the above equation results in

$$\varphi_2 \left[\frac{q^2}{k^2} - \left(\eta^2 + \frac{p^2}{\sigma^2} \right) \right] \sigma^2 = -F_2 + \frac{\sigma^2}{k^2}(q \varphi_{10} + \varphi'_{10}) \qquad \ldots(2.12)$$

where

$$\varphi'_{10} = \left[\frac{\partial}{\partial z} \varphi_1(\eta, z) \right]_{z=0}$$

and F_2 is the result of repeated applications of Fourier transform and Laplace transform.

The boundary condition is

$$\Delta f_y = 0 \text{ at } z = 0$$

where Δf_y is the incremental boundary force per unit initial area along y-direction and is given by

$$\Delta f_y = s_{23} \qquad \ldots(2.13)$$

i.e.

$$\left(\mu + \mu' \frac{\partial}{\partial t}\right) \frac{\partial v}{\partial z} = 0 \quad \text{at } z = 0$$

i.e.

$$\left[\frac{\partial \varphi}{\partial z}\right]_{z=0} = 0.$$

On applying Fourier transform (2.10), the above condition reduces to

$$\varphi'_{10} = 0.$$

Using this condition, eqn. (2.12) may be written as

$$\varphi_2 = -\frac{F_2}{M} + \frac{\sigma^2 q}{k^2 M} \varphi_{10} \qquad \ldots(2.14)$$

where

$$M = m^2 (q^2 - \beta_s^2) \qquad \ldots(2.15)$$

in which

$$m^2 = \frac{\sigma^2}{k^2} \text{ and } \beta_s^2 = k^2 \left(\eta^2 + \frac{p^2}{\sigma^2}\right).$$

Now, the displacement at the surface $z = 0$ due to body forces will be evaluated subject to the condition that the displacement tends to zero as $z \to \infty$.

The inversion formula for Laplace transform (2.11) and Fourier transform (2.10) are given respectively as

$$f_1(\eta, z) = \frac{1}{2\pi i} \int_{\nu - i\infty}^{\nu + i\infty} e^{qz} f_2(\eta, q)\, dq$$

and

$$f(x, z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} f_1(\eta, z)\ e^{i\eta x}\ d\eta.$$

Applying Laplace inverse transform to (2.14), one obtains

$$\varphi_1 = \frac{1}{2\pi i}\int_{\nu-i\infty}^{\nu+i\infty} e^{qz}\ \frac{m^2 q}{M}\ \varphi_{10}\, dq + \frac{1}{2\pi i}\int_{\nu-i\infty}^{\nu+i\infty} e^{qz}\left(-\frac{F_2}{M}\right) dq.$$

Since F_2 is the result of repeated application of *F. T.* and *L. T.* on the body force component $F(x, z)$, assuming it as the function of q, has no pole with positive real part and $F_2\, e^{qz} \to 0$ as $R \to \infty$ where $q = R\, e^{i\theta}$, $\pi/2 < \theta < 3\pi/2$. Hence, the poles of the integrand are $q = \pm\, \beta_s$. Since q should be positive (Laplace parameter), it is required to evaluate residue of the integrand at $q = +\, \beta_s$ only.

The residue of the integrand an $q = +\, \beta_s$ is

$$\frac{1}{2}\ \varphi_{10}\, e^{z\beta_s} - \frac{1}{2m^2\, \beta_s}[e^{z\beta_s}\, F_2(\eta, \beta_s)].$$

therefore,

$$\varphi_1 = \frac{1}{2}\ \varphi_{10}\, e^{z\beta_s} - \frac{1}{2m^2\, \beta_s}[e^{z\beta_s}\, F_2(\eta, \beta_s)]. \qquad \ldots(2.16)$$

Since the displacement tends to zero as $z \to \infty$, $\varphi_1 \to 0$ as $z \to \infty$, and the value of φ_{10} becomes

$$\varphi_{10} = \frac{F_2(\eta, \beta_s)}{m^2\, \beta_s}.$$

Taking inverse Fourier transform of the above, one gets

$$\varphi(x, 0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \frac{F_2(\eta, \beta_s)}{m^2\, \beta_s}\ e^{i\eta x}\, d\eta \qquad \ldots(2.17)$$

which gives the displacement at the surface and will now be calculated for different forms of F.

