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PLANE PROBLEM IN MICROPOLAR ELASTIC HALF-SPACE WITH STRETCH

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The present investigation is concerned with plane problem in micropolar elastic half space with stretch. The displacement components, force stress, couple stress and first moment are obtained for a half-space subjected to an arbitrary normal load and a particular case of an impulsive line source is studied in detail. The effects of stretch are illustrated numerically for specific models.

1. INTRODUCTION

Eringen and Suhubi[1] and Eringen[2] introduced the theories of microelastic solids and microfluids in which the micromotions of the material particles contained in a macrovolume element with respect to its centroid are taken into account in an average sense. Materials affected by such micromotions and microdeformations are called micromorphic materials[3].

Eringen[4,5] developed theories for a subclass of micromorphic materials which are called micropolar media and these materials show microrotational effects and microrotational inertia. Here, the material particles in a volume element can undergo only rigid rotational motions about their centre of mass. Micropolar media may represent the materials that are made up of dipole atoms or dumbbell molecules and are subjected to surface- and body-couples.

Later on, Eringen[6] extended his work to include the effect of axial stretch during the rotation of molecules and developed the theory of micropolar elastic solids with stretch. These solids respond to intrinsic rotational motions and spin inertia and therefore can support couple stresses and body couples. In addition, dipolar microelement of such solids can undergo axial intrinsic motions. The model introduced is conjectured to explain the motions of certain classes of granular and composite materials in which grains and fibers are elastic in the direction of their major axes.

The mechanical model underlying the theory of micropolar elastic solids with stretch can be envisioned as an elastic medium composed of a large number of short springs. These springs possess average inertia moments and can deform in axial direction. Thus material particles in this continuum possess not only classical translational degrees of freedom represented by the deformation vector field, but also intrinsic rotations and intrinsic axial stretch.

The plane problem in micropolar elasticity has been studied by several investigators. Nowacki[8,9] studied the first and second plane problem in micropolar elasticity. Later on, Nowacki[10] discussed the same problem in micropolar thermoelasticity. Iesan[7] discussed the first and second problems of plane coupled micropolar thermoelasticity.

In this investigation, we have discussed the plane problem of micropolar elastic body with stretch.

2. Basic Equations

The constitutive and field equations for micropolar elastic solid with stretch in the absence of body forces, body moments and stretch force are given by Eringen[6]

$$t_{ji} = \lambda\, u_{k,k}\, \delta_{ij} + (\mu-\alpha)\,(u_{i,j} + u_{j,i}) + 2\alpha\,(u_{i,j} - \epsilon_{kji}\,\omega_k) \qquad \ldots(2.1)$$

$$m_{ji} = \beta_0\, \epsilon_{kji}\, \Phi_{,k} + \beta \omega_{k,k}\, \delta_{ij} + (\gamma-\epsilon)\,\omega_{j,i} + (\gamma+\epsilon)\,\omega_{i,j} \qquad \ldots(2.2)$$

$$\lambda_j = \alpha_0\, \Phi_{,j} + \frac{1}{3}\, \beta_0\, \epsilon_{kji}\, \omega_{k,i} \qquad \ldots(2.3)$$

$$(\mu+\alpha)\nabla^2 \vec{u} + (\lambda + \mu - \alpha)\ \text{grad div}\ \vec{u} + 2\alpha\ \text{rot}\ \vec{\omega} = \rho\, \frac{\partial^2 \vec{u}}{\partial t^2} \qquad \ldots(2.4)$$

$$(\gamma+\epsilon)\,\nabla^2 \vec{\omega} + (\gamma+\beta-\epsilon)\ \text{grad div}\ \vec{\omega} - 4\alpha\, \vec{\omega} + 2\alpha\ \text{rot}\ \vec{u} = J\, \frac{\partial^2 \vec{\omega}}{\partial t^2} \qquad \ldots(2.5)$$

$$\alpha_0\, \nabla^2 \Phi - \eta_0\, \Phi = \frac{J}{2}\, \frac{\partial^2 \Phi}{\partial t^2} \qquad \ldots(2.6)$$

where λ, μ, α, β, γ, α_0, β_0, η_0 are material constants, t_{ji}, m_{ji} and λ_j are the components of the force stress, couple stress and first moment respectively, ρ the density, J the rotational inertia, $\vec{u}$ the displacement vector, $\vec{\omega}$ the microrotation vector and Φ the scalar microstretch.

3. Formulation of the Problem

We consider a micropolar elastic, homogeneous, isotropic and centrosymmetric body which can stretch. Let us take Oxyz as the frame of reference in cartesian

coordinates, the origin 0 being any point on the plane boundary and Ox pointing normally into the medium. Let the arbitrary load $p(y)$ be acting in the positive direction of x-axis at the plane face $x = 0$ of the elastic half space $x \geq 0$.

Since we are interested in solving the plane problem, we assume that all causes and effects depend only on two variables x, y.

Let us take $\vec{u} = (u_1, u_2, 0)$, $\vec{\omega} = (0, 0, \omega_3)$.

Moreover, we are discussing static problem, hence

$$\frac{\partial^2 \vec{u}}{\partial t^2} = \frac{\partial^2 \vec{\omega}}{\partial t^2} = \frac{\partial^2 \Phi}{\partial t^2} = 0. \qquad \ldots(3.1)$$

We introduce potentials ϕ, ψ which are related to the displacements as follows

$$\left.\begin{aligned} u_1 &= \frac{\partial \phi}{\partial x} + \frac{\partial \psi}{\partial y} \\ u_2 &= \frac{\partial \phi}{\partial y} - \frac{\partial \psi}{\partial x} \end{aligned}\right\}. \qquad \ldots(3.2)$$

