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SOME FINITE AND INFINITE SUMMATION FORMULAE FOR THE *H*-FUNCTION OF SEVERAL COMPLEX VARIABLES

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In the present paper we establish some finite and infinite series summation formulae for the *H*-function of several complex variables defined by Srivastava and Panda as a multiple Mellin-Barnes contour integral.

1. INTRODUCTION AND NOTATIONS

The *H*-function of several complex variables, which was introduced and studied systematically in a series of recent papers by Srivastava and Panda (1975, 1976, 1978, 1979), is an extension of the *G*-function of several complex variables. This *H*-function of several variables includes Fox's *H*-functions of one and two variables, Meijer's *G*-functions of one and two variables, the generalized Lauricella function of Srivastava and Daoust (1969, p. 454), Appell functions, the Whittaker functions, etc. Therefore, the results established in this paper are of a general character and hence encompass several cases of interest.

The object of this paper is to establish some finite and infinite summation formulae for the *H*-function of several complex variables and recurrence relations based upon them. These formulae will yield a number of new and known results including the results of Sharma (1972, 1973).

The *H*-function of several complex variables defined by Srivastava and Panda (1975, 1976) and represented in the following manner (Srivastava *et al.* 1982, p. 251):

$$H\,[Z_1,\ldots,Z_r[\equiv H^{0,\,n\,:\,m_1,\,n_1;\ldots;m_r,\,n_r}_{p,\,q\,:\,p_1,\,q_1;\ldots;\,p_r,\,q_r}$$

$$\times\left[\begin{matrix}Z_1\\ \vdots\\ Z_r\end{matrix}\middle|\begin{matrix}(a_j;\ \alpha'_j,\ldots,\alpha_j^{(r)})_{1,p}\,:\,(c'_j,\,\gamma_j)_{1,p_1};\ \ldots;\ (c_j^{(r)},\,\gamma_j^{(r)})_{1,p_r}\\ (b_j;\ \beta'_j,\ldots,\beta_j^{(r)})_{1,q}\,:\,(d'_j,\,\delta'_j)_{1,q_1};\ \ldots;\ (d_j^{(r)},\,\delta_j^{(r)})_{1,q_r}\end{matrix}\right]$$

$$=\frac{1}{(2\pi\omega)^r}\int_{\alpha_1}\cdots\int_{\alpha_r}\phi_1(\xi_1)\ldots\phi_r(\xi_r)\,\psi(\xi_1,\ldots,\xi)_r\,Z_1^{\xi_1}\ldots Z_r^{\xi_r}\,d\xi_1\ldots d\xi_r,\qquad\ldots(1.1)$$

where

$$\omega = \sqrt{-1}$$

$$\phi_I(\xi_I) = \frac{\prod_{j=1}^{m_i} \Gamma(d_j^{(I)} - \delta_j^{(I)}\xi_I) \prod_{j=1}^{n_i} \Gamma(1 - c_j^{(I)} + \gamma_j^{(I)}\xi_I)}{\prod_{j=m_i+1}^{q_i} \Gamma(1 - d_j^{(I)} + \delta_j^{(I)}\xi_I) \prod_{j=n_i+1}^{p_i} \Gamma(c_j^{(I)} - \gamma_j^{(I)}\xi_I)} \qquad \ldots(1.2)$$

$$\psi(\xi_1, \ldots, \xi_r) = \frac{\prod_{j=1}^{n} \Gamma(1 - a_j + \sum_{i=1}^{r} \alpha_j^{(i)}\xi_i)}{\prod_{j=n+1}^{p} \Gamma(a_j - \sum_{i=1}^{r} \alpha_j^{(i)}\xi_i) \prod_{j=1}^{q} \Gamma(1 - b_j + \sum_{i=1}^{r} \beta_j^{(i)}\xi_i)} \qquad \ldots(1.3)$$