Case 1—Consider a point force on z-axis at a depth h below the free surface : i. e.

$$F = R\, \delta(x)\, \delta(z-h) \qquad \ldots(2.18)$$

where R is a constant and $\delta(x)$, $\delta(z-h)$ are Dirac's delta functions.

Applying Fourier transform (2.10) and then Laplace transform (2.11) in succession to (2.18), one obtains,

$$F_2 = R\, e^{-qh}. \qquad \ldots(2.19)$$

Substitution of (2.19) in (2.17) gives

$$\varphi(x, 0) = \frac{R}{2\pi m^2} \int_{-\infty}^{\infty} \frac{e^{-\beta_s h}\, e^{i\eta x}}{\beta_s}\, d\eta$$

$$= \frac{R}{\pi m^2 k} \int_{0}^{\infty} \frac{e^{-hk}\sqrt{\eta^2 + \frac{p^2}{\sigma^2}}}{\sqrt{\eta^2 + \frac{p^2}{\sigma^2}}} \cos \eta x\, d\eta$$

$$= \frac{R}{\pi m^2 k} K_0\left[\frac{p}{\sigma}\sqrt{h^2 k^2 + x^2}\right], \text{ (Erdelyi[8], p. 17)}$$

where $K_0(x)$ is the modified Bessel function of order zero. Therefore, the displacement at the free surface is obtained as

$$v(x, 0) = e^{-pt}\, \varphi(x, 0)$$

$$= \frac{R\, e^{-pt}}{\pi \beta^2} \frac{1}{\sqrt{1-I}\left(1 - p\frac{\mu'}{\mu}\right)} K_0\left[\frac{p}{\sqrt{1-I}\sqrt{\left(1 - \frac{\mu'}{\mu}p\right)}} \frac{\sqrt{h^2(1-I) + x^2}}{\beta}\right] \qquad \ldots(2.20)$$

where

$$\beta^2 = \mu/\rho.$$

Case 2—Consider a point torque on z-axis at a depth h below the free surface : i.e.

$$F = R\, \frac{\partial}{\partial z}[\delta(x)\, \delta(z-h)] \qquad \ldots(2.21)$$

then it is easy to calculate

$$F_2 = R\, q\, e^{-qh}. \qquad \ldots(2.22)$$

From (2.17) and (2.22)

$$\varphi(x, 0) = \frac{R}{2\pi} \int_{-\infty}^{\infty} \frac{e^{-\beta_s h}\, e^{i\eta x}}{m^2}\, d\eta$$

(equation continued on p. 316)

$$= \frac{R}{\pi m^2} \int_0^\infty e^{-hk} \sqrt{\eta^2 + \frac{p^2}{\sigma^2}} \cos \eta x \, d\eta$$

$$= \frac{R}{\pi m^2} \frac{hk \frac{p}{\sigma}}{\sqrt{h^2 k^2 + x^2}} K_1 \left[\frac{p}{\sigma} \sqrt{h^2 k^2 + x^2} \right], \text{(Erdelyi[5], p.16)}$$

which gives the displacement $v(x, 0)$ at the surface as

$$\frac{v \pi \beta^4 e^{pt}}{R h} = \left[\frac{p}{\left(1 - \frac{\mu'}{\mu} p\right)^{3/2} \frac{\sqrt{h^2 (1-I) + x^2}}{\beta}} \right]$$

$$\times K_1 \left[\frac{p}{\sqrt{1-I} \sqrt{1 - \frac{\mu'}{\mu} p}} \frac{\sqrt{h^2 (1-I) + x^2}}{\beta} \right]. \quad \ldots(2.23)$$

Section II

The model consists of a viscoelastic layer of thickness h lying over a viscoelastic half space. The x-axis is taken along the interface and + ve z-axis along vertically downwards in the half space. Both the media have been considered under a constant initial compressive stress P along x-axis (Fig. 2). The shearing stress discontinuity is supposed to be created suddenly at the interface at a depth h from the free surface. The displacement is assumed to be continuous at the interface and bounded everywhere.

