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UNSTEADY ROTATING FLOW OVER AN INFINITE ROTATING DISK WITH AN APPLIED MAGNETIC FIELD

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The flow of an incompressible rotating electrically conducting fluid over an infinite disk with an applied magnetic field has been studied when the disk rotates with angular velocity which continuously varies with time. The partial differential equations governing the flow have been solved numerically using an implicit finite-difference scheme. The flow fields are found to be more sensitive to time when the angular velocity of the disk decreases with time than when it increases. The velocity profiles in the absence of the magnetic field are found to be oscillatory and the oscillations are suppressed by the magnetic field.

1. INTRODUCTION

The rotating flows over a surface find applications in natural phenomena as well as in industry. The analogous hydromagnetic case with magnetic field has got applications in cosmical and geophysical fluid dynamics. Also, it has industrial applications such as in MHD generators, gaseouscore nuclear reactors etc. Steady rotating flow over a stationary infinite disk with or without magnetic field has been studied by several investigators (Bödewadt 1940, King and Lewellen 1964, King and King 1965, Kuo 1971, Nath and Venkatachala 1977, Stewartson and Troesch 1977 and Venkatachala and Nath 1980). However, most flows of practical interest are unsteady. The unsteadiness in the flow field is caused by the rotation of the disk or fluid or both. Several authors including Chawla (1972, 1973), Debnath and Mukherjee (1973, 1974), Debnath (1974, 1975a, 1975b), Loper and Benton (1970) and Loper (1971) have made investigations of the unsteady hydromagnetic rotating fluid flows in various geometric configurations. Bodonyi (1978) has considered the unsteady motion of a rotating fluid over a disk without magnetic field where the unsteadiness is caused by impulsively rotating the disk.

The aim of the present study is to consider the flow of an incompressible rotating electrically conducting fluid over an infinite disk which rotates with angular

velocity which varies continuously with time. Such a situation has not been considered before. The partial differential equations governing the flow have been solved numerically using an implicit finite-difference scheme. The results have been compared with those available in the literature.

2. Governing Equations

We consider a laminar incompressible electrically conducting fluid which is rotating with a constant angular velocity ω over an infinite insulated disk which rotates with time dependent angular velocity $(\omega_1 T)$. We apply a magnetic field B perpendicular to its plane and assume that the magnetic Reynolds number is small. Hence, the induced magnetic field can be neglected as compared to the applied magnetic field without introducing any appreciable error in the analysis. The flow is assumed to be constant. The Hall effect has also been neglected. The equations governing the flow under foregoing assumptions in dimensionless form can be expressed as (King and Lewellen 1964, King and King 1965), Nath and Venkatachala 1977, Stewartson and Troesch 1977, and Venkatachala and Nath 1980)

$$F'' - HF' + F^2/2 - MF - 2G^2 - F_T + 2 = 0 \qquad ...(1)$$

$$G'' - HG' + GF - M(G - 1) - G_T = 0. \qquad ...(2)$$

The boundary conditions are given by

$$\left.\begin{aligned} &F = 0,\ G = \epsilon T \text{ at } \eta = 0 \\ &F \to 0,\ G \to 1 \text{ as } \eta \to \infty \end{aligned}\right\} \text{ for } T \geqslant 0 \qquad ...(3)$$

where

$$\epsilon = \omega_1/\omega,\ \omega_1 \ll \omega. \qquad ...(4)$$

The initial conditions (i.e., at $T = 0$) can be obtained from (1) and (2) by putting $F_T = G_T = 0$ and they are

$$F'' - HF' + F^2/2 - MF - 2G^2 + 2 = 0 \qquad ...(5)$$

$$G'' - HG' + GF - M(G - 1) = 0 \qquad ...(6)$$

where

$$\left.\begin{aligned} &\eta = (\omega/\nu)^{1/2} z,\ T = \omega t,\ u = -2^{-1} r\omega F \\ &v = r\omega G,\ w = (\nu\omega)^{1/2} H,\ M = \sigma B^2/\rho\omega \end{aligned}\right\} \qquad ...(7)$$

$$H = \int_0^{\eta} F d\eta.$$

Here r, θ and z are the radial, tangential and axial directions, respectively; u, v and w are the dimensional velocity components in the r, θ and z directions, respectively; η is the dimensionless axial distance; t and T are the dimersional and dimensionless times, respectively; ω and ω_1 are the angular velocities of the fluid and

body, respectively; ϵ is a dimensionless constant; ρ, ν and σ are the density, kinematic viscosity and electrical conductivity, respectively; F, G and H are the dimensionless radial, tangential and axial velocities, respectively; M is the dimensionless magnetic parameter; prime denotes derivatives with respect to η; and subscript T denotes differentiation with respect to T.

