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LOVE WAVES IN A LIQUID LOCKED LAYERED POROELASTIC HALF SPACE

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The equation determining the phase velocity and attenuation in propagation of Love waves in a liquid locked isotropic homogeneous layered poroelastic half space is obtained. Following the procedures adopted by Banghar [*Proc. Indian. Acad. Sci.* **88A** (1979), 133-46] and Deresiewicz [*Bull. Seism. Soc. Am.* **51** (1961), 51-59] approximate values of phase, group velocities and attenuation are evaluated. Also the results of Banghar and Deresiewicz are obtained as particular cases.

INTRODUCTION

Various attempts are made to explain the attenuation of seismic waves in the Earth. Consideration is given to some of the following aspects : (a) non-homogeneity of the medium, (b) nature of contact between the various layers, (c) poroelastic nature and others. Banghar *et al.*[2], considered a liquid layer between two homogeneous isotropic elastic half spaces and studied propagation of Rayleigh waves. Banghar[1] separately studied propagation of Love waves in a liquid locked layered half space.

Natural products like petroleum are locked between the earth's crust and its interior. Treating the crust and the interior to be a homogeneous isotropic poroelastic layer and a half space respectively the propagation and attenuation of Love waves is studied in the following. As in any problem of this type in a poroelastic body, the equation for phase velocity is sufficiently complicated. As a particular case, neglecting the effects of pores the period equation of Banghar[1] can be obtained. Following his way of approximation, the results for the problem of propagation of Love waves in (a) a half space with overlying liquid layer, (b) layered half space (absence of liquid in between) can be obtained. Neglecting the effects of pores in the half space, the results of Deresiewicz[4] can be obtained.

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Following the procedure of approximation, as given by Deresiewicz, approximate values of phase velocity, group velocity and attenuation of the problem are obtained and presented graphically.

Formulation and Solution of the Problem

The free surface of the layer is taken as the *xoy* plane, *x*-axis along the direction of propagation of waves, *z* axis into the medium and *y* axis suitably in the free surface such that *oxyz* is a right handed system. Let the interface between the liquid and the layer be $z = h_1$ and the liquid and the half space by $z = h_1 + h_2$. Let $\rho_{11}^{(1)}$, $\rho_{12}^{(1)}$, $\rho_{22}^{(1)}$, $N^{(1)}$, b_1 be the dynamic mass densities, shear modulus, coefficient of dissipation of the layer and similar quantities of the half space are denoted with a superfix or a suffix of '3'. Let η_2 and ρ_2 be the coefficient of viscosity and mass densities of the viscous liquid layer of thickness h_2.

The governing equations of motion for a poroelastic medium as given by Biot[3] are

$$N \nabla^2 \bar{u} + \text{grad}\,[(A + N)\, e + Q\, \epsilon] = \frac{\partial^2}{\partial t^2}(\rho_{11}\, \bar{u} + \rho_{12}\, \bar{U}) + b\, \frac{\partial}{\partial t}(\bar{u} - \bar{U})$$

$$\text{grade}\,(Qe + R\epsilon) = \frac{\partial^2}{\partial t^2}(\rho_{12}\, \bar{u} + \rho_{22}\, \bar{U}) - b\, \frac{\partial}{\partial t}(\bar{u} - \bar{U}) \qquad \ldots(1)$$

where a bar over a symbol indicates its vectorial nature.

In the case of Love waves propagating in the *x*-direction, the displacement vectors for solid and liquid phases respectively are

$$\bar{u} = \bar{j} v\,(z) \exp i\,(\omega t - \gamma x) \qquad \ldots(2)$$

$$\bar{U} = \bar{j} V\,(z) \exp i\,(\omega t - \gamma x)$$

where i, j, k are unit vectors along the coordinate axes. From equations (1) and (2), we get

$$v_1 = [A_1 \cos(\nu_1 z) + B_1 \sin(\nu_1 z)] \exp i\,(\omega t - \gamma x) \qquad \ldots(3)$$

$V_1 = \alpha_0^{(1)}\, v_1$ for the solid and liquid phases of the poroelastic layer.

$$v_3 = A_3\, e^{-\nu_3 z} \exp i\,(\omega t - \gamma x) \qquad \ldots(4)$$

$V_3 = \alpha_0^{(3)}\, v_3$ for the half space,

where

$$\nu_1^2 = (\delta_1^2 - \gamma^2) \text{ and } \nu_3^2 = (\gamma^2 - \delta_3^2)$$