In the absence of body forces, the equation of motion (2.1) reduces to

$$\frac{\partial s_{21}}{\partial x} + \frac{\partial s_{23}}{\partial z} - P \frac{\partial \omega_2}{\partial x} = \rho \frac{\partial^2 v}{\partial t^2}$$

and the equation (2.9) takes the form

$$p^2 \varphi = \sigma^2 \left(\frac{\partial^2 \varphi}{\partial x^2} + \frac{1}{k^2} \frac{\partial^2 \varphi}{\partial z^2} \right) \quad \ldots(2.24)$$

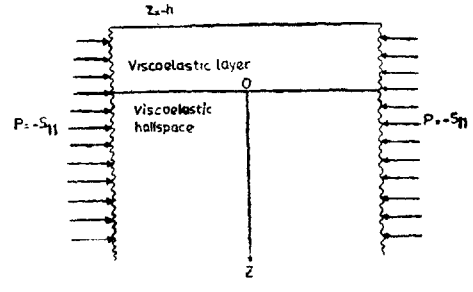

FIG. 2. Viscoelastic layer over a viscoelastic half-space with underforce-stress P.

where σ and k are defined in Section I.

The Fourier transform $\bar{\varphi}(\eta, z)$ of $\varphi(x, z)$ is defined by

$$\bar{\varphi}(\eta, z) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \varphi(x, z)\, e^{-i\eta x}\, dx \qquad \ldots(2.25)$$

where

$$\varphi(x, z) = \int_{-\infty}^{\infty} \bar{\varphi}(\eta, z)\, e^{i\eta x}\, d\eta.$$

Taking Fourier transform of (2.24), one gets

$$\frac{\partial^2 \bar{\varphi}}{\partial z^2} - k^2\left(\eta^2 + \frac{p^2}{\sigma^2}\right)\bar{\varphi} = 0. \qquad \ldots(2.26)$$

The solution for the layer may be taken as

$$\bar{\varphi}_1 = A \cosh \beta_s z + B \sinh \beta_s z \qquad \ldots(2.27)$$

and the solution for the half space as

$$\bar{\varphi}_2 = C\, e^{-\beta_s z} \qquad \ldots(2.28)$$

where

$$\beta_s = + k \sqrt{\eta^2 + \frac{p^2}{\sigma^2}}$$

and A, B, C are constants.

The inverse Fourier transform of (2.27) and (2.28), give

$$\varphi_1(x, z) = \int_{-\infty}^{\infty} (A \cosh \beta_s z + B \sinh \beta_s z)\, e^{i\eta x}\, d\eta \qquad \ldots(2.29)$$

and

$$\varphi_2(x, z) = \int_{-\infty}^{\infty} C\, e^{-\beta_s z}\, e^{i\eta x}\, d\eta. \qquad \ldots(2.30)$$

From geometry of the problem, the boundary conditions may be taken as

(i) The displacements are continuous at the interface for all x and t i.e.

$\varphi_1 = \varphi_2$ at $z = 0$ for all x and t.

(ii) The incremental boundary force per unit initial area along y-direction vanishes at the free surface for all x and t, i.e.

$$(\Delta f_y)_1 = 0 \text{ at } z = -h \text{ for all } x \text{ and } t.$$

(iii) At the interface $z = 0$, the shearing stress discontinuity is a function of x which decays with respect to t, i.e.

$$(\Delta f_y)_1 - (\Delta f_y)_2 = L(x)\, e^{-pt} \text{ at } z = 0 \text{ for all } x \text{ and } t$$

where Δf_y is defined (2.13).