It may be noted that our steady-state equations (5) and (6) for $M = 0$ reduce to those of Bödewadt (1940) and Kuo (1971) and for $M > 0$ to those of King and Lewellen (1964), King and King (1965), Nath and Venkatachala (1977), Stewartson and Troesch (1977) and Venkatachala and Nath (1980) if we put $H = -2f$ in (5) and (6). However, equations (5) and (6) are identical with those of Nath and Venkatachala (1977) and Venkatachala and Nath (1980). Further, (5) and (6) for $M = 0$ reduce to those of Kuo (1971) when $H = 2f$. Also, the unsteady-state equations (1) and (2) for $M = 0$ reduce to those of Bodonyi (1978) who has solved the analogous impulsive flow problem using the following boundary conditions.

$$\left.\begin{aligned} &F = 0,\ G = 1 \text{ at } T = 0 \text{ for all } \eta \\ &F \to 0,\ G \to 1 \text{ as } \eta \to \infty \text{ for all } T \\ &F = G = 0 \text{ at } \eta = 0 \text{ for all } T > 0. \end{aligned}\right\} \quad \ldots(8)$$

The skin-friction coefficients in the r and θ directions can be expressed as

$$\left.\begin{aligned} C_f &= 2\tau_r/(\rho r^2 \omega^2) = -2^{-1/2} Re^{-1/2} F'_w \\ \bar{C}_f &= 2\tau_\theta/(\rho r^2 \omega^2) = Re^{-1/2} G'_w \end{aligned}\right\} \quad \ldots(9a)$$

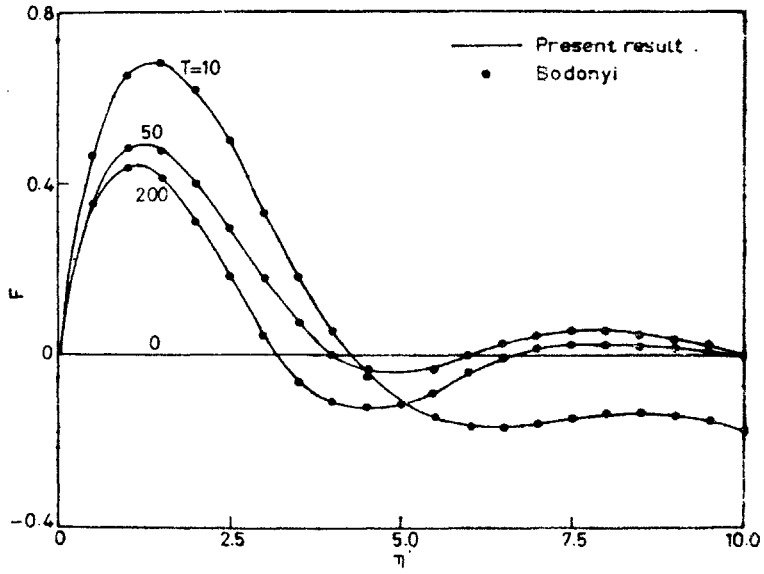

FIG. 1. Comparison of velocity profile in the radial direction F with that of Bodonyi.

where

$$\tau_r = \mu\,(\partial u/\partial z)_w,\ \tau_\theta = \mu\,(\partial v/\partial z)_w,\ Re = r\,(\omega/\nu)^{1/2}. \qquad ...(9b)$$

Here C_f and $\bar{C}_f$ are the skin-friction coefficients in the r and θ directions, respectively; τ_r and τ_θ are the shear stresses in the r and θ directions, respectively; μ is the coefficient of viscosity; Re in the Reynolds number; F'_w and G'_w are the skin-friction parameters in the r and θ directions, respectively.

3. Results and Discussion

The nonlinear coupled partial differential equations (1) and (2) under boundary conditions (3) and initial conditions (5) and (6) have been solved with the help of an implicit finite-difference scheme. First these equations were converted to the difference form using a central difference scheme in η-direction and a backward difference scheme in T-direction. The resulting nonlinear algebraic equations were linearized using Newton-Raphson's method. This system of linear equations was solved using block elimination method (Varga 1962). The effect of step size $\Delta\eta$ and ΔT and the edge of the boundary layer η_∞ on the solution was studied and the results presented here are independent of $\Delta\eta$, ΔT and η_∞ at least up to 4th decimal place. Hence, the computations were carried out with $\Delta T = \Delta\eta = 0.01$ and $\eta_\infty = 25$.