δ_1 and δ_3 are complex quantities given by

$$\delta_1^2 = \beta_1^{(1)} - i\,\beta_2^{(1)},\ \delta_3^2 = \beta_1^{(3)} - i\,\beta_2^{(3)}$$

and

$$\alpha_0^{(1)} = -\frac{r_{12}^{(1)}\,\varphi^{(1)} + i}{r_{22}^{(1)}\,\varphi^{(1)} - i}$$

$$\beta_1^{(1)} = \frac{R^{(1)}\,\omega^2}{(C_r^{(1)})^2},\ \beta_2^{(1)} = \frac{S^{(1)}\,\omega^2}{(C_r^{(1)})^2},\ C_r^{(1)} = \left(\frac{N^{(1)}}{P^{(1)}}\right)^{1/2}$$

$$R^{(1)} = \frac{C^{(1)}\,r_{22}^{(1)}\,\varphi^{(1)2} + 1}{r_{22}^{(1)^2}\,\varphi^{(1)2} + 1},\ \varphi^{(1)} = \frac{\rho^{(1)}\omega}{b^{(1)}}$$

$$S^{(1)} = \frac{(r_{22}^{(1)} - C^{(1)})\,\varphi^{(1)}}{(r_{22}^{(1)^2}\,\varphi^{(1)2} - 1)},\ C^{(1)} = r_{11}^{(1)}\ r_{22}^{(1)} - r_{12}^{(1)^2}$$

$$r_{ij}^{(1)} = \frac{\rho_{ij}^{(1)}}{\rho^{(1)}},\ \rho^{(1)} = \rho_{11}^{(1)} + 2\rho_{12}^{(1)} + \rho_{22}^{(1)}.$$

The above quantities with superfix (or suffix) '1' refer to the layer and similar quantities of superfix (or suffix) '3' are understood for the half space.

The equations of motion for the liquid layer is given[1] by

$$(\nabla^2 + \delta_2^2)\,v_2 = 0 \qquad \ldots(5)$$

where v_2 is the transverse horizontal velocity; $\delta_2^2 = i\omega/\nu'$, $\nu' = -\eta_2/\rho_2$; the coefficient of kinematic viscosity, η_2 the coefficient of viscosity; and ρ_2 is the density of the liquid.

A solution, suitable to the present context of (5) is

$$v_2 = i\omega\,[A_2 \cos(\nu_2 z) + B_2 \sin(\nu_2 z)] \exp i(\omega t - \gamma x) \qquad \ldots(6)$$

where $\nu_2^2 = \delta_1^2 - \gamma^2$. The boundary conditions are

$$\text{(i)}\ \sigma_{yz}^{(1)} = 0 \text{ at } z = 0 \qquad \ldots(7)$$

$$\left.\begin{aligned} &\text{(ii) } i\omega v_1 = v_2 \\ &\sigma_{yz}^{(1)} = \sigma_{yz}^{(2)} \end{aligned}\right\} \text{ at } z = h_1 \qquad \ldots(8)$$

and

$$\left.\begin{aligned} &\text{(iii) } \sigma_{yz}^{(2)} = \sigma_{yz}^{(3)} \\ &v_2 = i\omega v_3 \end{aligned}\right\} \text{ at } z = h_1 + h_2 \qquad \ldots(9)$$

where

$$\sigma_{yz}^{(1)} = 2N^{(1)} \frac{\partial v_1}{\partial z}, \quad \sigma_{yz}^{(2)} = \eta_2 \frac{\partial v_2}{\partial z}$$

all the remaining stress components vanish identically throughout the body.

From (3), (4), (6)-(9) the frequency equation is given by

$$[i\omega\eta_2 \nu_2 + N^{(3)} \nu_3 \tan(\nu_2 h_2)] N^{(1)} \nu_1 \tan(\nu_1 h_1)$$
$$- \nu_2^2 \omega^2 \eta_2^2 \tan(\nu_2 h_2) = i\omega \eta_2 \nu_2 N^{(3)} \nu_3. \qquad \ldots(10)$$