Conditions (i), (ii) and (iii) give respectively

$$A = C \qquad \ldots(2.31)$$

$$A \sinh \beta_s h = B \cosh \beta_s h \qquad \ldots(2.32)$$

and

$$\frac{\partial \varphi_1}{\partial z} - \frac{\partial \varphi_2}{\partial z} = \frac{K}{\sigma^2 \rho} L(x) = U(x) \text{ (say)}. \qquad \ldots(2.33)$$

Let $\bar{U}(\eta)$ be the Fourier transform of $U(x)$ defined by

$$\bar{U}(\eta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} U(x)\, e^{-i\eta x}\, dx$$

i.e.

$$\bar{U}(\eta) = \frac{1}{2\pi} \int_{-\infty}^{\infty} \frac{L(x)}{(\sigma^2 \rho / k)}\, e^{-i\eta x}\, dx$$

therefore

$$\frac{L(x)}{(\sigma^2 \rho / k)} = \int_{-\infty}^{\infty} \bar{U}(\eta)\, e^{i\eta x}\, d\eta. \qquad \ldots(2.34)$$

From (2.31), (2.32) and (2.33), the constants A, B, C are obtained as

$$A = C = \frac{1}{\beta_s} \frac{\bar{U}(\eta)}{\sinh \beta_s h + \cosh \beta_s h} \cosh \beta_s h$$

and

$$B = \frac{\bar{U}(\eta)}{\beta_s} \frac{\sinh \beta_s h}{\sinh \beta_s h + \cosh \beta_s h}.$$

Substituting the values of A and B in (2.27), the displacement in the layer at the free surface is

$$\varphi_1(x,-h) = \int_{-\infty}^{\infty} \frac{\bar{U}(\eta)}{\beta_s} e^{(i\eta x - \beta_s h)}\, d\eta. \qquad \ldots(2.35)$$

The displacement $\varphi_1(x,-h)$ at the free surface will now be calculated for different forms of $L(x)$.

Case I—Constant shearing stress over a limited region

Let

$$L(x) = R \text{ for } 0 \leqslant x \leqslant a$$
$$= 0 \text{ elsewhere} \qquad \ldots(2.36)$$

where R is a constant.

In this case $\bar{U}(\eta)$ reduces to

$$\bar{U}(\eta) = \frac{1}{2\pi} \int_0^a e^{-i\eta x} \frac{R}{(\sigma^2 \rho/k)}\, dx$$

$$= \frac{R}{2\pi(\sigma^2\rho/k)} \int_0^a e^{-i\eta x}\, dx$$

$$= \frac{Ri}{2(\sigma^2\rho/k)} \frac{e^{-i\eta a} - 1}{\eta}.$$

Substituting the value of $\bar{U}(\eta)$ in (2.35), results in

$$\varphi_1(x,-h) = \int_{-\infty}^{\infty} \frac{Ri\,[e^{i\eta(x-a)} - e^{i\eta x}]\, e^{-\beta_s h}}{2\pi\left(\frac{\sigma^2\rho}{k}\right)\beta_s\eta}\, d\eta$$

$$= \frac{R}{\pi(\sigma^2\rho/k)} \left[\int_0^{\infty} \frac{e^{-\beta_s h}}{\beta_s\eta} \sin\eta(x-a)\, d\eta \right.$$

$$\left. - \int_0^{\infty} \frac{e^{-\beta_s h}}{\beta_s\eta} \sin\eta x\, d\eta \right]$$

$$= \frac{R}{\pi(\sigma^2\rho/k)} [N_1 - N] \text{ (say)}. \qquad \ldots(2.37).$$

The integrals N_1 and N can be evaluated as follows :

$$N = \frac{1}{k} \int_0^\infty \frac{e^{-kh}\sqrt{\eta^2 + \frac{p^2}{\sigma^2}}}{\sqrt{\eta^2 + \frac{p^2}{\sigma^2}}} \frac{\sin \eta x}{\eta} \, d\eta.$$

