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COMBINED FREE AND FORCED CONVECTION IN VERTICAL CIRCULAR POROUS CHANNEL

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The combined free and forced convection in a saturated vertical porous tube of circular cross section with a uniform heat source is examined in this paper. The governing equations are solved for velocity and temperature fields in the form of Fourier Bessel series.

1. INTRODUCTION

Convective heat transfer from a heated impermeable surface in a saturated porous media has important applications in the assessment of geothermal resources and in the design of under ground energy storage systems. This problem is also of interest in nuclear power reactor engineering.

The combined convective heat transfer in long vertical channels of circular cross section has been studied earlier by several authors : Hallman[4], Lu[6] and Tao[9]. Later, Cheng and Minkowycz[3] and Merkin[7] discussed free convection on an impermeable vertical plate embedded in a saturated porous media. Kaneko *et al.*[5] experimentally studied free convection in porous media. Cheng[2] studied free and forced convection in porous media above inclined surfaces whereas Burns *et al.*[1] studied combined convection through an insulated vertical slot.

The aim of the present investigation is to examine combined free and forced convection in a saturated porous vertical tube of circular cross section. The medium is assumed to be fully saturated and the bounding wall impermeable. The porous medium is assumed to be non-Darcian in the sense that the momentum equations include viscous stresses and interia terms as proposed by Yamamoto and Yoshida[10] and Yamamoto and Iwamura[11].

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2. Nomenclature

Z = axial coordinate

R, ϕ, Z = cylindrical coordinate

w = fluid velocity in the axial direction

p = fluid pressure

μ = coefficient of viscosity

ρ = fluid density

ρ_0 = standard density

ρ_d = density of the fluid at the wall

k = permeability parameter (dimensional)

σ = permeability parameter (non-dimensional)

t = temperature of the fluid

t_0 = standard temperature

t_d = wall temperature = $t_d(Z)$

T = temperature difference or temperature = $t - t_d$ (dimensional)

c_p = specific heat of the fluid

k_1 = thermal conductivity of the saturated fluid

g = acceleration due to gravity

β = coefficient of thermal expansion

f = internal heat source (dimensional)

F = internal heat source (non-dimensional)

C_1 = axial temperature gradient (constant) = $\partial t/\partial Z$

θ = temperature difference (non-dimensional)

W_m = mean velocity (dimensional)

W_M = mean velocity (non-dimensional)

W = axial velocity = w/W_m

E = pressure gradient parameter

R_a = Rayleigh number

3. Mathematical Formulation

Consider a steady fully developed (both hydrodynamically and thermally) laminar flow of a viscous incompressible fluid in a long vertical porous channel of circular cross section. The channel wall is assumed to be impermeable while the flow region is assumed to be highly porous. The fluid receives uniform heat input per unit length of the tube. Further, a uniformly distributed constant heat source is assumed to be present in the flow region. Consider a cylindrical coordinate system (R, ϕ, Z) : where R is the distance from the Z-axis which incidentally coincides with the axis of the tube and is directed vertically upwards. The fluid properties such as specific that (c_p), the coefficient of viscosity (μ) as well as the medium properties such as the permeability (k) and the thermal conductivity (k_1) are all constants. The dissipation effects in the energy equation are ignored. Boussinesq approximation is employed amounting to the neglect of the density variation except in the buoyancy term. The equation of state then reduces to

$$\rho = \rho_0 [1 - \beta (t - t_0)]. \qquad ...(1)$$

Due to the fully developed laminarity of the flow, the wall and the fluid temperature gradients in the axial direction are equal and constants. Under these conditions, the steady flow characterized by velocity $(0, 0, w)$ and the temperature (T) are governed by the following equations :

Continuity equation :

$$\frac{\partial w}{\partial Z} = 0 . \qquad ...(2)$$

Momentum equation :

$$0 = -\left(\frac{\partial p}{\partial Z} + \rho_d g\right) + \mu \nabla^2 w - \frac{\mu}{k} w + \rho g \beta T \qquad ...(3)$$

and energy equation :

$$\rho c_p w C_1 = k_1 \nabla^2 T + f \qquad ...(4)$$

with the boundary conditions

$$w = 0, \; T = 0 \qquad ...(5)$$

on the boundary wall $R = a$; a being tube radius.

The following non-dimensional scheme is employed :

$$\left.\begin{aligned} r &= \frac{R}{a} \\ W &= \frac{w}{W_m}, \quad \theta = \frac{k_1 T}{\rho c_p C_1 W_m a^2} \\ E &= \left(\frac{\partial p}{\partial Z} + \rho_d g\right) \frac{a^2}{\mu W_m}, \quad F = \frac{f}{\rho c_p C_1 W_m} \\ R_a &= \frac{\rho^2 g c_p C_1 \beta a^4}{k_1 \mu}, \quad \sigma^2 = \frac{a^2}{k}. \end{aligned}\right\} \qquad \ldots(6)$$

The basic equations (3) and (4) for the velocity and temperature now reduce to non-dimensional form

$$\frac{d^2 W}{dr^2} + \frac{1}{r}\frac{dW}{dr} - \sigma^2 W + R_a \theta = E \qquad \ldots(7)$$

$$\frac{d^2 \theta}{dr^2} + \frac{1}{r}\frac{d\theta}{dr} - W = -F \qquad \ldots(8)$$

together with the boundary conditions

$$W(1) = 0, \ \theta(1) = 0. \qquad \ldots(9)$$

E and F occurring in eqns. (7) and (8) are constants characterizing the pressure gradient and the heat source in the channel.