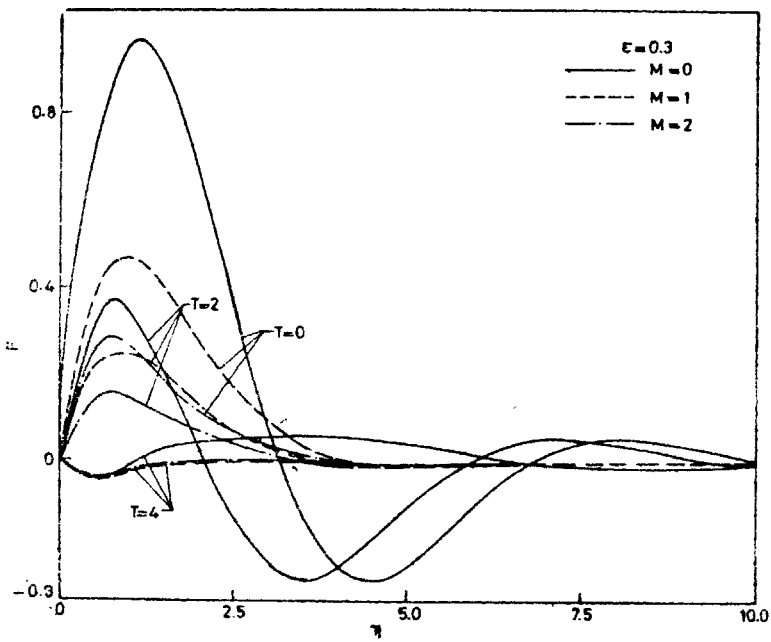

Fig. 2. Effect of magnetic field M on the radial velocity profile F ($\epsilon = 0.3$).

In order to assess the accuracy of our method, we have compared our steady-state results ($T = 0$) with those of Bödewadt (1940), King and Lewellen (1964), Kuo (1971) and Nath and Venkatachala (1977) and the unsteady-state results for $M = 0$ (no magnetic field) with those of Bodonyi (1978) and found them in very good agreement. However, for the sake of brevity, the comparison is shown here only with those King and Lewellen (1964) and Nath and Venkatachala (1977) in Table I and with those of Bodonyi in Figure 1.

Figures 2–4, respectively, give the radial, tangential and axial velocity profiles (F, G, H) at different times T and at different magnetic parameter M when the

TABLE I

Comparison of steady-state results

	Present results		King and Lewellen		Nath and Venkatachala	
M	F'_w	G'_w	F'_w	G'_w	F'_w	G'_w
0	1.8839	0.7727	1.884	0.773	1.8839	0.7729
1	1.2207	1.1119	1.220	1.112	1.2198	1.1119
2	0.9183	1.4606	0.918	1.460	0.9183	1.4602

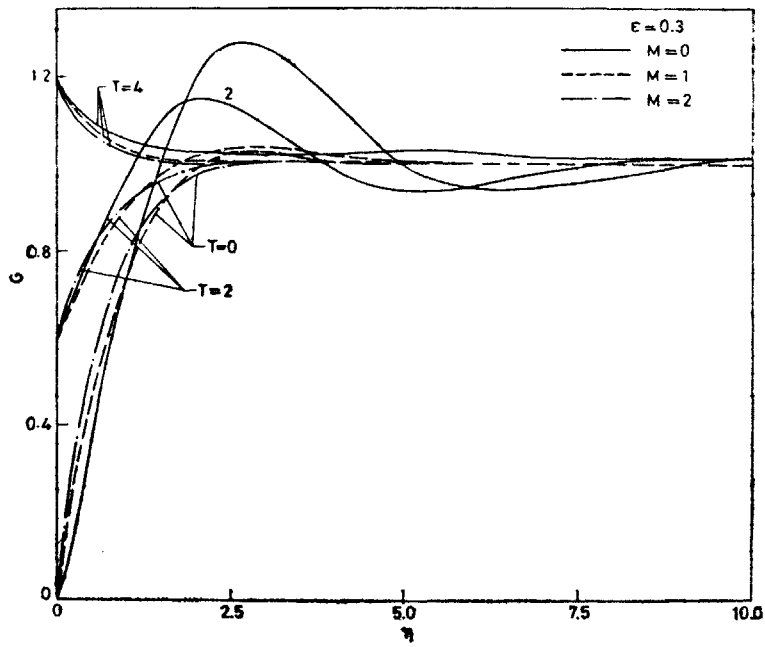

FIG. 3. Effect of magnetic field M on the tangential velocity profile G ($\epsilon = 0.3$).