Substituting (3.2) in (2.4)-(2.6) and after using (3.1), we get

$$(\lambda + 2\mu) \frac{\partial}{\partial x} (\nabla^2 \phi) + \frac{\partial}{\partial y} [(\mu + \alpha) \nabla^2 \psi + 2\alpha\omega_3] = 0 \qquad \ldots(3.3)$$

$$(\lambda + 2\mu) \frac{\partial}{\partial y} (\nabla^2 \phi) - \frac{\partial}{\partial x} [(\mu + \alpha) \nabla^2 \psi + 2\alpha\omega_3] = 0 \qquad \ldots(3.4)$$

$$[(\gamma + \epsilon) \nabla^2 - 4\alpha] \omega_3 - 2\alpha \nabla^2 \psi = 0 \qquad \ldots(3.5)$$

$$\alpha_0 \nabla^2 \Phi - \eta_0 \Phi = 0. \qquad \ldots(3.6)$$

The boundary conditions are

$$t_{11} = -p(y), \; t_{12} = m_{13} = \lambda_1 = 0, \text{ at } x = 0. \qquad \ldots(3.7)$$

In items of ϕ and ψ

$$\left.\begin{aligned} t_{11} &= \lambda \nabla^2 \phi + 2\mu \left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \psi}{\partial x \, \partial z} \right) \\ t_{12} &= \mu \left[2 \frac{\partial^2 \phi}{\partial x \, \partial y} + \frac{\partial^2 \psi}{\partial y^2} - \frac{\partial^2 \psi}{\partial x^2} \right] - \alpha (\nabla^2 \psi + 2\omega_3) \\ m_{13} &= (\gamma + \epsilon) \frac{\partial \omega_3}{\partial x} - \beta_0 \frac{\partial \Phi}{\partial y} \\ \lambda_1 &= \alpha_0 \frac{\partial \Phi}{\partial x} + \frac{1}{3} \beta_0 \frac{\partial \omega_3}{\partial y}. \end{aligned}\right\} \qquad \ldots(3.8)$$

4. Solution ot the Equations

Equations (3.3)-(3.6) have been solved by Fourier integral transform defined by

$$\left.\begin{aligned} \bar{f}(x, \xi) &= \int_{-\infty}^{\infty} f(x, y)\, e^{i\xi y}\, dy \\ \text{and its inverse by} \quad & \\ f(x, y) &= \frac{1}{2\pi} \int_{-\infty}^{\infty} \bar{f}(x, \xi)\, e^{-i\xi y}\, d\xi. \end{aligned}\right\} \quad \ldots(4.1)$$

Applying transformation (4.1) to (3.3)-(3.6), we get

$$\left(\frac{d^2}{dx^2} - \xi^2\right)^2 \bar{\phi} = 0 \quad \ldots(4.2)$$

$$\left(\frac{d^2}{dx^2} - \xi^2\right)\left(\frac{d^2}{dx^2} - \eta^2\right)\bar{\omega}_3 = 0 \quad \ldots(4.3)$$

$$\frac{d}{dx}\left(\frac{d^2}{dx^2} - \xi^2\right)\bar{\phi} = \frac{2i\,\mu\, m^2\, \xi}{\lambda + 2\mu}\left(\frac{d^2}{dx^2} - \eta^2\right)\bar{\omega}_3 \quad \ldots(4.4)$$

$$\left(\frac{d^2}{dx^2} - \zeta^2\right)\bar{\Phi} = 0 \quad \ldots(4.5)$$

where

and

$$\left.\begin{aligned} & \eta^2 = \xi^2 + m_1^2,\ \zeta^2 = \xi^2 + m_2^2 \\ & m_1^2 = \frac{4\alpha\mu}{(\mu + \alpha)(\gamma + \epsilon)},\ m_2^2 = \eta_0/\alpha_0 \end{aligned}\right\}. \quad \ldots(4.6)$$

Since, all stresses and displacements should tend to zero as $x \to \infty$, we seek the solutions of (4.2), (4.3) and (4.5) in the form

$$\bar{\phi} = (A + Bx)\, e^{-\xi x} \quad \ldots(4.7)$$

$$\bar{\omega}_3 = C\, e^{-\xi x} + De^{-\eta x} \quad \ldots(4.8)$$

$$\bar{\Phi} = E\, e^{-\zeta x}. \quad \ldots(4.9)$$

It is auumed that Re (ξ), Re (η) and Re (ζ) are all nonnegative. Substituting (4.7) and (4.8) in (4.4) and comparing the coefficients of $e^{-\xi x}$, we get

$$C = \frac{i(\lambda + 2\mu)}{\mu}\, \xi\, B. \quad \ldots(4.10)$$

Thus

$$\bar{\omega}_3 = \frac{i(\lambda + 2\mu)}{\mu} \xi B e^{-\xi x} + D e^{-\eta x}. \qquad \ldots(4.11)$$

From (3.5), (4.1) and (4.11) we get

$$\psi = \frac{i(\lambda + 2\mu)}{\mu} x B e^{-\xi x} - \left(\frac{\gamma + \epsilon}{2\mu} \right) D e^{-\eta x}. \qquad \ldots(4.12)$$

From (3.7), (3.8), (4.1), (4.7), (4.9), (4.11) and (4.12), we get the following set of equations

$$2\mu \xi^2 A - i(\gamma + \epsilon) \eta \xi D = -\bar{p}(\xi) \qquad \ldots(4.13)$$

$$2i\mu \xi A + 2i(\lambda + \mu) B + (\gamma + \epsilon) \xi D = 0 \qquad \ldots(4.14)$$

$$i(\lambda + 2\mu)(\gamma + \epsilon) \xi^2 B + \mu(\gamma + \epsilon) \eta D - i\mu B_0 \xi E = 0 \qquad \ldots(4.15)$$

$$(\lambda + 2\mu) \beta_0 \xi^2 B - i\mu \beta_0 \xi D - 3\alpha_0 \mu \zeta E = 0 \qquad \ldots(4.16)$$

where

$$\bar{p}(\xi) = \int_{-\infty}^{\infty} p(y) e^{i\xi y} dy. \qquad \ldots(4.17)$$