The sequence of functions $\{J_0(\lambda_n r)\}$, where J_0 represents the zeroth order Bessel function of the first kind and λ_n's are the successive roots of $J_0(\lambda)=0$, vanishes on the tube boundary $r = 1$. Following Pipes and Harvill[8], we obtain the Fourier-Bessel series :

$$1 = 2 \sum_{n=1}^{\infty} \frac{1}{\lambda_n J_1(\lambda_n r)} J_0(\lambda_n r). \qquad \ldots(10)$$

The choice

$$W(r) = \sum_{n=1}^{\infty} r_n J_0(\lambda_n r) \qquad \ldots(11)$$

$$\theta(r) = \sum_{n=1}^{\infty} s_n J_0(\lambda_n r) \qquad \ldots(12)$$

for the velocity and temperature evidently satisfies the boundary conditions (9) for all constants r_n and s_n. Substituting eqns. (10) to (12) in eqns. (7) and (8), we obtain, after some simplification and comparing the coefficients of like terms :

$$-(\sigma^2 + \lambda_n^2)\, r_n + R_a\, s_n = \frac{2E}{\lambda_n J_1(\lambda_n)} \qquad \ldots(13)$$

$$r_n + \lambda_n^2\, s_n = \frac{2F}{\lambda_n J_1(\lambda_n)}. \qquad \ldots(14)$$

Solving eqns. (13) and (14) we obtain r_n and s_n.

We thus have velocity and temperature fields :

$$W = 2 \sum_{n=1}^{\infty} \frac{[F R_a - E \lambda_n^2]}{[R_a + \lambda_n^2 (\sigma^2 + \lambda_n^2)]} \frac{J_0(\lambda_n r)}{\lambda_n J_1(\lambda_n)} \qquad \ldots(15)$$

$$\theta = 2 \sum_{n=1}^{\infty} \frac{[F(\sigma^2 + \lambda_n^2) + E]}{[R_a + \lambda_n^2 (\sigma^2 + \lambda_n^2)]} \frac{J_0(\lambda_n r)}{\lambda_n J_1(\lambda_n)}. \qquad \ldots(16)$$

Mean velocity (non-dimensional form) is

$$W_M = 4 \sum_{n=1}^{\infty} \frac{[F R_a - E]}{\lambda_n^2 [R_a + \lambda_n^2 (\sigma^2 + \lambda_n^2)]}. \qquad \ldots(17)$$

Mean temperature difference (non-dimensional form) is

$$\theta_m = \frac{1}{\pi} \int_0^1 2\pi r \theta \, dr$$

$$= 4 \sum_{n=1}^{\infty} \frac{[F(\sigma^2 + \lambda_n^2) E]}{\lambda_n^2 [R_a + \lambda_n^2 (\sigma^2 + \lambda_n^2)]}. \qquad \ldots(18)$$

Mean mixed temperature difference (non-dimensional form) is

$$\theta_{m_v} = \frac{\int_{r=0}^{1} W \theta \, r \, dr}{\int_{r=0}^{1} W r \, dr}$$

$$= \frac{2 \sum_{n=1}^{\infty} \frac{[F R_a - E \lambda_n^2][F(\sigma^2 + \lambda_n^2) + E]}{\lambda_n^2 [R_a + \lambda_n^2 (\sigma^2 + \lambda_n^2)]^2}}{\sum_{n=1}^{\infty} \frac{[F R_a - E \lambda_n^2]}{\lambda_n^2 [R_a + \lambda_n^2 (\sigma^2 + \lambda_n^2)]}}. \qquad \ldots(19)$$

REFERENCES

1. J. Burns, L. C. Chowl, and C. L. Tien, *Int. J. Heat Mass Transfer* **20** (1977) 919.
2. P. Cheng, *Int. J. Heat Mass Transfer* **20** (1977) 807.
3. P. Cheng, and W. T. Minkowycz, *J. Geophys. Res.* **82** (1977) 2040.
4. T. M. Hallman, *J. Heat Transfer* **78** (1956) 1831.
5. T. Kaneko, M. F. Mohtadi and K. Aziz, *Int. J. Heat Mass Transfer* **17** (1974) 485.
6. P. C. Lu, *J. Heat Transfer* **82** (1960) 227.
7. J. H. Merkin, *Int. J. Heat Mass Transfer* **21** (1978) 1499.
8. L. A. Pipes, and L. R. Harvill, *Applied Mathematics for engineers and Physicist*, 3rd edition, McGraw-hill Book Co., Inc, New York, (1970) p. 795–97.
9. L. N. Tao, *Appl. Sci. Res.* A **9** (1960) 357.
10. K. Yamamoto, and Z. Yoshida, *J. Phys. Soc. Japan* **37** (1974) No. 3, 774.
11. K. Yamamoto, and N. Iwamura, *J. Engng. Math.* **10** (1976) No. 1, 41.