Let $Q = h_2/\eta_2$, constant. Dividing (10) by $i\omega \eta_2 \nu_2$ introducing Q; and approximating $\tan(\nu_2 h_2)$ by $\nu_2 h_2$, we get

$$\left[1 - \frac{iQN^{(3)}}{\omega} \nu_3\right] \tan(\nu_1 h_1) = \frac{N^{(3)} \nu_3}{N^{(1)} \nu_1} \qquad \ldots(11)$$

where

$$\nu_3^2 = \left(\gamma^2 - \frac{\rho^{(3)} \omega^2}{N^{(3)}}\right).$$

Particular cases

Case 1—When the effects of pores are neglected, then

$$r_{12}^{(1)} = r_{22}^{(1)} = C^{(1)} = S^{(1)} = 0, \quad R^{(1)} = 1$$

and

$$\beta_1^{(1)} = \frac{\rho_{11}^{(1)} \omega^2}{N^{(1)}}$$

$$\nu_1 = \delta_1^2 - \gamma^2 = \beta_1^{(1)} - \gamma^2$$

with these substitutions eqn. (10) coincides with the eqn. (33) of Banghar[1] for the propagation of Love waves in viscous liquid locked elastic layered half space.

Case 2—If effects of pores are neglected in the half space only and in the absence of the liquid layer the results of Deresiewicz[4] can be obtained as shown below.

On taking the limit as $Q \to 0$, eqn. (11) reduces to

$$\tan (\nu_1 h_1) = \frac{N^{(3)} \nu_2}{N^{(1)} \nu_1}$$

which coincides with equation (21) of Deresiewicz[4].

Case 3—If the effects due to pores are negligible in the layer also, then

$$\nu_1^2 = \left(\frac{\rho_{(1)} \omega^2}{N^{(1)}} - \gamma^2 \right)$$

and

$$\tan (\nu_1 h) = \frac{N^{(3)} \nu_3}{N^{(1)} \nu_1}$$

which coincides with the result given in Ewing *et al.*[5]

Phase Velocity and Attenuation

We adopt the procedure of Deresiewicz[4] in getting successive approximate values for phase velocity and attenuation of Love waves for the period equation (11). Since δ_1^2, δ_2^2 and δ_3^2 are complex, let the square of the wave number, γ^2, be

$$\gamma^2 = a_1 - ia_2, \; a_1 > 0, \; a_2 \geqslant 0. \qquad \ldots(12)$$

For a poroelastic medium, it is shown[4] to b $a_2 \ll a_1$ and $\beta_2 \ll \beta_1$. Let

$$\xi = \left(\frac{\beta_2 - a_2}{\beta_1 - a_1} \right), \; \zeta = \left(\frac{a_2 - \beta_4}{a_1 - \beta_3} \right) \qquad \ldots(13)$$

ξ and ζ are both very small when compared with unity. Let

$$\nu_1 = k_1 - ik_2, \; \nu_3 = k_3 - ik_4. \qquad \ldots(14)$$

From the definitions of ν_1^2, ν_2^2, ν_3^2 and (12)-(14), we get

$$k_1 = (\beta_1^{(1)} - a_1)^{1/2} \left(1 + \frac{1}{8} \xi^2 \right)$$

$$k_2 = (\beta_1^{(1)} - a_1)^{1/2} \frac{\xi}{2}$$

$$k_3 = (a_1 - \beta_1^{(3)})^{1/2} \left(1 + \frac{1}{8} \zeta^2 \right) \qquad \ldots(15)$$

$$k_4 = (a_1 - \beta_1^{(3)})^{1/2} \frac{1}{2} \zeta.$$

From eqns. (11)-(14), the period equation simplifies to

$$\tan (k_1 - ik_2)\, h_1 = M_1 - iM_2 \qquad \ldots(16)$$

where

$$M_1 = \left(\frac{N^{(3)}\,\omega}{N^{(1)}} \right) \frac{(k_1 k_3 + k_2 k_4)\,(\omega - N^{(1)} k_4 Q) - N^{(3)} Q k_3\,(k_2 k_3 - k_1 k_4)}{(k_1^2 + k_2^2)\,[(\omega - N^{(3)} k_4 Q)^2 + k_3^2 N^{(3)} Q^2]}$$

$$M_2 = \left(- \frac{N^{(3)}\,\omega}{N^{(1)}} \right) \frac{[N^{(3)} k_3 Q\,(k_1 k_3 + k_2 k_4) + (k_2 k_3 - k_1 k_4)\,(\omega - N^{(3)} k_4 Q]}{(k_1^2 + k_2^2)\,[\omega - N^{(3)} k_4 Q)^2 + k_3^2 N^{(3)2} Q_2]} \qquad \ldots(17)$$