Solving eqns. (4.13)-(4.16), for A, B, D and E, we get

$$A = -\frac{\bar{p}(\xi)}{2\mu \xi^2 \Delta} \left[\left(1 - S \frac{\xi}{\eta} \right) - \epsilon_0 \frac{\xi^3}{\eta} (1 - S) \right] \qquad \ldots(4.18)$$

$$B = \frac{\bar{p}(\xi)}{\epsilon^2 \xi \Delta} \left(1 - S \frac{\xi}{\eta} \right) \qquad \ldots(4.19)$$

$$D = -\frac{i(\lambda + 2\mu) \bar{p}(\xi)}{\mu \epsilon_2 \Delta} \frac{\xi}{\eta} (1 - S) \qquad \ldots(4.20)$$

$$E = \frac{\epsilon_0 \beta_0 \bar{p}(\xi)}{\epsilon_1 \Delta} \frac{\xi}{\zeta} \left(1 - \frac{\xi}{\eta} \right) \qquad \ldots(4.21)$$

where

$$\left.\begin{aligned} &\epsilon_1 = 3\alpha_0 (\gamma + \epsilon), \ \epsilon_2 = 2(\lambda + \mu), \ \epsilon_3 = (\gamma + \epsilon)(\lambda + 2\mu) \\ &S = \frac{\beta_0^2 \xi}{\epsilon_1 \zeta}, \ \epsilon_0 = \frac{\epsilon_3}{\mu \epsilon_2} \\ \text{and} \quad &\Delta = \left(1 - S \frac{\xi}{\eta} \right) + \epsilon_0 \xi^2 (1 - S) \left(1 - \frac{\xi}{\eta} \right). \end{aligned}\right\} \qquad \ldots(4.22)$$

Thus

$$\bar{\phi} = \frac{\bar{p}(\xi)}{\xi \Delta}\left[-\frac{1}{2\mu\xi}\left\{\left(1-S\frac{\xi}{\eta}\right)-\epsilon_0\frac{\xi^3}{\eta}(1-S)\right\}+\frac{1}{\epsilon_2}\left(1-S\frac{\xi}{\eta}\right)x\right]e^{-\xi x} \quad \ldots(4.23)$$

$$\bar{\psi} = \frac{i\bar{p}(\xi)}{\mu\Delta}\left[\frac{(\lambda+2\mu)}{\epsilon_2}\left(1-S\frac{\xi}{\eta}\right)\frac{x}{\xi}e^{-\xi x}+\frac{\epsilon_0}{2}\frac{\xi}{\eta}(1-S)e^{-\eta x}\right] \quad \ldots(4.24)$$

$$\bar{\omega}_3 = \frac{i(\lambda+2\mu)\bar{p}(\xi)}{\mu\epsilon_2\Delta}\left[\left(1-S\frac{\xi}{\eta}\right)e^{-\xi x}-\frac{\xi}{\eta}(1-S)e^{-\eta x}\right] \quad \ldots(4.25)$$

$$\bar{\Phi} = \frac{\epsilon_0\beta_0\bar{p}(\xi)}{\epsilon_1\Delta}\frac{\xi}{\zeta}\left(1-\frac{\xi}{\gamma_1}\right)e^{-\xi x}. \quad \ldots(4.26)$$

Applying Fourier transform (4.1) to (3.2) and (3.8) and using (4.23)-(4.26) and taking Fourier inverse transform, we get the displacements, force stresses, couple stress and first moment in the following forms :

$$u_1 = \frac{1}{4\pi\mu}\int_{-\infty}^{\infty}\frac{p(\xi)}{\Delta}\left[\left(1-S\frac{\xi}{\eta}\right)\left\{x+2(1-\nu)\frac{1}{\xi}\right\}e^{-\xi x}\right.$$

$$\left.+\epsilon_0(1-S)\frac{\xi^2}{\gamma_1}(e^{-\eta x}-e^{-\xi x})\right]e^{-i\xi y}\,d\xi \quad \ldots(4.27)$$

$$u_2 = \frac{i}{4\pi\mu}\int_{-\infty}^{\infty}\frac{\bar{p}(\xi)}{\Delta}\left[\left(1-S\frac{\xi}{\eta}\right)\left\{x+(2\nu-1)\frac{1}{\xi}\right\}e^{-\xi x}\right.$$

$$\left.+\epsilon_0\xi(1-S)\left\{e^{-\eta x}-\frac{\xi}{\eta}e^{-\xi x}\right\}\right]e^{-i\xi y}\,d\xi \quad \ldots(4.28)$$

$$\omega_3 = \frac{i(\lambda+2\mu)}{2\pi\mu\epsilon_2}\int_{-\infty}^{\infty}\frac{\bar{p}(\xi)}{\Delta}\left[\left(1-S\frac{\xi}{\eta}\right)e^{-\xi x}-\frac{\xi}{\eta}(1-S)e^{-\eta x}\right]e^{-i\xi y}\,d\xi \quad \ldots(4.29)$$

$$\Phi = \frac{\epsilon_0\beta_0}{2\pi\epsilon_1}\int_{-\infty}^{\infty}\frac{\bar{p}(\xi)}{\Delta}\frac{\xi}{\zeta}\left(1-\frac{\xi}{\eta}\right)e^{-(\xi x+i\xi y)}\,d\xi \quad \ldots(4.30)$$

and

$$t_{11} = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\bar{p}(\xi)}{\Delta}\left[\left(1-S\frac{\xi}{\eta}\right)(1+\xi x)e^{-\xi x}+\epsilon_0\xi^2(1-S)\left(e^{-\eta x}\right.\right.$$

$$\left.\left.-\frac{\xi}{\eta}e^{-\xi x}\right)\right]e^{-i\xi y}\,d\xi \quad \ldots(4.31)$$

$$t_{12} = -\frac{i}{2\pi}\int_{-\infty}^{\infty}\frac{\bar{p}(\xi)}{\Delta}\left[\left(1-S\frac{\xi}{\eta}\right)\xi x e^{-\xi x} + \epsilon_0\frac{\xi^3}{\eta}(e^{-\eta x}-e^{-\xi x})\right]e^{-i\xi y}\,d\xi \qquad \ldots(4.32)$$

