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THE IMPULSIVE MOTION OF A FLAT PLATE IN A VISCOELASTIC FLUID IN THE PRESENCE OF A TRANSVERSE MAGNETIC FIELD

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In this note the MHD flow near a wall suddenly set in motion has been studied for a particular class of non-Newtonian viscoelastic fluids. For the description of such a fluid one has used the Rivlin-Ericksen constitutive equation only the first three material constants have been taken into consideration. It is observed that velocity decreases with increasing magnetic field. Due to the presence of the elastic property of these elastico-viscous fluids smaller velocities are observed in a thin liquid layer than that for the Newtonian fluids whereas the effect of elastic forces beyond this thin liquid layer is just the reverse. The shearing stress increases with magnetic field but decreases with elasticity.

1. INTRODUCTION

Teipel[1] studied the problem of the impulsive motion of a flat plate in a viscoelastic fluid. In the present study it is proposed to analyse the hydromagnetic flow of an electrically conducting Rivlin-Ericksen liquid near an infinite horizontal flat plate started impulsively from rest in its own plane with constant velocity and is subjected to an applied uniform transverse magnetic field.

Basic Equation

The second order approximation of the general constitutive equation given by Rivlin and Ericksen[2] can be written in the following form[3]

$$T = -pI + \mu A_1 + \alpha A_1^2 + \beta A_2 \qquad \ldots(1.1)$$

where T is the stress tensor, p the pressure, I the unit tensor, A_1 and A_2 the first two Rivlin-Ericksen tensors and μ, α and β three matirial constants. A_1 and A_2 are given by symmetric matrices defined by

$$A_1 = \frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i}$$

$$A_2 = \frac{\partial}{\partial x_j}\left(\frac{Dv_i}{Dt}\right) + \frac{\partial}{\partial x_i}\left(\frac{Dv_j}{Dt}\right) + 2\frac{\partial v_m}{\partial x_i}\frac{\partial v_m}{\partial x_j} \qquad \ldots(1.2)$$

$$(i, j, m = 1, 2, 3).$$

For the boundary condition it is assumed that there is no slip at the wall. The flow field near a wall suddenly set in motion prescribes that the velocity component u along the wall is only a function of time and of the co-ordinate perpendicular to the wall. Let a constant impulsive velocity U be given to the plate in its own plane and a uniform magnetic field H_0 be applied transversely to the plate. The liquid is assumed to be of low conductivity, such that the induced magnetic field is negligible and the Lorentz force is $-\sigma H_0^2 u$.

Hence the flow field is governed by the equation,

$$\frac{\partial u}{\partial t} = \nu \frac{\partial^2 u}{\partial y^2} + \beta \frac{\partial^3 u}{\partial t \, \partial y^2} - mu \qquad \ldots(1.3)$$

where $m = \sigma H_0^2/\rho$.

ρ being the density and σ the electrical conductivity of the fluid, β the kinematic coefficient of viscoelasticity and ν the kinematic coefficient of viscosity.

The initial conditions are,

$$t \leqslant 0, u = 0, y > 0. \qquad \ldots(1.4)$$

The boundary conditions are

$$t > 0, u = U \text{ at } y = 0$$

$$u = 0 \text{ as } y \to \infty. \qquad \ldots(1.5)$$

2. Solution and Results

It is noticed in eqn. (1.3) that the presence of elasticity yields a third order differential equation, where as in the viscous case ($\beta = 0$) the order of the equation is two. Therefore, for a unique solution, three boundary conditions are necessary. But there are only two boundary conditions. Therefore to overcome this difficulty following Beard and Walters[4], we assume the solution as

$$u = u_0 + \beta u_1 \qquad \ldots(2.1)$$

which is valid for small values of β.

Substituting (2.1) in (1.3) and (1.5), equating the coefficients of different powers of β and neglecting those of β^2, we get

$$\frac{\partial u_0}{\partial t} = \nu \frac{\partial^2 u_0}{\partial y^2} - m u_0 \qquad \ldots(2.2)$$

$$\frac{\partial u_1}{\partial t} = \nu \frac{\partial^2 u_1}{\partial y^2} + \frac{\partial^3 u_0}{\partial y^2 \partial t} = m u_1 \qquad \ldots(2.3)$$

and the boundary conditions will be

$$\left.\begin{aligned} u_0 &= U, u_1 = 0 \text{ at } y = 0 \\ u_0 &= 0, \; u_1 = 0 \text{ as } y \to \infty. \end{aligned}\right\} \qquad \ldots(2.4)$$

