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LINE CRACK IN AN ELASTIC PLATE SUBJECT TO VARYING INTERNAL PRESSURE IN NONLOCAL ELASTICITY

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The problem of a line crack in an elastic plate subject to an internal pressure varying along the length of the crack using the nonlocal theory of elasticity is solved. The stress intensity factor is calculated and the corresponding results of the classical theory are obtained as particular cases.

1. INTRODUCTION

A systematic approach to nonlocal elasticity was presented by Eringen and Edelen[1], through both the balance laws and the thermodynamic and nonlocal variational principles. A few problems that have been studied recently using the nonlocal theory of elasticity include stress concentration at the tip of crack[2], line crack subject to shear[3], and line crack subject to antiplane shear[4].

The theory of cracks in a two dimensional elastic medium was first developed by Griffith[8]. Sneddon and Elliott[12] solved the problem of finding the distribution of stress in the neighbourhood of a Griffith crack that is subject to an internal pressure which may vary along the length of crack by considering the corresponding boundary value problem for a semi-infinite two dimensional medium. Lowengrub and Srivastava[9] derived the expressions for the stress intensity factors and the crack energy.

In this paper an attempt has been made to solve the problem of a line crack in an elastic plate subject to an internal pressure varying along the length of the crack using the nonlocal theory of elasticity. It is interesting to observe that for a varying internal pressure there appears a singularity in the stress distribution and thus the stress intensity factor is obtained. Further the singularity in the stress distribution disappears if the internal pressure is taken as constant. This position tallys with the earlier results[5].

2. BASIC EQUATIONS OF NONLOCAL ELASTICITY

Basic equations of linear, homogeneous, isotropic, nonlocal elastic solids with vanishing body inertia forces, are

$$t_{kl,k} = 0 \qquad \ldots(2.1)$$

$$t_{kl} = \int_V \left[\lambda' \left(| \hat{x}' - \hat{x} | \right) e_{rr} \left(\hat{x}' \right) \delta_{kl} + 2\mu' \left(| \hat{x}' - \hat{x} | \right) e_{kl} \left(\hat{x} \right) \right] dv \left(\hat{x}' \right). \qquad \ldots(2.2)$$

$$e_{kl} = \frac{1}{2} \left(u_{k,l} + u_{l,k} \right) \qquad \ldots (2.3)$$

where the only difference from classical elasticity is in the stress constitutive equations (2.2) in which the stress $t_{kl} (\hat{x})$ at a point $\hat{x}$ depends on the strains $e_{kl} (\hat{x}')$ at all points of the body. For homogeneous and isotropic solids there exists only two material moduli $\lambda' (|\hat{x}' - x|)$ and $\mu' (|\hat{x}' - \hat{x}|)$ which are functions of the distance $|\hat{x}' - \hat{x}|$. The integral in (2.2) is over the volume V of the body enclosed within the surface ∂V.

Throughout this paper we employ cartesian coordinates x_k with the usual convention that a free index takes the values (1, 2, 3) and repeated indices are summed over the range (1, 2, 3). Indices following a comma represent partial differentiation. e.g.,

$$u_{k,l} = \frac{\partial u_k}{\partial x_l} .$$

Eringen[1,5] has obtained the forms of $\lambda' (|\hat{x}' - \hat{x}|)$ and $\mu' (|\hat{x}' - \hat{x}|)$ for which the dispersion curves of Plane waves coincide within the entire Brillouin Zone with those obtained in the Born-Von Karman theory of lattice dynamics. Accordingly

$$(\lambda', \mu') = (\lambda, \mu)\, \alpha \left(|\hat{x}' - \hat{x}| \right)$$

$$\alpha \left(|\hat{x}' - \hat{x}| \right) = \begin{cases} \alpha_0 \left(a - |\hat{x}' - \hat{x}| \right) & |\hat{x}' - \hat{x}| \leqslant a \\ 0 & |\hat{x}' - \hat{x}| > a \end{cases} \qquad \ldots(2.4)$$

where a is the lattice parameter, λ and μ are classical Lamé constants and α_0 is normalization constant to be determined from

$$\int_V \alpha \left(|\hat{x}' - \hat{x}| \right) dv \left(\hat{x}' \right) = 1. \qquad \ldots(2.5)$$