From (15), (17) and taking only terms of ξ^2, $\xi\zeta$, ζ^2 the approximate values of M_1 and M_2 are

$$M_1 = \frac{s}{1+\eta_1^2}\left[1 - \frac{3\xi^2}{8} + \frac{\xi\zeta}{4} + \frac{\xi^2}{8} - \frac{\eta_1 \xi}{2} + \frac{\eta_1 \zeta}{1 + \eta_1^2} - \frac{\eta_1^2\,\xi\zeta}{2\,(1 + \eta_1^2)} + \frac{\eta_1^2\,(1 + \eta_1^2)\,\zeta^2}{2\,(1 + \eta_1^2)^2}\right]$$

$$M_2 = -\frac{s}{1+\eta_1^2}\left[\eta_1 + \frac{\xi}{2} + \frac{(\eta_1^2 - 1)\,\zeta}{2\,(1 + \eta_1^2)} - \frac{3\eta_1\,\xi_2}{8} + \frac{\eta_1\,\xi\zeta}{2\,(1 + \eta_1^2)} + \frac{\eta_1^3\,\zeta^2}{(1 + \eta_1^2)^2}\right] \qquad \ldots(18)$$

with

$$s = \frac{N^{(3)}}{N^{(1)}}\left(\frac{a_1 - \beta_1^{(3)}}{\beta_1^{(1)} - s_1} \right), \quad r = h_1\,(\beta_1^{(1)} - a_1)^{1/2}$$

$$\eta_1 = \frac{N^{(3)}\,Q}{\omega}\,(a_1 - \beta_1^{(3)})^{1/2}.$$

From the real and imaginary parts of (16), we get

$$\text{taa}\; k_1 h_1 = \frac{M_1}{1 - M_2 k_2 h_1} \qquad \ldots(19)$$

$$k_2 h_1\,(1 + M_1 \tan k_1 h_1) = M_2. \qquad \ldots(20)$$

Substituting the values of M_1 and M_2 from (18), eqn. (19) reduces to

$$\tan r = \frac{s}{1+\eta_1^2}\,(1 + \psi) \qquad \ldots(21)$$

where

$$\psi = -\frac{\eta_1 \xi}{2} - \frac{rs\,\eta_1 \xi}{2(1+\eta_1^2)} + \frac{\eta_1 \zeta}{(1+\eta_1^2)} + \xi\left\{-\frac{3}{4} - \frac{r(1+\eta_1^2)}{s} - \frac{2s(\eta_1^2-1)}{4(1+\eta_1^2)}\right\} + \xi\zeta\left\{\frac{1}{4} - \frac{\eta_1^2}{2(1+\eta_1^2)} - \frac{rs(3\eta_1^2-1)}{4(1+\eta_1^2)^2}\right\}. \quad \ldots(22)$$

Similarly proceeding with (2), ζ is given by

$$\frac{(\eta_1^2-1)\zeta}{2(1+\eta_1^2)} = -\eta_1 - \frac{\xi}{2}\left[1 + \frac{r(1+\eta_1^2)}{s}\left(1 + \frac{s^2}{(1+\eta_1^2)^2}\right)\right]. \quad (23)$$

An approximate value of a_2 i.e. imaginary part of wavenumber γ, on using in (19) and (23), is obtained to be

$$Ga_2 = \alpha_4 - \frac{2(1+\eta_1^2)\eta_1(a_1-\alpha_3)}{(\eta_1^2-1)} - \frac{(1+\eta_1^2)\alpha_2}{(\eta_1^2-1)}\frac{N^{(1)2}s^2}{N^{(3)2}} \times \left\{1 + \frac{r(1+\eta_1^2)}{s} + \frac{rs}{1+\eta_1^2}\right\} \quad \ldots(24)$$

where

$$G = 1 - \frac{s^2 N^{(1)2}(1+\eta_1^2)}{N^{(3)2}}\left\{1 + \frac{(1+\eta_1^2)r}{s} + \frac{rs}{1+\eta_1^2}\right\} \quad \ldots(25)$$

Finally to obtain the period equation, attenuation coefficient in a convenient form suitable for successive iterations, let

$$\lambda = \frac{C_r^{(3)}}{C_r^{(1)}},\ q = \frac{h_1\,\omega}{C_r^{(1)}}\ \nu = \frac{C_r^{(1)}}{C_p},\ \theta = \frac{\rho^{(3)}}{\rho^{(1)}}$$

$$\gamma = \gamma_r - i\gamma_I,\ Cg = Cp + a_1\frac{dC_p}{da_1}. \quad \ldots(26)$$