and the integral (1.1) converges absolutely if

$$\Omega \equiv \sum_{j=1}^{n} \alpha_j^{(i)} - \sum_{j=n+1}^{p} \alpha_j^{(I)} + \sum_{j=1}^{n_i} \gamma_j^{(I)} - \sum_{j=n_i+1}^{p_i} \gamma_j^{(I)} - \sum_{j=1}^{q} \beta_j^{(I)} + \sum_{j=1}^{m_i} \delta_i^{(I)} - \sum_{j=m_i+1}^{q_i} \delta_j^{(I)} > 0,$$

$$i \in \{1, \ldots, r\}, \ |\arg(z_i)| < \frac{1}{2}\Omega_i \pi.$$

The multiplication formula for the gamma function is

$$\Gamma(mz) = (2\pi)^{1/2(1-m)} m^{mz-1/2} \prod_{j=0}^{m-1} \Gamma\left(z + \frac{j}{m}\right) \qquad \ldots(1.4)$$

where m is a positive integer.

From (1.4), if N is a positive integer, the formulae

$$\prod_{j=0}^{m-1} \Gamma\left(\frac{\alpha + N + j}{m}\right) = m^{-N}(\alpha)_N \prod_{j=0}^{m-1} \Gamma\left(\frac{\alpha + j}{m}\right) \qquad \ldots(1.5)$$

$$\prod_{j=0}^{m-1} \Gamma\left(\frac{\alpha - N + j}{m}\right) = \frac{(-m)_N}{(1+\alpha)_N} \prod_{j=0}^{m-1} \Gamma\left(\frac{\alpha + j}{m}\right) \qquad \ldots(1.6)$$

can easily be derived.

2. Infinite Summation Formulae

The infinite summation formulae to be established are

$$\sum_{N=0}^{\infty} \frac{(-1)^N (\alpha)_N \left(\frac{1}{2} + \alpha\right)_N}{N! \left(\frac{1}{2}\alpha\right)_N} H_{p,q:\, p_1+\delta m, q_1; \ldots; p_r, q_r}^{0, n:\, m_1, n_1; \ldots; m_r, n_r}$$

(*equation continued on p.* 411)

$$\times \left[\begin{matrix} Z_1 \\ \vdots \\ Z_r \end{matrix} \middle| \begin{matrix} (a_j,\ \alpha_j', \ldots,\ \alpha_j^{(r)})_{1,p} : (c_j',\ \gamma_j')_{1,p_1}, \\ (b_j;\ \beta_j', \ldots,\ \beta_j^{(r)})_{1,q} : (d_j',\ \delta_j')_{1,q_1} \end{matrix} \right.$$

$$(\Delta(m, 1+\alpha-\beta+N), h), (\Delta(m, 1+\alpha-\gamma+N), h), (\Delta(m, 1-\beta-N), h),$$

$$(\Delta(m, 1-\gamma-N), h); \ldots;$$

$$\left. \begin{matrix} (c_j^{(r)},\ \gamma_j^{(r)})_{1,p_r} \\ (d_j^{(r)},\ \delta_j^{(r)})_{1,q_r} \end{matrix} \right] = \frac{m^{\alpha+1/2}\,\Gamma\left(\frac{1}{2}\right)}{2^{\alpha-\beta-\gamma}\,\Gamma(\alpha+1)} H^{0,\,n\ :\ m_1;\ n_1;\ldots;\ m_r,\ n_r}_{p,q\ :\ p_1+4m,q;\ldots;\ p_r,\ q_r}$$

$$\times \left[\begin{matrix} 2^{2mh}\ Z_1 \\ \vdots \\ Z_r \end{matrix} \middle| \begin{matrix} (a_j;\ \alpha_j', \ldots,\ \alpha_j^{(r)})_{1,p} : (c_j',\ \gamma_j')_{1,p_1}, \\ (b_j;\ \beta_j', \ldots,\ \beta_j^{(r)})_{1,q} : (d_j',\ \delta_j')_{1,q_1}; \end{matrix} \right.$$

$$\left. \begin{matrix} (\Delta(m, 1-\beta), h), (\Delta(m, 1-\gamma), h), (\Delta(2m, (1+\alpha-\beta-\gamma), h); \ldots; (c_j^{(r)},\ \gamma_j^{(r)})_{1,p_r} \\ ; \ldots; (d_j^{(r)},\ \delta_j^{(r)})_{1,q_r} \end{matrix} \right]$$