While this simple and elegant result is useful in many calculations, it is not the only one that approximates the dispersion curves of lattice dynamics. In fact a very useful one is

$$\alpha \left(|\hat{x}' - \hat{x}| \right) = \alpha_0 \exp \left[- \left(\frac{\beta}{a} \right)^2 (x_k' - x_k) \; (x_k' - x_k) \right] \qquad \ldots(2.6)$$

where β is a constant. For the two dimensional case (2.6) has the specific form

$$\alpha\,(|\hat{x}' - \hat{x}|) = \frac{1}{\pi}\left(\frac{\beta}{a}\right)^2 \exp\left\{-\left(\frac{\beta}{a}\right)^2\left[(x_1' - x_1)^2 + (x_2' - x_2)^2\right]\right\}. \qquad \ldots(2.7)$$

Employing (2.4) in (2.2) we write

$$t_{kl} = \int_V \alpha\,(|\hat{x}' - \hat{x}|)\,\sigma_{kl}\,(\hat{x}')\,dv\,(\hat{x}') \qquad \ldots(2.8)$$

where

$$\begin{aligned}\sigma_{kl}\,(\hat{x}') &= \lambda e_{rr}\,(\hat{x}')\,\delta_{kl} + 2\mu\, e_{kl}\,(\hat{x}') \\ &= \lambda u_{r,r}\,(\hat{x}')\,\delta_{kl} + \mu\,[u_{k,l}\,(\hat{x}') + u_{l,k}\,(\hat{x}')] \qquad \ldots(2.9)\end{aligned}$$

is the classical Hooke's Law.

Substituting (2.8) into (2.1) and using Green-Gauss theorem, we obtain

$$\int_V \alpha\,(|\hat{x}' - \hat{x}|)\,\sigma_{kl,k}\,(\hat{x}')\,dv\,(\hat{x}') - \oint_{\partial V} \alpha\,(|\hat{x}' - \hat{x}|)\,\sigma_{kl}\,(\hat{x}')\,da_k\,(\hat{x}') = 0.$$

Here the surface integral may be dropped if the only surface of the body is at infinity.

3. Crack Under Varying Internal Pressure

Consider a plate in $(x_1 = x;\ x_2 = y)$ plane weakened by a line crack of length $2c$ along the x-axis. The plate is subjected to an internal varying pressure $p\,(x)$ along the surface of the crack Fig. 1. For the plane strain problem (2.10) takes the form

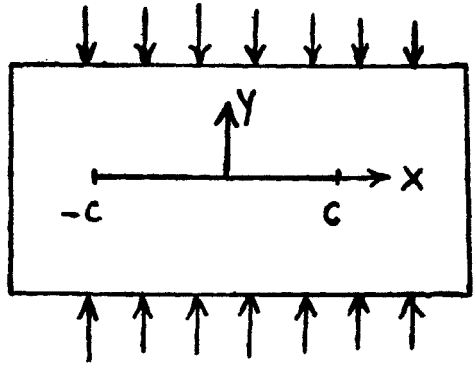

Fig. 1.

$$\int\!\!\!\!\!\!-_R \alpha\,(|\hat{x}' - \hat{x}|)\,\sigma_{kl,k}(x', y')\,dx'\,dy' - \int_{-c}^{c} \alpha\,(|\hat{x}' - \hat{x}|)\,[\sigma_{2l}(x', 0)]\,d\hat{x}' = 0 \quad \ldots(3.1)$$

where the integral with a slash is over the two dimensional infinite space excluding the crack line ($|x| < c, y = 0$). A boldface bracket indicates a jump at the crack line.