On integration of the standard relation

$$\int_0^\infty \frac{e^{-hk}\sqrt{\eta^2 + \frac{p^2}{\sigma^2}}}{\sqrt{\eta^2 + \frac{p^2}{\sigma^2}}} \cos \eta x \, d\eta = K_0\left[\frac{p}{\sigma}\sqrt{h^2 k^2 + x^2}\right]$$

(*cf.* Erdelyi[5], p. 17)

with respect to x from 0 to x, one gets

$$\int_0^\infty \frac{e^{-hk}\sqrt{\eta^2 + \frac{p^2}{\sigma^2}}}{\sqrt{\eta^2 + \frac{p^2}{\sigma^2}}} \frac{\sin \eta x}{\eta} \, d\eta = \int_z^x K_0\left[\frac{p}{\sigma}\sqrt{h^2 k^2 + x^2}\right] dx.$$

Hence, value of N is obtained as

$$N = \frac{1}{k} \int_0^x K_0\left[\frac{p}{\sigma}\sqrt{h^2 k^2 + x^2}\right] dx.$$

Similarly, N_1 may be calculated as

$$N_1 = \frac{1}{k} \int_0^{x-a} K_0\left[\frac{p}{\sigma}\sqrt{h^2 k^2 + x^2}\right] dx.$$

Therefore, the displacement component at the free surface is given by

$$v(x,-h) = \frac{R\, e^{-pt}}{\pi(\mu - \mu' p)(1 - I)}\left[\int_0^x K_0\left[\frac{p\sqrt{h^2(1 - I) + x^2}}{\beta\sqrt{1-I}\sqrt{1 - \frac{\mu'}{\mu}p}}\right] dx\right.$$

$$\left. + \int_0^{r-a} K_0\left[\frac{p\sqrt{h^2(1 - I) + x^2}}{\beta\sqrt{1-I}\sqrt{1 - \frac{\mu'}{\mu}p}}\right] dx\right] \qquad \ldots(2.38)$$

where $\beta^2 = \mu/\rho$.

Case 2 : Let

$$L(x) = R\,\delta(x) \qquad \ldots(2.39)$$

then

$$\bar{U}(\eta) = \frac{1}{2\pi\left(\frac{\sigma^2\rho}{k}\right)} \int_{-\infty}^{\infty} R\,\delta(x)\, e^{-i\eta x}\, dx$$

$$= \frac{R}{2\pi(\sigma^2\rho/k)}.$$

Equation (2.35) now becomes

$$\varphi_1(x,-h) = \int_{-\infty}^{\infty} \frac{R}{2\pi(\sigma^2\rho/k)} \frac{e^{-\beta_s h}}{\beta_s} e^{i\eta x}\, d\eta$$

$$= \frac{R}{\pi(\sigma^2\rho/k)} \text{ Real part } \int_0^{\infty} \frac{e^{-\beta_s h}}{\beta_I} e^{i\eta x}\, d\eta$$

$$= \frac{R}{\pi(\sigma^2\rho/k)} K_0\left[\frac{p}{\sigma}\sqrt{h^2k^2 + x^2}\right] \text{(Erdelyi[5], p. 17).}$$

Therefore,

$$v(x,-h) = \frac{R\,e^{-pt}}{\pi(\mu-\mu' p)\sqrt{1-I}} K_0\left[\frac{p\sqrt{h^2(1-I)+x^2}}{\beta\sqrt{1-I}\sqrt{1-\frac{\mu'}{\mu}p}}\right]. \qquad \ldots(2.40)$$

Discussions

The equations (2.20), (2.23), (2.28) and (2.40) show that the surface displacement due to different types of body forces and also due to shearing stress discontinuity are much influenced by initial stresses present in the medium as well as the viscoelastic property of the medium. The presence of I, in particular outside the arguments of modified Bessel functions K_0 and K_1 explicity indicate that the initial stress factor present in the medium deserves its consideration in the problem. From equations (2.20) and (2.40) it is also to be noted that the stress discontinuity at a point and the point source at that very point produce equivalent displacements on the free boundary of the semi-infinite viscoelastic medium.

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