$$\mathrm{Re}(\alpha - 2\beta - 2\gamma) > -2; \qquad \ldots(2.1)$$

$$\text{(ii)} \quad \sum_{N=0}^{\infty} \frac{(\alpha)_N (\beta)_N}{N! \left(\alpha+\beta+\frac{1}{2}\right)_N} H^{0,\,n\ :\ n_1+m,\ n_1;\ldots;\ m_r,n_r}_{p,\ q\ :\ p_1+m,\ q_1+m;\ldots;\ p_r,q_r}$$

$$\times \left[\begin{matrix} Z_1 \\ \vdots \\ Z_r \end{matrix} \middle| \begin{matrix} (a_j;\ \alpha_j', \ldots;\ \alpha_j^{(r)})_{1,p} : (c_j',\ \gamma_j')_{1,p_1}, \\ (b_j;\ \beta_j', \ldots;\ \beta_j^{(r)})_{1,q} : (\Delta(m, \gamma+N), h), \end{matrix} \right.$$

$$\left. \begin{matrix} (\Delta(m, \frac{1}{2}+\gamma+N), h); \ldots; (c_j^{(r)},\ \gamma_j^{(r)})_{1,p_r} \\ (d_j',\ \delta_j')_{1,q_1} \qquad\qquad ; \ldots;\ (d_j^{(r)},\ \delta_j^{(r)})_{1,qr} \end{matrix} \right]$$

$$= \frac{\Gamma\left(\frac{1}{2}\right)\Gamma\left(\alpha+\beta+\frac{1}{2}\right)}{\Gamma\left(\alpha+\frac{1}{2}\right)\Gamma\left(\beta+\frac{1}{2}\right)} H^{0,n\ :\ m_1+2m,\ n_1\ ;\ldots;\ m_r,\ n_r}_{p,\ q\ :\ p_1+2m;\ q_{1+}{}^{2m}\ ;\ldots;\ p_r,\ q}$$

$$+ \left[\begin{matrix} Z_1 \\ \vdots \\ Z_r \end{matrix} \middle| \begin{matrix} (a_j;\ \alpha_j', \ldots,\ \alpha_j^{(r)})_{1,p} : (c_j',\ \gamma_j')_{1,p_1}, \\ (b_j;\ \beta_j', \ldots;\ \beta_j^{(r)})_{1,q} : (\Delta(m, \gamma+\tfrac{1}{2}), h), \end{matrix} \right.$$

(*equation continued on p.* 412)

$$\left.\begin{matrix}(\Delta\,(m,\,\frac{1}{2}+\gamma-\alpha),\,h),\,(\Delta(m,\,\frac{1}{2}+\gamma-\beta),\,h);\ldots;(c_j^{(r)},\,\gamma_j^{(r)})_{1,p_r}\\ (\Delta\,(m,\,\gamma-\alpha-\beta+\tfrac{1}{2}),\,h),\,(d_j',\,\delta_j')_{1,q_1}\qquad\qquad\qquad;\ldots;(d_j^{(r)},\,\delta_j^{(r)})_{1,q_r}\end{matrix}\right]$$

...(2.2)

PROOF : Expressing the H-function of several complex variables on the left-hand side of (2.1) by Mellin-Barnes contour integral (1.1), changing the order of summation and integration as permissible by absolute convergence for Re $(\alpha-2\beta-2\gamma) < -2$ and the above stated conditions, finally using (1.5) and (1.6), and simplifying the expression with the help of result (MacRobert 1966, p. 368), the right-hand side of (2.1) obtained.

Similarly, eqn. (2.2) can be obtained by using the result given by MacRobert (1958) in place of MacRobert (1966, p. 368).