$$m_{13} = -\frac{i\epsilon_0}{2\pi}\int_{-\infty}^{\infty}\frac{\xi\bar{p}(\xi)}{\Delta}\left[(e^{-\xi x}-e^{-\eta x}) + S(e^{-\eta x}-e^{-\zeta x}) + S\frac{\xi}{\eta}(e^{-\zeta x}-e^{-\xi x})\right]e^{-i\xi y}\,d\xi \qquad \ldots(4.33)$$

$$\lambda_1 = -\frac{(1-\nu)\beta_0}{6\pi\mu}\int_{-\infty}^{\infty}\frac{\xi\bar{p}(\xi)}{\Delta}\left[(e^{-\zeta x}-e^{-\xi x}) + \frac{\xi}{\eta}(e^{-\eta x}-e^{-\zeta x}) + S\frac{\xi}{\eta}(e^{-\xi x}-e^{-\eta x})\right]e^{-i\xi y}\,d\xi \qquad \ldots(4.34)$$

where ν is the Poisson's ratio.

If we make the terms corresponding to stretch tend to zero, we get the results for micropolar elastic solid without stretch as

$$u_1 = \frac{1}{4\pi\mu}\int_{-\infty}^{\infty}\frac{\bar{p}(\xi)}{\Delta_0}\left[\left\{x+2(1-\nu)\frac{1}{\xi}\right\}e^{-\xi x} + \epsilon_0\frac{\xi^2}{\eta}(e^{-\eta x}-e^{-\xi x})\right]e^{-i\xi y}\,d\xi \qquad \ldots(4.35)$$

$$u_2 = \frac{i}{4\pi\mu}\int_{-\infty}^{\infty}\frac{\bar{p}(\xi)}{\Delta_0}\left[\left\{x+(2\nu-1)\frac{1}{\xi}\right\}e^{-\xi x} + \epsilon_0\,\xi\left\{e^{-\eta x} - \frac{\xi}{\eta}e^{-\xi x}\right\}\right] e^{-i\xi y}d\xi \qquad \ldots(4.36)$$

$$\omega_3 = \frac{i(\lambda+2\mu)}{\mu\epsilon_2}\int_{-\infty}^{\infty}\frac{\bar{p}(\xi)}{\Delta_0}\left[e^{-\xi x} - \frac{\xi}{\eta}e^{-\eta x}\right]e^{-i\xi y}\,d\xi \qquad \ldots(4.37)$$

and

$$t_{11} = -\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{\bar{p}(\xi)}{\Delta_0}\left[(1+\xi x)e^{-\xi x} + \epsilon_0\xi^2(e^{-\eta x} - \frac{\xi}{\eta}e^{-\xi x})\right]e^{-i\xi y}\,d\xi \qquad \ldots(4.38)$$

$$t_{12} = -\frac{i}{2\pi}\int_{-\infty}^{\infty}\frac{\bar{p}(\xi)}{\Delta_0}\left[\xi x e^{-\xi x} + \epsilon_0\frac{\xi^3}{\eta}(e^{-\eta x}-e^{-\xi x})\right]e^{-i\xi y}\,d\xi \qquad \ldots(4.39)$$

$$m_{13} = -\frac{i\epsilon_0}{2\pi}\int_{-\infty}^{\infty}\frac{\bar{p}(\xi)}{\Delta_0}(e^{-\xi x}-e^{-\eta x})\,e^{-i\xi y}\,d\xi \qquad \ldots(4.40)$$

where

$$\Delta_0 = 1 + \epsilon_0 \xi^2 \left(1 - \frac{\xi}{\eta}\right). \qquad \ldots(4.41)$$

These results agree with those obtained by Nowacki[8].

5. Particular Case

Let us take an arbitrary load $p(y)$ as an impulsive line source

$$p(y) = \frac{F_0}{2\pi} \delta(y) \qquad \ldots(5.1)$$

where F_0 is the magnitude of the force and $\delta(y)$ the Dirace-delta function.

Applying Fourier transform on (5.1), we get

$$\bar{p}(\xi) = \frac{F_0}{2\pi}. \qquad \ldots(5.2)$$

Thus from (4.27)–(4.34) and (5.2) we get the displacement components, microstretch, force stress, couple stress and first moment as

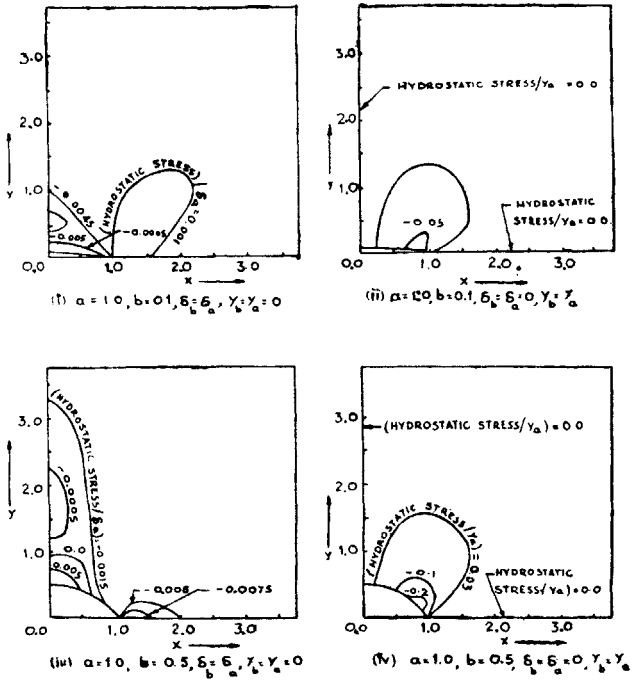

FIG. 1 Lines of Hydrostatic Streess in Matrix.