angular velocity of the disk increases with time T. The velocity profiles F, G and H in the absence of the magnetic field ($M = 0$) are oscillatory in character and these oscillations are damped out by the application of magnetic field. The physical basis for these oscillations is the surplus convection of angular momentum present in the boundary layer. The detailed explanation of this phenomenon is presented by King and Lewellen (1964) and hence, it is not repeated here. Also, the maximum value of the velocity profiles F, G and H decrease with time T.

Figures 5 and 6 present the effect of the magnetic field M on the surface skin-friction parameters (F'_w, G'_w), when the angular velocity of the disk increases as well as decreases with time. The skin-friction parameters F'_w and G'_w increase with time T when the angular velocity of the disk increases with time whereas the effect of increasing the angular velocity of the disk on F'_w and G'_w is just reverse. Also, it is observed that the magnetic field M reduces F'_w but increases G'_w. As time T progresses, F'_w and G'_w for the non-magnetic case $(M = 0)$ seem to oscillate when the

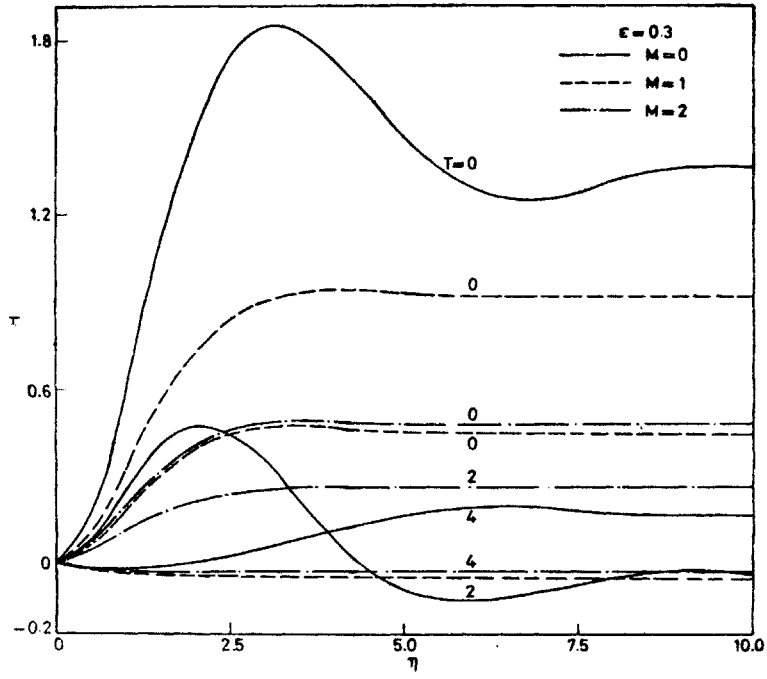

FIG. 4. Effect of magnetic field M on the axial velocity profile H ($\epsilon = 0.3$).

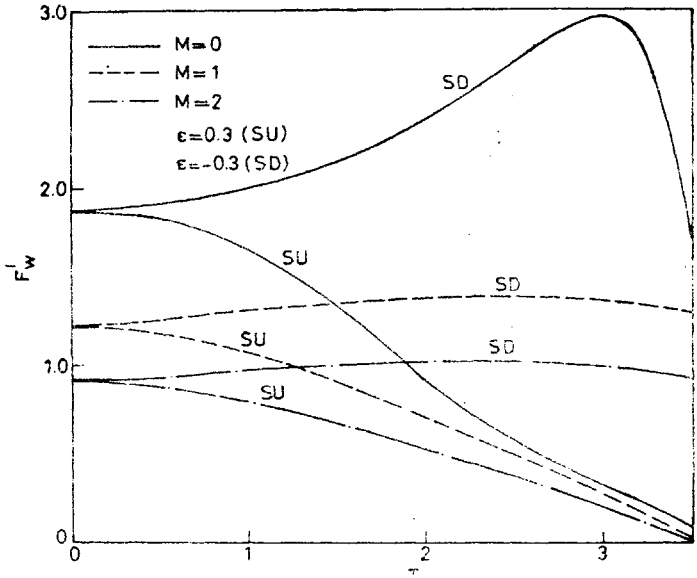

FIG. 5. Effect of magnetic field M on the skin-friction parameter in the radial direction F'_w ($\epsilon = \pm 0.3$).