Under the applied internal pressure on the unopened surfaces of the crack, the displacement field possess the following symmetry regulations.

$$u(x, -y) = u(x, y);\ v(x, -y) = -v(x, y). \quad \ldots(3.2)$$

Employing this in (2.9) we get

$$[\sigma_{2l}(x, 0)] = 0 \quad |x| > c. \quad \ldots(3.3)$$

Hence the limits $(-c, c)$ in the second integral of (3.1) may be replaced by $(-\infty, \infty)$. The Fourier transform of (3.1) with respect to x' gives

$$\int_{-\infty}^{\infty} \bar{\alpha}\,(\xi, |y' - y|)\,[-i\xi\,\bar{\sigma}_{1l}(\xi, y') + \frac{d}{dy'}\,\bar{\sigma}_{2l}(\xi, y')]\,dy'$$

$$- \bar{\alpha}\,(\xi, |y|)\,[\bar{\sigma}_{2l}(\xi, 0)] = 0. \quad \ldots(3.4)$$

Here superposed bar indicates the Fourier transform, e. g.

$$\bar{f}(\xi, y) = 2\pi^{-1/2} \int_{-\infty}^{\infty} f(x, y) \exp(i\xi x)\,dx.$$

If we take the Fourier transform of (3.4) with respect to y and solve for the factor of $\bar{\alpha}$ in the integrand of (3.4) upon inversion we obtain

$$-i\xi\,\bar{\sigma}_{1l}(\xi, y) + \frac{d}{dy}\,\bar{\sigma}_{2l}(\xi, y)$$

$$= (2\pi)^{-1/2}\,[\bar{\sigma}_{2l}(x, 0)] \int_{-\infty}^{\infty} \exp(-i\eta y)\,d\eta$$

since the left-hand side is defined for all y except $y = 0$. Thus we obtain

$$-i\xi\,\bar{\sigma}_{1l} + \frac{d}{dy}\,\bar{\sigma}_{2l} = 0, \quad l = 1, 2. \quad \ldots(3.5)$$

Substituting this into (3.4) we get

$$[\sigma_{2l}(x, 0)] = 0, \quad l = 1, 2. \quad \ldots(3.6)$$

Hence we have shown that the general solution of (3.4) is the same as that of the system (3.5) and (3.6).

The jump condition (3.6) for $|x| > c$ is satisfied identically. For $|x| < c$ we also have $[\sigma_{2l}(x, 0)] = 0$, on account of (3.2). We see that $t_{xy}(x, 0) = 0$ for $|x| < c$. Thus the shearing stress condition on the crack surface is satisfied identically. Considering the continuity requirement of the displacement field satisfying (3.3), we find that the boundary conditions at $y = 0$ are

$$\begin{aligned} &\sigma_{xy}(x, 0) = 0 \text{ for all } x \\ &t_{yy}(x, 0) = -p(x) \quad 0 \leqslant x \leqslant c \\ &v(x, 0) = 0 \quad |x| > c. \end{aligned} \qquad \ldots (3.7)$$

In addition we must have

$$u = v = 0 \text{ as } y \to \infty. \qquad \ldots (3.8)$$

Consequently we must obtain the solution of (3.5) subject to (3.7) and (3.8). Equations (3.5) are none other than the Fourier transforms of the Navier's equations in two dimensions, namely,

$$\begin{aligned} &\mu \bar{u}_{,yy} - (\lambda + 2\mu)\xi^2 \bar{u} - i\xi(\lambda + \mu)\bar{v}_{,y} = 0 \\ &-i\xi(\lambda + \mu)\bar{u}_{,y} + (\lambda + 2\mu)\bar{v}_{,yy} - \xi^2 \mu \bar{v} = 0. \end{aligned} \qquad \ldots (3.9)$$