The Laplace transform is defined as

$$\bar{u} = \int_0^\infty e^{-st} u(y, t)\, dt, \; (s > 0)$$

then eqns. (2.2) and (2.4) transform to

$$\frac{d^2 \bar{u}_0}{dy^2} = \frac{s + m}{\nu} \bar{u}_0 \qquad \ldots(2.5)$$

and

$$\left.\begin{aligned} \bar{u}_0 &= U/s \text{ at } y = 0 \\ \bar{u} &= 0 \quad \text{as } y \to \infty. \end{aligned}\right\} \qquad \ldots(2.6)$$

The solution of (2.5) subject to (2.6) is

$$\bar{u}_0 = \frac{U}{s} e^{-y\sqrt{(s+m)/\nu}}. \qquad \ldots(2.7)$$

On taking Laplace of (2.3) and using the conditions $\bar{u}_1 = 0$ at $y = 0, \infty$, and $\bar{u}_0$ from (2.7) we get

$$\bar{u}_1 = \frac{Uy}{2} \sqrt{\left(\frac{s+m}{\nu}\right)} \exp\left(-y\sqrt{(s+m)/\nu}\right). \qquad \ldots(2.8)$$

Taking Laplace inverse of equations (2.1), (2.7) and (2.8), we get

$$u = \frac{U}{2}\left[e^{-y\sqrt{m/\nu}} \operatorname{erfc}\left(\frac{y}{2\sqrt{\nu t}} - \sqrt{(mt)}\right) + \exp\left(y\,(m/\nu)^{1/2}\right) \right.$$

$$\left. \times \operatorname{erfc}\left(\frac{y}{2\sqrt{\nu t}} + (mt)^{1/2}\right) + \frac{\beta y}{4\nu^2 t^2} \frac{y^2 - 2\nu t}{\sqrt{(\pi \nu t)}} \exp\left(-mt - y^2/4\nu t\right) \right]. \qquad \ldots(2.9)$$

Introducing the following non-dimensional quantities

$$\eta = \frac{y}{2\sqrt{\nu t}}, \; M = \sqrt{(mt)}, \; \beta^* = \beta/\nu t. \qquad \ldots(2.10)$$

The non-dimensional form of (3.9) is

$$u^* = \frac{u}{U} = \frac{1}{2}\Big[e^{-2M\eta}\,\mathrm{erfc}\,(\eta - M) + e^{-2M\eta}\,(\eta + M) - \frac{\beta^*}{\sqrt{\pi}}(1 - 2\eta^2)\, e^{-(\eta^2 + M^2)} \Big].$$

The shearing stress at the plate is given by

$$Txy = -\left[\mu \frac{\partial u}{\partial y} + \beta \frac{\partial^2 u}{\partial y\, \partial t} \right]_{y=0}. \qquad \ldots(2.11)$$

Using (2.9) and (2.10) in (2.11) and neglecting coefficients of β^2 we obtain

$$T^*_{xy} = \frac{Txy}{\rho U \sqrt{\nu/t}} = M\,\mathrm{erf}\,M + \frac{1}{\sqrt{\pi}}\left(1 - \frac{\beta^*}{4}\right) e^{-M^2}. \qquad \ldots(2.12)$$

3. Interpretation and Discussion of Results

In this section we shall study the effect of elastic elements and magnetic field on the velocity distribution and skin friction and give the results in the tabular forms. We make the numerical calculations for

$\beta^* = 0.00,\ 0.10,\ 0.20,\ 0.30,\ 0.40$

$M = 0.00\ \ 1.00\ \ 2.00\ \ 3.00\ \ 4.00$

$\eta = 0.20,\ 0.40\ \ 0.60\ \ 0.80\ \ 1.00,\ 1.20,\ 1.40, \ldots, 2.20.$

Table I

Values of U when $\eta = 0.20$ *and* $M = 1.00$

β^*	U
0.00	0.65078051
0.10	0 64894589
0.20	0.64711127
0.30	0.64527664
0.40	0.64344202

In an elastico-viscous fluid, from Table I, it is observed that the presence of elastic elements in an elastico-viscous fluid decreases the velocity profile and an increase in elastic parameter also decreases the velocity profile.