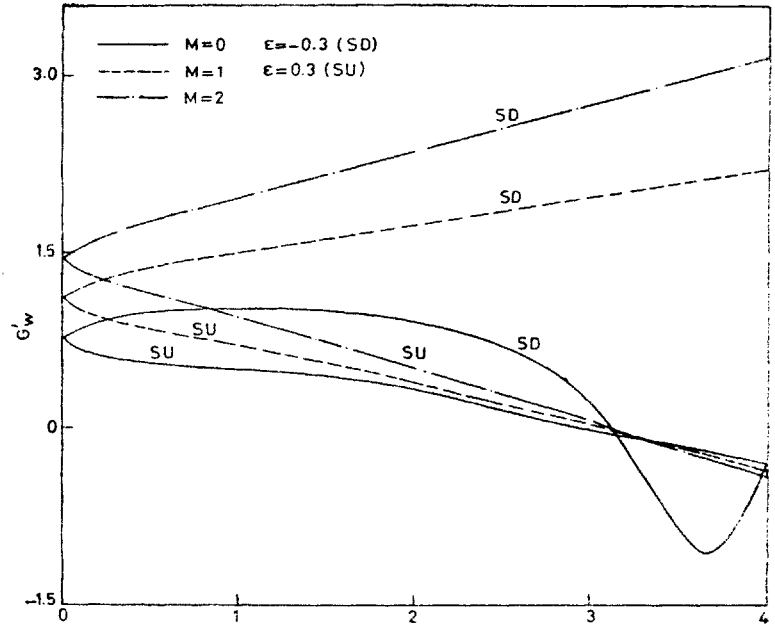

FIG. 6. Effect of magnetic field M on the skin-friction parameter in the tangential direction G'_w ($\epsilon = \pm 0.3$).

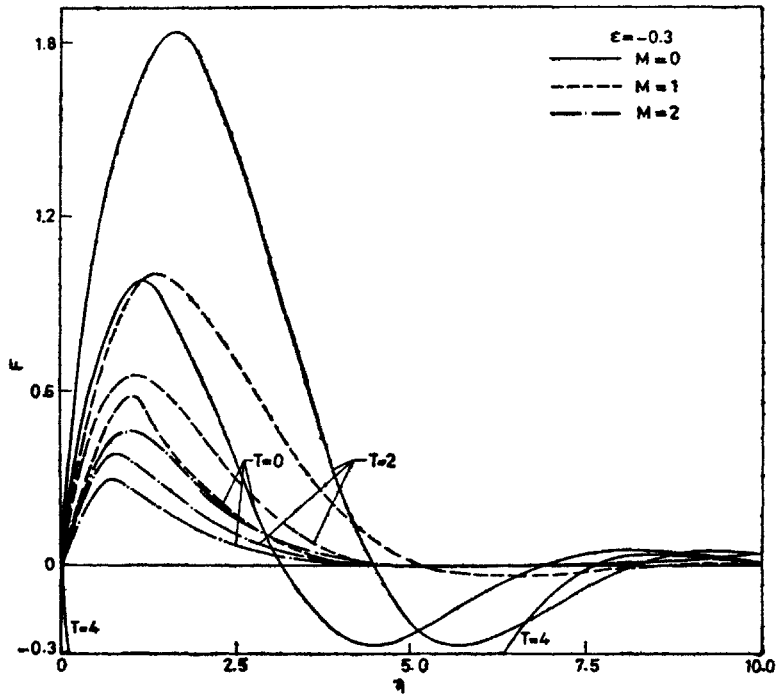

FIG. 7. Effect of magnetic field M on the radial velocity profile $F(\epsilon = -0.3)$.