The general solution of this set (for $y > 0$) satisfying (3.8) is

$$u = (2\pi)^{-1/2} \int_{-\infty}^{\infty} i\xi^{-1} \left[|\xi| A(\xi) + \left(|\xi| y - \frac{\lambda + 3\mu}{\lambda + \mu} \right) B(\xi) \right] \times \exp(|\xi| y - i\xi x)\, d\xi \qquad \ldots (3.10)$$

$$v = (2\pi)^{-1/2} \int_{-\infty}^{\infty} [A(\xi) + y B(\xi)] \exp(-|\xi| y - i\xi x)\, d\xi$$

where $A(\xi)$ and $B(\xi)$ are to be determined from the boundary conditions (3.7). Using (2.9) we calculate

$$\bar{\sigma}_{xy} = \mu i \left[-\xi A(\xi) + B(\xi) \left(\frac{\lambda + 2\mu}{\lambda + \mu} - \xi y \right) \right] \exp(-|\xi| y). \qquad \ldots (3.11)$$

According to $(3.7)_1$ this must vanish at $y = 0$.

Hence

$$B(\xi) = \frac{\xi(\lambda + \mu)}{\lambda + 2\mu} A(\xi). \qquad \ldots(3.12)$$

Hence the displacement field may be put in the form

$$\left.\begin{aligned} u(x, y) &= \left(\frac{2}{\pi}\right)^{1/2} \frac{\lambda + \mu}{\lambda + 2\mu} \int_0^\infty A(\xi) \left[-\frac{\mu}{\lambda + \mu} + |\xi| y\right] \\ &\qquad \times \exp(-|\xi| y) \sin(\xi x)\, d\xi \\ v(x, y) &= \left(\frac{2}{\pi}\right)^{1/2} \frac{\lambda + \mu}{\lambda + 2\mu} \int_0^\infty A(\xi) \left[\frac{\lambda + 2\mu}{\lambda + \mu} + \xi y\right] \\ &\qquad \times \exp(-|\xi| y) \cos(\xi x)\, d\xi. \end{aligned}\right\} \qquad \ldots(3.13)$$

For σ_{kl} through (2.9) and (3.12) we obtain ($y > 0$)

$$\begin{aligned} \sigma_{xx}(x, y) &= \sigma_{xx}(x, -y) \\ &= -2\left(\frac{2}{\pi}\right)^{1/2} \mu \frac{\lambda + \mu}{\lambda + 2\mu} \int_0^\infty A(\xi) [\xi - \xi^2 y] \exp(-|\xi| y) \\ &\qquad \times \cos(\xi x)\, d\xi \qquad \ldots(3.14a) \end{aligned}$$

$$\begin{aligned} \sigma_{xy}(x, y) &= -\sigma_{xy}(x, -y) \\ &= -2\left(\frac{2}{\pi}\right)^{1/2} \mu \frac{\lambda + \mu}{\lambda + 2\mu} \int_0^\infty A(\xi) \xi^2 y \exp(-|\xi| y) \sin(\xi x)\, d\xi. \end{aligned} \qquad \ldots(3.14b)$$

$$\begin{aligned} \sigma_{yy}(x, y) &= \sigma_{yy}(x, -y) \\ &= -2\left(\frac{2}{\pi}\right)^{1/2} \mu \frac{\lambda + \mu}{\lambda + 2\mu} \int_0^\infty A(\xi) [\xi + \xi^2 y] \exp(-|\xi| y) \\ &\qquad \times \cos(\xi x)\, d\xi. \qquad \ldots(3.14c) \end{aligned}$$