3. FINITE SUMMATION FORMULAE

The finite summation formulae to be established are

$$\text{(i)}\ \sum_{u=0}^{N}\binom{N}{u}\frac{m^{2u}}{(1+k)_u\,(1-k-N)_u}\,H_{p,\,q\,:\,p_1+m,\,q_1+m;\ldots;\,p_r,\,q_r}^{0,\,n\,:\,m_1+m,\,n_1;\ldots;\,m_r,\,n_r}$$

$$\times\left[\begin{matrix}Z_1\\ \vdots\\ Z_r\end{matrix}\middle|\begin{matrix}(a_j;\,\alpha_j',\ldots;\,\alpha_j^{(r)})_{1,p}:\\ (b_j;\,\beta_j',\ldots;\,\beta_j^{(r)})_{1,q}:\end{matrix}\right.$$

$$\left.\begin{matrix}(c_j',\,\gamma_j')_{1,p_1},\,(\Delta\,(m,\,-u+\alpha),\,h);\ldots;(c_j^{(r)},\,\gamma_j^{(r)})_{1,p_r}\\ (\Delta\,(m,\,\alpha+u),\,h),\,(d_j',\,\delta_j')_{1,q_1}\ ;\ldots;(d_j^{(r)},\,\delta_j^{(r)})_{1,q_r}\end{matrix}\right]$$

$$=\frac{(-1)^N\,m^{2N}\,\Gamma(k+1)\,\Gamma(k)}{\Gamma(k+N)\,\Gamma(k+1+N)}\,H_{p,\,q\,:\,p_1+2m,\,q_1+2m;\ldots;\,p_r,\,q_r}^{0,\,n\,:\,m_1+2m,n_1;\ldots;\,m_r,\,n_r}$$

$$\times\left[\begin{matrix}Z_1\\ \vdots\\ Z_r\end{matrix}\middle|\begin{matrix}(a_j;\,\alpha_j',\ldots;\,\alpha_j^{(r)})_{1,p}:\\ (b_j;\,\beta_j',\ldots;\,\beta_j^{(r)})_{1,q}:\end{matrix}\right.$$

$$\left.\begin{matrix}(c_j',\,\gamma_j')_{1,p_1},\,(\Delta\,(m,\,\alpha+k),\,h),\,(\Delta\,(m,\,\alpha-k-N),\,h);\ldots;\,(c_j^{(r)},\,\gamma_j^{(r)})_{1,p_r}\\ (\Delta\,(m,\,\alpha+k+N),\,h),\,(\Delta\,(m,\,\alpha-k),\,h),\,(d_j',\,\delta_j')_{1,q_1};\ldots;(d_j^{(r)},\,\delta_j^{(r)})_{1,q_r}\end{matrix}\right],$$

...(3.1)

$$\text{(ii)}\ \sum_{u=0}^{N}\binom{N}{u}(k)_u(-k+N)_u\,m^{-2u}\,H^{0,\,n\,:\,m_1+m,\,n_1;\ldots;\,m_r,\,n_r}_{p,\,q\,:\,p_1+m,\,q_1+m;\ldots;p_r,\,q_r}$$

$$\times\left[\begin{matrix}Z_1\\ \vdots\\ Z_r\end{matrix}\middle|\begin{matrix}(a_j;\ \alpha'_j,\ldots;\ \alpha_j^{(r)})_{1,p}:(c'_j,\ \gamma'_j)_{1,p_1},\\ (b_l;\ \beta'_j,\ldots;\ \beta_j^{(r)})_{1,q}:(\Delta(m,\alpha-u),h),\end{matrix}\right.$$

$$\left.\begin{matrix}(\Delta(m,\alpha+u),h);\ldots;\ (c_j^{(r)},\ \gamma_j^{(r)})_{1,p_r}\\ (d'_j,\ \delta'_j)_{1,q_1}\qquad;\ldots;\ (d_j^{(r)},\ \delta_j^{(r)})_{1,q_r}\end{matrix}\right]$$

$$=H^{0,\,n\,:\,m,\,n_1+3m;\ldots;\,m_r,\,n_r}_{p,\,q\,:\,p_1+3m,\,q_1+3m;\ldots;\,p_r,q_r}\left[\begin{matrix}Z_1\\ \vdots\\ Z_r\end{matrix}\middle|\begin{matrix}(a_j;\ \alpha'_j,\ldots;\ \alpha_j^{(r)})_{1,p}:\\ (b_j;\ \beta'_j,\ldots;\ \beta_j^{(r)})_{1,q}:\end{matrix}\right.$$