$$u_1 = \frac{F_0}{8\pi^2\mu}\int_{-\infty}^{\infty}\frac{1}{\Delta}\left[\left(1-S\,\frac{\xi}{\eta}\right)\left\{x+2(1-\nu)\,\frac{1}{\xi}\right\}e^{-\xi x}\right.$$

$$\left.+\,\epsilon_0\,\frac{\xi^2}{\eta}\,(1-S)\,(e^{-\eta x}-e^{-\xi x})\right]e^{-i\xi y}\,d\xi \qquad \ldots(5.3)$$

$$u_2 = \frac{i\,F_0}{8\pi^2\,\mu}\int_{-\infty}^{\infty}\frac{1}{\Delta}\left[\left(1-S\,\frac{\xi}{\eta}\right)\left\{x+(2\nu-1)\right\}e^{-\xi x}\right.$$

$$\left.+\,\epsilon_0\,\xi\,(1-S)\left\{e^{-\eta x}-\frac{\xi}{\eta}\,e^{-\xi x}\right\}\right]e^{-i\xi y}\,d\xi \qquad \ldots(5.4)$$

$$\omega_3 = \frac{i\,(\lambda+2\mu)\,F_0}{4\pi^2\,\mu\,\epsilon_2}\int_{-\infty}^{\infty}\frac{1}{\Delta}\left[\left(1-S\,\frac{\xi}{\eta}\right)e^{-\xi x}-\frac{\xi}{\eta}\,(1-S)\,e^{-\eta x}\right]e^{-i\xi y}d\xi \qquad \ldots(5.5)$$

$$\Phi = \frac{\epsilon_0\,\beta_0\,F_0}{4\pi^2\,\epsilon_1}\int_{-\infty}^{\infty}\frac{1}{\Delta}\,\frac{\xi}{\eta}\left(1-\frac{\xi}{\eta}\right)e^{-(\xi x+i\xi y)}\,d\xi \qquad \ldots(5.6)$$

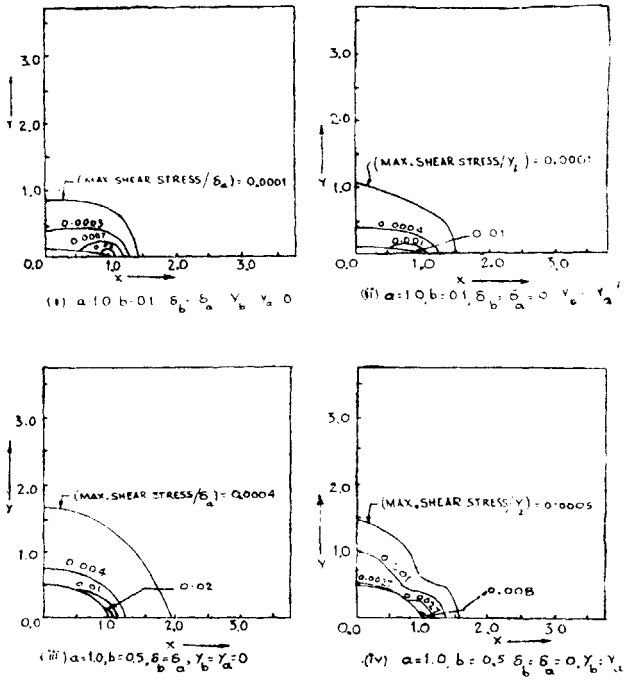

FIG. 2. Lines of Max. Shear Stress in Matrix.

and

$$t_{11} = -\frac{F_0}{4\pi^2}\int_{-\infty}^{\infty}\frac{1}{\Delta}\left[\left(1-S\,\frac{\xi}{\eta}\right)(1+\xi x)\,e^{-\xi x} + \epsilon_0\,\xi^2\,(1-S)\,(e^{-\eta x}-\frac{\xi}{\eta}\,e^{-\xi x})\right]e^{-i\xi y}\,d\xi \qquad ...(5.7)$$

$$t_{12} = -\frac{i\,F_0}{4\pi^2}\int_{-\infty}^{\infty}\frac{1}{\Delta}\left[\left(1-S\,\frac{\xi}{\eta}\right)\xi x\,e^{-\xi x} + \epsilon_0\,\frac{\xi^3}{\eta}\,(1-S)\,(e^{-\eta x}-e^{-\xi x})\right] \times e^{-i\xi y}\,d\xi \qquad ...(5.8)$$

$$m_{13} = -\frac{i\,\epsilon_0\,F_0}{4\pi^2}\int_{-\infty}^{\infty}\frac{\xi}{\Delta}\left[(e^{-\xi x}-e^{-\eta x}) + S\,(e^{-\eta x}-e^{-\zeta x}) + S\,\frac{\xi}{\eta}\,(e^{-\zeta x}-e^{-\xi x})\right]e^{-i\xi y}\,d\xi \qquad ...(5.9)$$

$$\lambda_1 = -\frac{(1-\nu)\,\beta_0\,F_0}{12\,\pi^2\,\mu}\int_{-\infty}^{\infty}\frac{\xi}{\Delta}\left[(e^{-\zeta x}-e^{-\xi x}) + \frac{\xi}{\eta}\,(e^{-\eta x}-e^{-\zeta x}) + S\,\frac{\xi}{\eta}\,(e^{-\xi x}-e^{-\zeta x})\right]e^{-i\xi y}\,d\xi. \qquad ...(5.10)$$

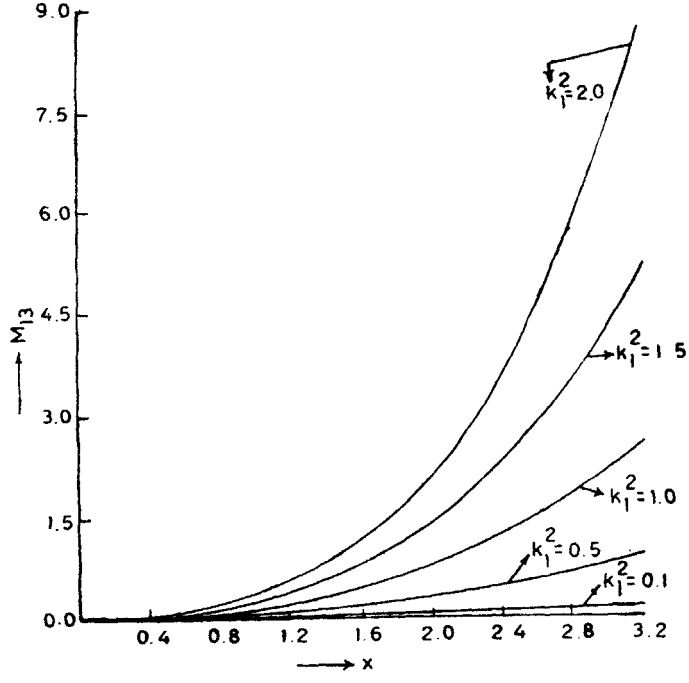

FIG. 3. Couple stress M_{13} with stretch $M_{13} = -\pi F_0^{-1} m_{13}$.