The stress field according to (2.8) is then given by

$$t_{xx}(x, y) = \int_0^\infty \left(\int_{-\infty}^\infty \sigma_{xx}(x', y') [\alpha(|x' - x|, |y' - y|) + \alpha(|x' - x|, |y' + y|)]\, dx' \right) dy'$$

$$t_{xy}(x, y) = \int_0^\infty \left(\int_{-\infty}^\infty \sigma_{xy}(x', y') [\alpha(|x'-x|, |y'-y|) - \alpha(|x'-x|, |y'+y|)] \, dx' \right) dy'$$

$$t_{yy}(x, y) = \int_0^\infty \left(\int_{-\infty}^\infty \sigma_{yy}(x', y') [\alpha(|x'-x|, |y'-y|, + \alpha(|x'-x|, |y'+y|)] \, dx' \right) dy'. \quad \ldots(3.15)$$

Substituting for α from (2.7), the integrations may be performed with respect to x', and y' by noting the integrals[7]

$$I_1 = \int_{-\infty}^\infty \exp(-Px'^2) \begin{Bmatrix} \sin \xi(x + x') \\ \cos \xi(x + x') \end{Bmatrix} dx'$$

$$= \left(\frac{\pi}{P}\right)^{1/2} \exp\left(-\frac{\xi^2}{4P}\right) \begin{Bmatrix} \sin(\xi x) \\ \cos(\xi x) \end{Bmatrix}$$

$$I_2 = \int_0^\infty \exp(-Py' - \xi y') \, dy' = \frac{1}{2}\left(\frac{\pi}{P}\right)^{1/2} \exp\left(\frac{\xi^2}{4P}\right)\left[1 - \Phi\left(\frac{\xi}{2\sqrt{P}}\right)\right] \quad \ldots(3.16)$$

$$I_3 = \int_0^\infty y' \exp(-Py'^2 - \gamma y') \, dy'$$

$$= \frac{1}{2P} - \frac{\gamma}{4P}\left(\frac{\pi}{P}\right)^{1/2} \exp\left(\frac{\gamma^2}{4P}\right)\left[1 - \phi\left(\frac{\gamma}{2\sqrt{P}}\right)\right].$$

Hence

$$t_{xx} = -\left(\frac{2}{\pi}\right)^{1/2} \mu \frac{\lambda + \mu}{\lambda + 2\mu} \int_0^\infty \xi A(\xi) \cos(\xi x) \left\{ e^{-\xi y}\left[1 + \frac{\xi}{2P}(\xi - 2Py)\right] \right.$$

$$\left. \times \left[1 - \phi\left(\frac{\xi - 2Py}{2\sqrt{P}}\right)\right] + e^{\xi y}\left[1 + \frac{\xi}{2P}(\xi + 2Py)\right]\left[1 - \phi\left(\frac{\xi + 2Py}{2\sqrt{P}}\right)\right] \right\} d\xi$$

$$+ 2\left(\frac{2}{\pi}\right)^{1/2} \mu \frac{\lambda + \mu}{\lambda + 2\mu} (\pi P)^{-1/2} e^{-Py^2} \int_0^\infty \xi^2 A(\xi) e^{-\xi^2/4P} \cos(\xi x) \, d\xi,$$

$$t_{xy} = \left(\frac{2}{\pi}\right)^{1/2} \mu \frac{\lambda + \mu}{\lambda + 2\mu} \int_0^\infty \xi^2 A(\xi) \sin(\xi x) \left\{ e^{-\xi y}\left[\frac{\xi - 2Py}{2P}\right] \right.$$

$$\left. \times \left[1 - \phi\left(\frac{\xi - 2Py}{2\sqrt{P}}\right)\right] - e^{\xi y}\left[\frac{\xi + 2Py}{2P}\right]\left[1 - \phi\left(\frac{\xi + 2Py}{2\sqrt{P}}\right)\right] \right\} d\xi. \quad \ldots(3.17)$$

$$t_{yy} = \left(\frac{2}{\pi}\right)^{1/2} \mu \frac{\lambda+\mu}{\lambda+2\mu} \int_0^\infty \xi A(\xi) \cos(\xi x) \left\{ e^{-\xi y}\left[1 - \frac{\xi}{2P}(\xi - 2Py)\right] \right.$$