In the above C_p, C_g are the phase and group velocities respectively. Then

$$\gamma_r \equiv \frac{\omega}{C_p} = a_1^{1/2}\left(1 + \frac{a_2^2}{8a_1^2}\right) \quad \ldots(27)$$

$$\gamma_I \simeq \frac{a_2}{2a_1^{1/2}}. \quad \ldots(28)$$

From (27) the first approximation of a_1 is

$$a_1 = a_1^{(1)} = \frac{\omega^2}{C_p^2} \quad \ldots(29)$$

and

$$\tan q\,(R^{(1)} - \nu_1^2)^{1/2} = \lambda\theta \left(\frac{\lambda^2 \nu_3^2 - R^{(3)}}{R^{(1)} - \nu_1^2} \right) \frac{1}{1 + \eta_1^2} \,. \qquad \ldots(30)$$

The attenuation coefficient per cycle obtained from (27) and (28), is given by

$$\frac{2\pi\gamma_I}{\gamma_r} = \frac{8\pi a_1 a_2}{8a_1^2 + a_2^2} \qquad \ldots(31)$$

Proceeding as in Deresiewicz[4] higher order approximations can be obtained.

Discussion

For $Q = 0, 1, 2, 5$ the non-dimensional phase velocity (by dividing C_p by C_r^w) is evaluated for the frequency equation (30). $Q = 0$ corresponds to absence of fluid layer. As Q increases, phase velocity is steeply falling in the first half of the interval (0, 1) of the non-dimensional wavenumber X $(= a_1 h_1)$ (Fig. 1). The non-dimensional

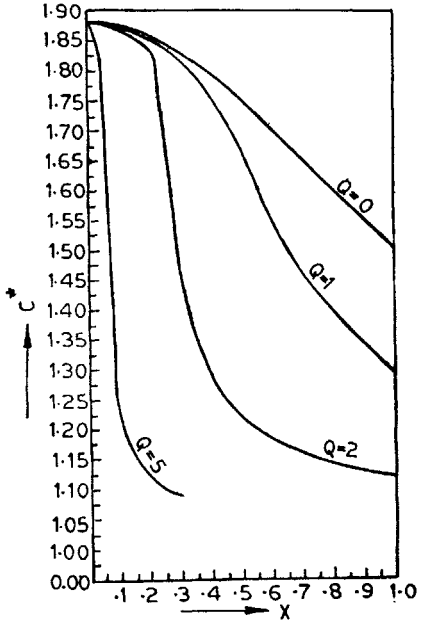

Fig. 1. Variation of non dimensional phase velocity (C_p/C_r^w) as a function of non dimensional wavenumber $(a_1 h_1)$ for some values of the parameter Q.

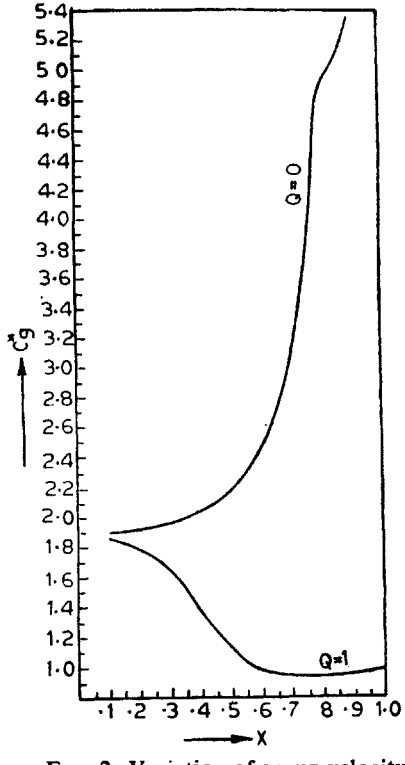

Fig. 2. Variation of group velocity as a function of wave number.

TABLE I

Attenuation Versus Wavenumber

	X	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9	1.0
$Q = 0$	$\frac{2\pi\gamma_i}{\gamma_r}$	1.563	1.555	1.539	1.475	1.434	1.186	0.933	0.625	0.301	0.004
$Q = 1$	$\frac{2\pi\gamma_i}{\gamma_r}$	1.563	1.550	1.476	1.098	0.348	0.006	0.145	0.462	0.771	1.033

group velocity (by dividing C_g by $C_r^{(1)}$) is also evaluated for $Q = 0, 1$ and presented graphically (Fig. 2). $Q = 0$, as the wavenumber X increases the group velocity steadily increases and attains a maximum (at $X = 0.9$). The attenuation coefficient per cycle is obtained for the interval (0, 1) of non-dimensional wavenumber and presented in Table I. The numerical values are considered from Fatt[6] for the layer and Nowinski[7] for the half space.

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