6. Evalutation of the Integrals

The integrals appearing in the expressions (5.3)-(5.10) are difficult to evaluate analytically, but these can be evaluated either numerically or asymptotically. We evaluate these integrals for large ξ, expanding η, ζ, $\eta\Delta$ in terms of $1/\xi$, we obtain approximately

$$\eta = \xi + m_1^2/2\xi,\ \zeta = \xi + m_2^2/2\xi,\ \eta\Delta = RL$$

where

$$R = \left(1 + \frac{\epsilon_0 m_1^2}{2}\right),\ L = \left(1 - \frac{\beta_0^2}{\epsilon_1}\right). \qquad \ldots(6.1)$$

Hence, from (5.3)-(5.10), we get

$$u_1 = \frac{F_0}{8\pi^2 \mu R} \int_{-\infty}^{\infty} \left[Qx + \{2(1-\nu) + Q_1 x^2\} \frac{1}{\xi}\right] e^{-\xi(x+iy)}\, d\xi \qquad \ldots(6.2)$$

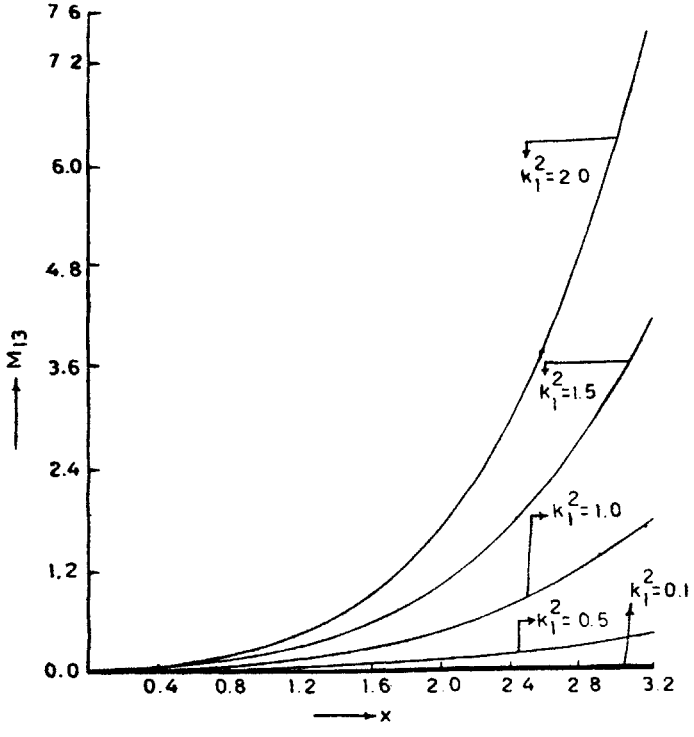

FIG. 4. Couple stress M_{13} without stretch $M_{13} = -\pi F_0^{-1} m_{13}$,

$$u_2 = \frac{i F_0}{8\pi^2 \mu R} \int_{-\infty}^{\infty} \left[Qx + \left\{ (2\nu - 1) + Q_2 \left(1 + \frac{m_1^2}{4} x^2 \right) \right\} \frac{1}{\xi} \right] e^{-\xi (x+iy)} \, d\xi \qquad \ldots(6.3)$$

$$\omega_3 = \frac{i(1-\nu) m_1^2 F_0}{8\pi^2 \mu R} \int_{-\infty}^{\infty} \left[\frac{x}{\xi} + \frac{1}{L\xi^2} - \frac{m_1^2}{4} \frac{x^2}{\xi^2} \right] e^{-\xi (x+iy)} \, d\xi \qquad \ldots(6.4)$$

$$\Phi = \frac{\beta_0 F_0 Q_2}{4\pi^2 \epsilon_1 RL} \int_{-\infty}^{\infty} \frac{1}{\xi^2} e^{-\xi (x+iy)} \, d\xi \qquad \ldots(6.5)$$

$$t_{11} = -\frac{F_0}{4\pi^2 R} \int_{-\infty}^{\infty} \left[Q \xi x + (R + Q_1 x^2) + (m_1^2 + QQ_3) \frac{x}{2L\xi} \right.$$

$$\left. - 2Q_1 \left(1 + \frac{m_1^2}{12} x^2 \right) \frac{x}{\xi} \right] e^{-\xi (x+iy)} \, d\xi \qquad \ldots(6.6)$$

$$t_{12} = -\frac{i F_0}{4\pi^2 R} \int_{-\infty}^{\infty} \left[Q \xi x + Q_1 x^2 + (m_1^2 + QQ_3) \frac{x}{2L\xi} \right.$$

$$\left. - Q_1 m_1^2 \frac{x^3}{6\xi} \right] e^{-\xi (x+iy)} \, d\xi \qquad \ldots(6.7)$$