$$\times \left[1 - \phi\left(\frac{\xi - 2Py}{2\sqrt{P}}\right)\right] + e^{\xi y}\left[1 - \frac{\xi}{2P}(\xi + 2Py)\right]$$

$$\left. \times \left[1 - \phi\left(\frac{\xi + 2Py}{2\sqrt{P}}\right)\right] + \frac{2\xi}{\sqrt{(P\pi)}} \exp\left(-Py^2 - \frac{\xi^2}{4P}\right)\right\} d\xi$$

where $P = \left(\frac{\beta}{a}\right)^2$, $\phi(z)$ is the error function defined by

$$\phi(z) = 2\pi^{-1/2} \int_0^z \exp(-t^2)\, dt.$$

The remaining two boundary conditions $(3.7)_2$, $(3.7)_3$ may now be expressed as

$$\left.\begin{aligned} &\int_0^\infty \zeta^{1/2} C(\zeta) K(\zeta, P) \cos(\zeta z)\, d\zeta = -\left(\frac{\pi}{2}\right)^{1/2} T_0, \quad 0 \leqslant z < 1 \\ &\int_0^\infty \zeta^{-1/2} C(\zeta) \cos(\zeta z)\, d\zeta = 0 \qquad\qquad \text{for } z > 1 \end{aligned}\right\} \quad \ldots(3.18)$$

where we have introduced

$$x/c \equiv z;\ \xi c = \zeta;\ A(\xi) \equiv \zeta^{-1/2} C(\zeta).$$

$$K(\zeta, P) \equiv \left[1 - \frac{\zeta^2}{2Pc^2}\right]\left[1 - \phi\left(\frac{\zeta}{2c\sqrt{P}}\right)\right] + \frac{\zeta}{c\sqrt{(P\pi)}} \exp\left(\frac{-\zeta^2}{4Pc^2}\right)$$

$$T_0 \equiv \frac{p(x)\, c^2 (\lambda + 2\mu)}{2\mu(\lambda + \mu)};\ P \equiv \left(\frac{\beta}{a}\right)^2. \quad \ldots(3.19)$$

To determine the unknown function $A(\xi)$, we must solve the dual integral equations (3.18) for $C(\zeta)$.

4. The Solution of Dual Integral Equations

By introducing $\cos(\zeta z) = \left(\frac{\pi \zeta z}{2}\right)^{1/2} J_{-1/2}(\zeta z)$ where $J_\nu(z)$ is the Bessel's function of the first kind of order ν, we write (3.18) as

$$\left.\begin{aligned} &\int_0^\infty \zeta\, C(\zeta)\,[1+k(\epsilon\zeta)]\, J_{-1/2}(\zeta z)\, d\zeta = -T_0\, z^{-1/2},\ 0 \leqslant z \leqslant 1 \\ &\int_0^\infty C(\zeta)\, J_{-1/2}(\zeta z)\, d\zeta = 0 \qquad\qquad \text{for } z > 1 \end{aligned}\right\} \quad \ldots(4.1)$$

where the Kernal function $k(\epsilon\zeta)$ is given by

$$k(\epsilon\zeta) = K(\zeta, P) - 1 = 2\epsilon^2\zeta^2\,[\phi(\epsilon\zeta) - 1] - \phi(\epsilon\zeta) + 2\pi^{-1/2}(\epsilon\zeta)\exp(-\epsilon^2\zeta^2) \quad \ldots(4.2)$$

and

$$\epsilon = \frac{1}{2c\,P^{1/2}} = \frac{a}{2\beta c}\ . \quad \ldots(4.3)$$

The solution of the dual integral equations (4.1) is not known. However it is possible to reduce the problem to the solution of a Fredholm equation[11].