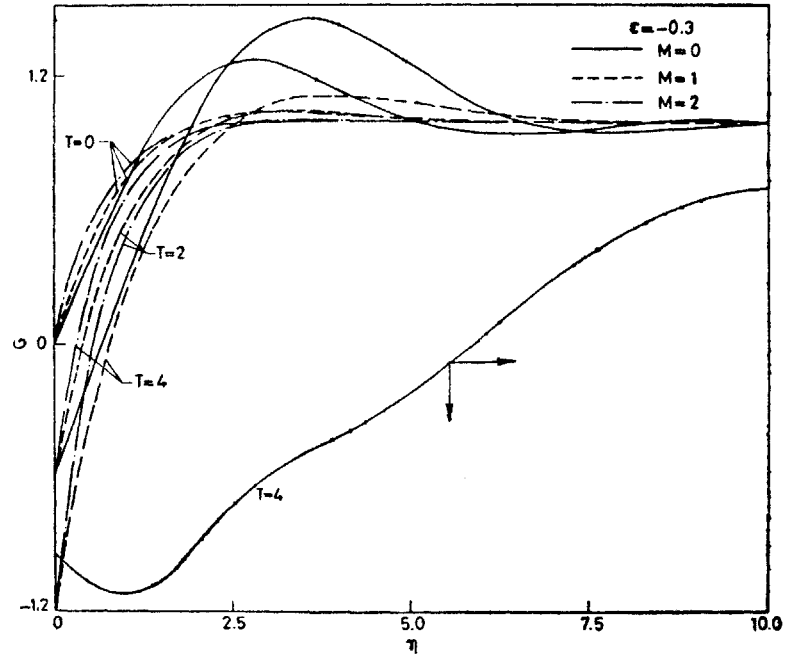

FIG. 8. Effect of magnetic field M on the tangential velocity profile G ($\epsilon = -0.3$)

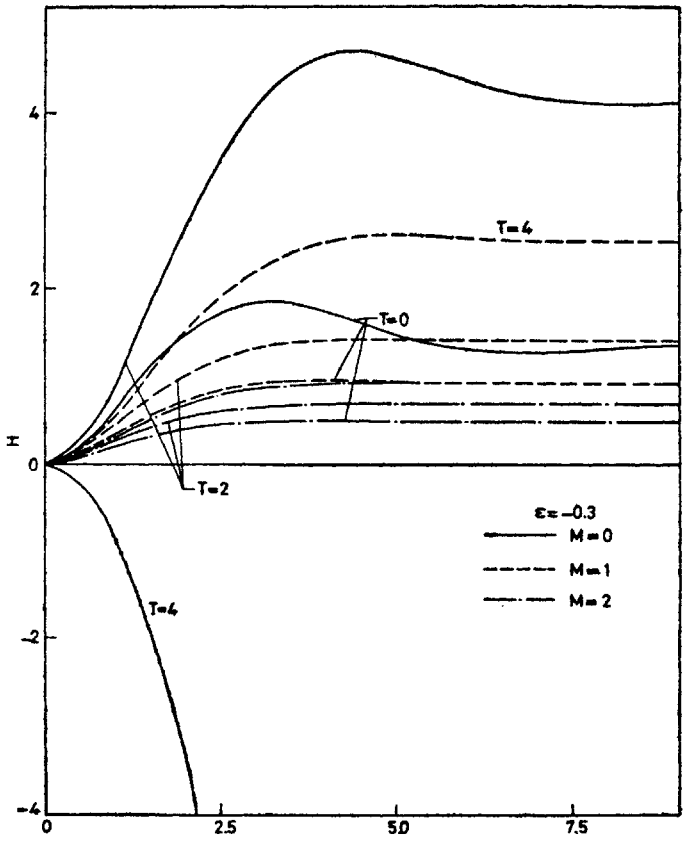

FIG. 9. Effect of magnetic field M on the axial velocity profile H ($\epsilon = -0.3$).

angular velocity of the disk decreases with time T while such oscillations are suppressed when a magnetic field is introduced.

The effect of the magnetic field M on the radial, tangential and axial velocity profiles (F, G, H), when the angular velocity of the disk decreases with time, is shown in Figs. 7–9. It is observed that these profiles show oscillatory characteristics in the absence of a magnetic field but these oscillations are damped out when a magnetic field is applied. The flow fields are found to be much more sensitive to the time T when the angular velocity of the disk decreases with time as compared to when it increases with time. This is true for both magnetic or non-magnetic case (Figs. 2–4 and 7–9).

4. CONCLUSIONS

The flow fields are found to be more sensitive to the time when the angular velocity of the disk decreases with time than when it increases. Therefore, they

cannot be regarded as symmetric reflection of each other. The velocity profiles are found to be oscillatory in the absence of the magnetic field and the magnetic field tends to suppress these oscillations.

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