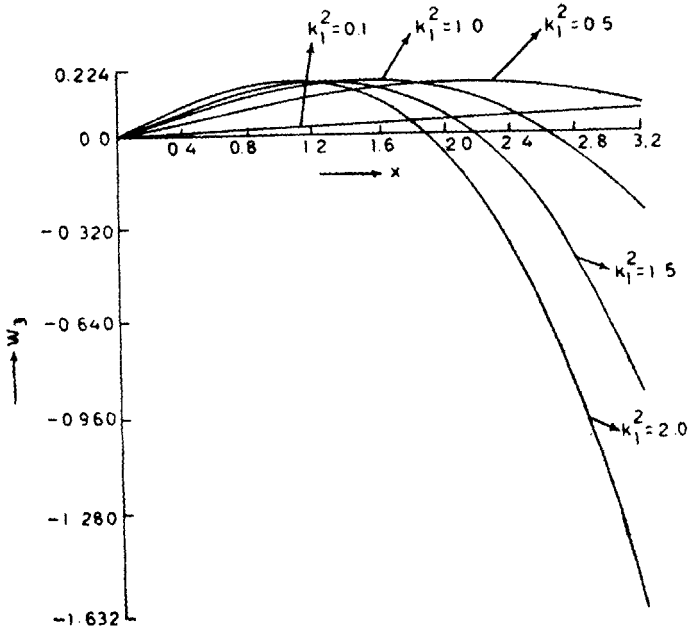

FIG. 5. Microrotation W_2 with stretch $W_3 = \pi \mu F_0^{-1} \omega_3$.

$$m_{13} = -\frac{i\,Q_2\,F_0}{4\pi^2\,R}\int_{-\infty}^{\infty}\left[1-m_1^2\,\frac{x^2}{4\xi}+\left\{\frac{m_1^2}{2}\left(1+\frac{m_1^2}{12}\,x^2\right)+\frac{Q_3}{L}\right\}\frac{x}{\xi^2}\right]e^{-\xi\,(\alpha+iy)}\,d\xi \qquad \ldots(6.8)$$

$$\lambda_1 = \frac{(1-\nu)\,m_1^2\,\beta_0\,F_0\,x}{24\pi^2\,\mu R}\int_{-\infty}^{\infty}\left(1-m_1^2\,\frac{x}{4\xi}\right)e^{-\xi\,(\alpha+iy)}\,d\xi \qquad \ldots(6.9)$$

where

$$Q = (1-Q_2),\ Q_1 = \epsilon_0\,m_1^4/8,\ Q_2 = \epsilon_0\,m_1^2/2,\ Q_3 = \beta_0\,m_3^2/\epsilon_1. \qquad \ldots(6.10)$$

Evaluating the integrals by the method of calculas of residues, we get

$$u_1 = -\frac{i\,F_0}{8\pi\,\mu R}\left[2\,(1-\nu)+Q_1\,x^2\right] \qquad \ldots(6.11)$$

$$u_2 = \frac{I_0}{8\pi\,\mu R}\left[(2\nu-1)+Q_2\left(1+\frac{m_1^2}{4}\,x^2\right)\right] \qquad \ldots(6.12)$$

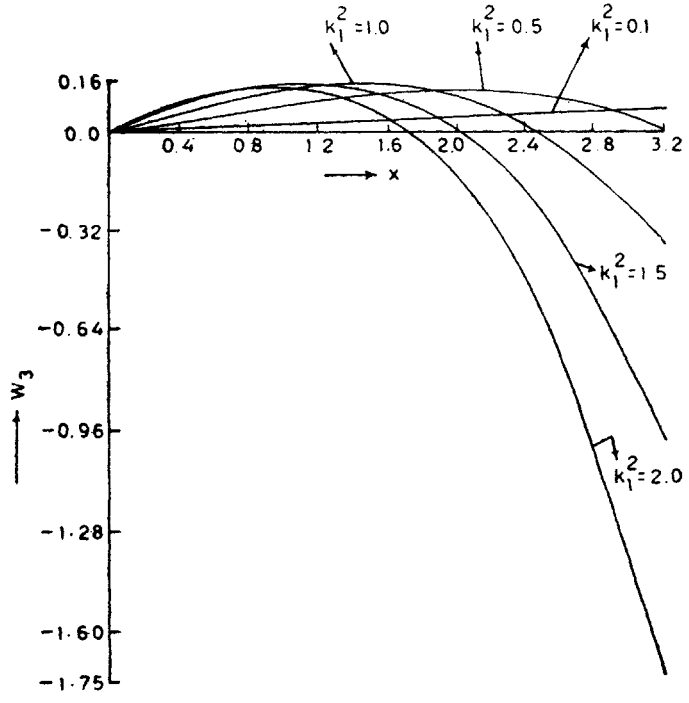

FIG. 6. Microrotation W_3 without stretch $W_3 = \pi\mu\,F_0^{-1}\omega_3$.

$$\omega_3 = \frac{(1-\nu)\, m_1^2\, F_0}{8\pi\, \mu R}\left[x - \left(\frac{m_1^2}{2}\, x_2 - \frac{2}{L}\right)(x+iy)\right] \qquad \ldots(6.13)$$

$$\Phi = \frac{\beta_0\, F_0\, Q_2}{2\pi\, \epsilon_1\, RL}\,(y-ix) \qquad \ldots(6.14)$$

and

$$t_{11} = \frac{iF_0\, x}{8\pi R}\left[(m_1^2 + QQ_3)\,\frac{1}{L} - 4Q_1\left(1 + \frac{m_1^2}{12}\, x^2\right)\right] \qquad \ldots(6.15)$$

$$t_{12} = -\frac{F_0\, x}{8\pi R}\left[(m_1^2 + QQ_3)\,\frac{1}{L} - Q_1\,\frac{m_1^2}{3}\, x^3\right] \qquad \ldots(6.16)$$

$$m_{13} = \frac{(1-\nu)\, F_0\, m_1^2\, x}{4\pi R}\left[\frac{m_1^2\, x}{4} - \left(\frac{m_1^4\, x^2}{12}\, x^2 + m_1^2 + \frac{2}{L}\, Q_3\right)(x+iy)\right] \qquad \ldots(6.17)$$

$$\lambda_1 = \frac{i\,(1-\nu)\,\beta_0\, m_1^4\, x^2\, F_0}{96\pi\, \mu R}. \qquad \ldots(6.18)$$