$$h(x) + \int_0^1 h(u)\, L(x, u)\, du = -\frac{1}{2}\,(\pi x)^{1/2}\, T_0 \quad \ldots(4.4)$$

for the function $h(x)$, where

$$L(x, u) = (xu)^{1/2} \int_0^\infty tk(\epsilon t)\, J_0(xt)\, J_0(ut)\, dt. \quad \ldots(4.5)$$

Once (4.4) is solved then $C(\zeta)$ is calculated by

$$C(\zeta) = (2\zeta)^{1/2} \int_0^1 x^{1/2}\, J_0(\zeta x)\, h(x)\, dx. \quad \ldots(4.6)$$

We see that for $\epsilon = 0$, we have $k = 0$ and the dual integral equations (4.1) reduces to those obtained in the classical elasticity. For this from (4.4)

$$h_0(x) = -\frac{T_0\,(\pi x)^{1/2}}{2}$$

and (4.6) gives the classical results.

$$C_0(\zeta) = -\left(\frac{\pi}{2}\right)^{1/2} \zeta^{1/2} \int_0^1 x\, T_0\, J_0(\zeta\, x)\, dx \quad \ldots(4.7)$$

or

$$A_0(\xi) = -\left(\frac{\pi}{2}\right)^{1/2} \int_0^1 f(t)\, J_0(\zeta t)\, dt \quad \ldots(4.8)$$

where

$$x\, T_0 = f(x).$$

Hence the solution for (4.1) is given by

$$C(\zeta) = -\left(\frac{\pi}{2}\right)^{1/2} \zeta^{1/2} \int_0^1 f(t)\, J_0(\zeta\, t)\, dt. \qquad \ldots(4.9)$$

Substituting (4.9) in $(4.1)_1$ we obtain

$$\int_0^1 f(t)\left(\int_0^\infty \zeta\, J_0(\zeta t)\,[1 + k(\epsilon\zeta)]\, \cos(\zeta z)\, d\zeta\right) dt = T_0. \qquad \ldots(4.10)$$

Due to the complexity of the integrand, in the first approximation we neglect the higher order terms in the expansion of $k(\epsilon\zeta)$, thus

$$k(\epsilon\zeta) = -2\epsilon^2 \zeta^2. \qquad \ldots(4.11)$$

If we use the fact

$$\int_0^\infty J_0(\xi t) \sin(\xi x)\, d\xi = \frac{H(x-t)}{\sqrt{(x^2-t^2)}}$$

and (4.11), where H is Heavy side unit function, we see that (4.10) is equivalent to

$$\int_0^1 f(t)\left[\frac{d}{dz}\frac{H(z-t)}{\sqrt{(z^2-t^2)}} + 2\epsilon^2 \frac{d^3}{dz^3}\frac{H(z-t)}{\sqrt{(z^2-t^2)}}\right] dt = T_0 \qquad \ldots(4.12)$$

and it can be written as

$$S + 2\epsilon^2 \frac{d^2 S}{dz^2} = T_0(z)$$

where

$$S = \frac{d}{dz}\int_0^z \frac{f(t)}{\sqrt{(z^2-t^2)}}\, dt.$$

This equation can be solved if the value of the variable pressure T_0 is prescribed.

Here

$$\frac{d}{dz}\int_0^z \frac{f(t)}{\sqrt{(z^2-t^2)}}\, dt = S(z)$$

is an Abel's integral equation whose solution is given by

$$f(t) = \frac{2t}{\pi} q(t) \text{ where } q(t) = \int_0^t \frac{S(u)}{\sqrt{(t^2-u^2)}}\, du. \qquad \ldots(4.13)$$

The expression for the normal component of Stress along the line of crack is given by the third of (3.17) and (4.9) as

$$t_{yy}(x, 0) = -\frac{d}{dx}\frac{2\mu(\lambda+\mu)}{\lambda+2\mu}\int_0^1 f(t)\left(\int_0^\infty J_0(\xi ct)\sin(\xi x)[1+k(\epsilon\zeta)]\, d\xi\right)dt. \qquad \ldots(4.14)$$

If $\epsilon = 0$, $k(\epsilon\zeta) = 0$ and the above equation reduces to the classical result[10].