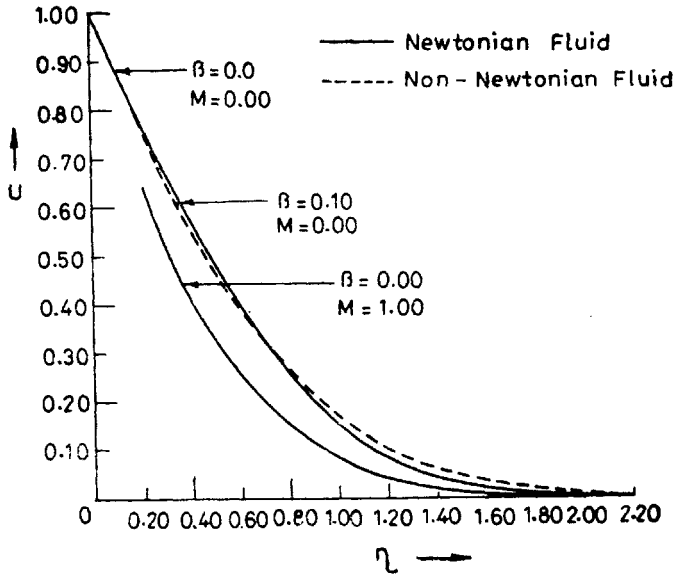

FIG. 1. Effect of elastic element and magnetic field on velocity profile.

From Fig. 1 it is found that in an elastico-viscous liquid an increase in the η decreases the velocity profile for fixed values of β and M. Figure 1 also shows that the largest velocities are observed in the absence of magnetic force and elastic element. Magnetic field decreases the velocity always rapidly but for the elastic element velocity changes as follows :

(i) For $\eta \leqslant 0.60$, due to the presence of the elastic property of these elastico-viscous fluid smaller velocities are observed whereas for $\eta \geqslant 0.80$ the effect of elastic force is just the reverse i.e. the greater velocities are observed.

From Table II it is found that in increase in magnetic field causes a decrease in the velocity profile.

TABLE II

($\eta = 0.20$, $\beta = 0.10$)

M	U
1.00	0.64894589
2.00	0.44885957
3.00	0.30119230
4.00	0.20189652

It can be easily seen from eqn. (3.12) that the shearing stress at the plate increases with the increasing value of magnetic field and shearing stress decreases with an increase in the value of elastic element of the elastico-viscous fluid.

§4. Furthermore we aim to study the non-magnetic steady flow of an elastico-viscous fluid parallel to an infinite plate with uniform suction. The basic equation for the present case i.e.

$$-\rho v_0 \frac{du}{dy} = \mu \frac{d^2u}{dy^2} - \beta v_0 \frac{d^3u}{dy^3} \qquad \ldots(4.1)$$

and the boundary conditions are

$$\left.\begin{aligned} &u = 0 \text{ on } \eta = 0 \\ &u \to U \text{ the main stream velocity, as } \eta \to \infty. \end{aligned}\right\} \qquad \ldots(4.2)$$

The attempt to provide the needed boundary conditions required for the solution of the (4.1) is due to Beard and Walters[4] in the form

$$f = \frac{u}{U} = [f_0(\eta) + \beta^* (f_1(\eta)] \qquad \ldots(4.3)$$

where

$$\eta = \frac{yv_0}{\nu}, u = fU, \beta^* = \frac{\beta v_0^2}{\rho \nu^2} \text{ and } \nu = \mu/\rho. \qquad \ldots(4.4)$$

In non-dimensional form eqns. (4.1) and (4.2) become

$$\beta^* f''' - f'' - f' = 0 \qquad \ldots(4.5)$$

and

$$\left.\begin{aligned} &f = 0 \text{ on } \eta = 0 \\ &f \to 1 \text{ as } \eta \to \infty \end{aligned}\right\} \qquad \ldots(4.6)$$

$( )'$ means the differentiation with respect to η.

We get two sets of differential equation by substituting (4.3) into (4.5) and equating the coefficients of different powers of β and neglecting those of β^2.

The solution obtained in a closed form is

$$f = \frac{u}{U} = [1 - e^{-\eta} = \beta^* \eta e^{-\eta}] \qquad \ldots(4.7)$$

$$f = 1 - e^{-(1-\beta^*)\eta} \text{ since } \beta^* \ll 1. \qquad ...(4.8)$$

From equation (4.8), it is clear that velocity never exceeds the main stream velocity.

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REFERENCES

1. I. Teipel, *Acta Mech.* **39** (1981) 277–79.
2. R. S. Rivlin, and J. L. Ericksen, *J. Rational Mech. Anal.* **4** (1955) 323–29.
3. G. Astarita, and G. Marrucci, *Principles of Non-Newtonian Fluid Mechanics*, McGraw-Hill Book Co., Inc. New York,
4. D. W. Beard, and K. Walters, *Proc. Camb. Phil. Soc.* **60** (1964) 667.