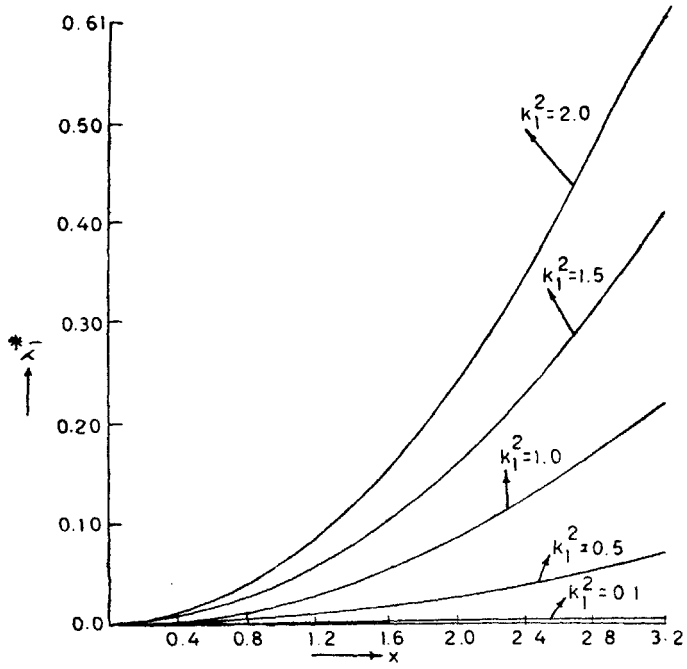

FIG. 7. First moment λ_1^*

$$\lambda_1^* = -\,i\,96\pi\mu\, F_0^{-1}\,\lambda_1$$

7. Numerical Results and Discussions

We have calculated normal displacement, microrotation, normal force stress, couple stress and first moment for various values of α in the plane $y = 0$, $\nu = 0.25$, $(\gamma + \epsilon)/2\mu = 1$, $\alpha_0 = 0.02$, $\beta_0 = 0.05$, $\eta_0 = 0.03$, $(\gamma + \epsilon) = 0.2$, As $k_1^2 = 2\alpha/(\mu+\alpha)$ we observe that the interval $0 \leqslant \alpha \leqslant \infty$ is transformed to the interval $0 \leqslant k_1^2 \leqslant 2$.

The variations of normal force stress with and without stretch are shown in Figs. 1 and 2. It is observed that normal force stress is a monotonically increasing function of x for $k_1^2 = 0.1, 0.5$ in both the cases. For $k_1^2 = 1.0$, normal force increases with x for $0 \leqslant x \leqslant 2.0$ (Fig. 1), and for $0 \leqslant x \leqslant 1.6$ (Fig. 2) and then it decreases in both of the cases as x increases further. From Fig. 1, it is evident that for $k_1^2 = 1.5$, normal force stress increases with x for $0 \leqslant x \leqslant 0.4$ and it decreases as x increases further, but Fig. 2 shows that it decreases monotonically. For $k_1^2 = 2.0$, normal force stress decreases monotonically (Figs. 1 and 2).

It is observed that couple stress with and without stretch increases monotonically with x for particular models corresponding to $k_1^2 = 0.1, 0.5, 1.0, 1.5, 2.0$. These variations have been shown graphically in Figs. 3 and 4.

The variations of microrotation with and without stretch are shown in Figs. 5 and 6. It is observed that microrotation increases as x increases for $k_1^2 = 0.1$ (Figs. 5 and 6). From, Figs. 5 and 6, we see that for $k_1^2 = 1.0$ microrotation increases for $0 \leqslant x \leqslant 1.6$ and decreases as x increases further. It is observed that for $k_1^2 = 1.5$,

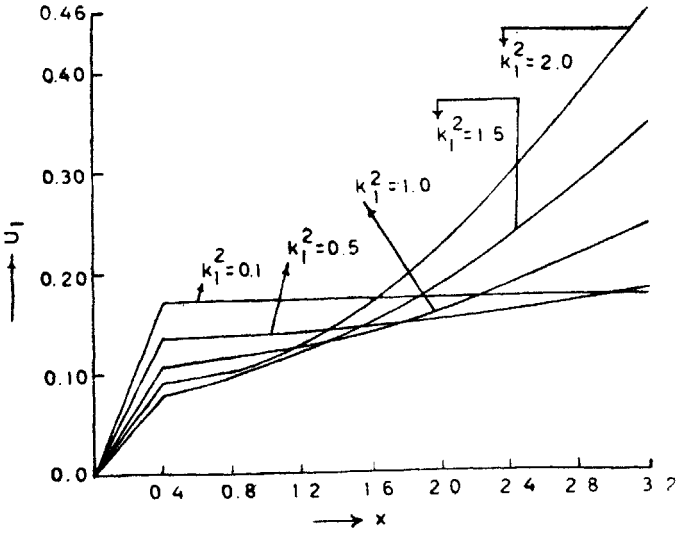

Fig. 8. Normal displacement U_1

$$U_1 = i\pi\mu F_0^{-1} U_1.$$

2.0, microrotation increases for $0 \leqslant x \leqslant 1.2$ and decreases as x increases further (Figs. 5 and 6).

It is also noticed that first moment increases as x increases for particular models corresponding to $k_1^2 = 0.1, 0.5, 1.0, 1.5, 2.0$ and these variations are shown in Fig. 7.

Figure 8 shows the variation of normal displacement. For $k_1^2 = 0.1$, normal displacement increases very rapidly for $0 \leqslant x \leqslant 0.4$, but for $x > 0.4$, it increases at a very slow rate. We observe that for $k_1^2 = 0.5$, normal displacement is smaller than that for the case $k_1^2 = 0.1$, for $x = 0.4$ and then it increases at a higher rate as x increases in comparison to the previous case. The same trend is observed for $k_1^2 = 1.0$, 1.5 and 2.0.

8. Conclusion

We conclude from above that displacement, microrotation, normal force stress and the couple stress are affected by stretch. Under the approximations used for the evaluation of the integrals, the stretch effect on the displacement vanishes. For the models discussed above, we observe that as x increases, the stretch effect increases on the normal force stress, couple stress and microrotation.

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