Using (4.12) and (4.13), (4.14) can be written as

$$t_{yy}(x, 0) = -\frac{2\mu(\lambda+\mu)}{c(\lambda+2\mu)}\left\{\frac{d}{dx}\int_0^c \frac{f(r/c)}{\sqrt{(x^2-r^2)}}\, dr + 2\epsilon^2 \frac{d^3}{dx^3}\int_0^c \frac{f(r/c)}{\sqrt{(x^2-r^2)}}\, dr\right\}. \qquad \ldots(4.15)$$

If we subsitute (4.13) into (4.15) and integrate by parts, we find on carrying out differentiation, that

$$t_{yy}(x, 0) = -\frac{4\mu(\lambda+\mu)}{c^2\pi(\lambda+2\mu)}\left\{-\frac{x}{\sqrt{(x^2-c^2)}} q(1) + x\int_0^c \frac{1}{\sqrt{(x^2-r^2)}} q'\left(\frac{r}{c}\right) dr + 2\epsilon^2\left[-\frac{3xc^2}{(x^2-c^2)^{5/2}} q(1) + 3x\int_0^c \frac{r}{(x^2-r^2)^{5/2}} q'\left(\frac{r}{c}\right) dr\right]\right\}. \qquad \ldots(4.16)$$

The local stress intensity factor K can be obtained as

$$K = \frac{4\mu(\lambda+\mu)}{\pi(\lambda+2\mu)}\frac{q(1)}{c\sqrt{(2c)}} \text{ where } q(1) = \int_0^1 \frac{T_0(u)}{\sqrt{(1-u^2)}}\, du \qquad \ldots(4.17)$$

which reduces to the classical result[10] if we take $c = 1$.

The nonlocal stress intensity factor K' is obtained by removing the singularity $\frac{1}{(x-c)^{5/2}}$ and is given by

$$K' = \epsilon^2 \frac{6\mu(\lambda+\mu)}{\pi(\lambda+2\mu)} \frac{q(1)}{c\sqrt{(2c)}} \quad \text{where } q(1) = \int_0^1 \frac{S(u)}{\sqrt{(1-u^2)}}\, du. \qquad \ldots(4.18)$$

References

1. A. C. Eringen, and D. G. B. Edelen, *Int. J. Engng. Sci.* **10** (1972) 233–48.
2. A. C. Eringen, and B. S. Kim, *Mech. Res. Commun.* **1** (1974) 233–37.
3. A. C. Eringen, *Int. J. Fracture* **14** (1978) No. 4, 367–79.
4. A. C. Eringen, *Engng. Fracture Mech.* **12** (1979) 211–19.
5. A. C. Eringen, "*Continum mechanics at the atomic scale*" Crystal Lattice Defects Vol. 7 (1977) pp. 109–30.
6. J. N. Goodier, *Fracture : An Advanced Treatise.* Vol. II. Mathematical Fundamentals (1968).
7. Gradshteyn, and I. W. Rhyzhik, *Tables of Integrals, Series and Products.* Academic Press New York (1965).
8. A. A. Griffith, *Phil. Trans.* A **22** (1921) 163.
9. M. Lowengrub, and K. N. Srivastava, *Int. J. Engng. Sci.* **6** (1968) 359.
10. I. N. Sneddon, and M. Lowengrub, *Crack Problems in the Classical Theory of Elasticity.* John Wiley and Sons, New York (1969) pp. 25–28.
11. I. N. Sneddon, *Mixed Boundary Value Problems in Potential Theory*, North-Holland Pub. Co., Amsterdam (1966) pp. 106–108.
12. I. N. Sneddon, and H. A. Elliott, *Quart. appl. Math.* (1946) 229.