$$\left.\begin{matrix}(\Delta(m,\alpha-k),h),(\Delta(m,\alpha+N),h).\\ (d'_j,\ \delta'_j)_{1,q_1},(\Delta(m,\alpha+k),h),\end{matrix}\right.$$

$$\left.\begin{matrix}(\Delta(m,\alpha+k-N),h),(c'_j,\ \gamma'_j)_{1,p_1}\qquad;\ldots;\ (c_j^{(r)},\ \gamma_j^{(r)})_{1,p_r}\\ (\Delta(m,\alpha-N),h),(\Delta(m,\alpha-k+N),h);\ldots;(d_j^{(r)},\ \delta_j^{(r)})_{1,q_r}\end{matrix}\right]\qquad\ldots(3.2)$$

$$\text{(iii)}\ \sum_{u=0}^{N}(-1)^u\binom{N}{u}\frac{(\alpha)_u\left(\frac{1}{2}\alpha+1\right)_u(\delta)_u(\epsilon)_u}{\left(\frac{1}{2}\alpha\right)_u(\alpha-\delta+1)_u(\alpha-\epsilon+1)_u(\alpha+N+1)_u}$$

$$H^{0,\,n\,:\,m_1+2m,\,n_1;\ldots;\,m_r,\,n_r}_{p,\,q\,:\,p_1+2m,\,q_1+2m;\ldots;\,p_r,\,q_r}\left[\begin{matrix}Z_1\\ \vdots\\ Z_r\end{matrix}\middle|\begin{matrix}(a_j;,\ \alpha_j,\ldots;\ \alpha_j^{(r)})_{1,p}:(c'_j,\ \gamma'_j)_{1,p_1}\\ (b'_j;\ \beta'_j,\ldots;\ \beta_j^{(r)})_{1,q}:(\Delta(m,\beta+u),h),\end{matrix}\right.$$

$$\left.\begin{matrix}(\Delta(m,1+\alpha-\gamma+u),h),(\Delta(m,1-\gamma-u),h);\ldots;\ (c_j^{(r)},\ \gamma_j^{(r)})_{1,p_r}\\ (\Delta(m,\beta-\alpha-u),h),\ (d'_j,\ \delta'_j)_{1,q_1}\qquad;\ldots;\ (d_j^{(r)},\ \delta_j^{(r)})_{1,q_r}\end{matrix}\right]$$

$$=\frac{(\alpha+1)_N(\alpha-\beta-\gamma+1)_N}{(\alpha-\delta+1)_N(\alpha-\beta-\gamma-\delta+1)_N}\,H^{0,n\,:\,m_1+4m,\,n_1;\ldots;\,m_r,\,n_r}_{p,\,q\,:\,p_1+4m,\,q_1+4m;\ldots;\,p_r,\,q_r}$$

$$\times\left[\begin{matrix}Z_1\\ \vdots\\ Z_r\end{matrix}\middle|\begin{matrix}(a_j;\ \alpha'_j,\ldots,\ \alpha_j^{(r)})_{1,p}:(c,\ \gamma'_j)_{1,p_1}\\ (b_j;\ \beta'_j,\ldots,\ \beta_j^{(r)})_{1,q}:(\Delta(m,\beta),h),\end{matrix}\right.$$

(*equation continued on p.* 414)

$$(\Delta(m, 1+\alpha-\gamma+N), h), (\Delta(m, 1-\gamma), h), (\Delta(m, 1+\alpha-\gamma-\delta), h),$$

$$(\Delta(m, \beta-\alpha-N),) h, \quad (\Delta(m, \beta+\delta-\alpha), h),$$

$$(\Delta(m, 1+\beta+\delta-\alpha-N), h;\ldots;$$

$$(\Delta(m, \alpha-\gamma-\delta+1+N), h), (d_j', \delta_j')_{1,q_1};\ldots;$$

$$\left.\begin{matrix}(c_j^{(r)}, \gamma_j^{(r)})_{1,p_r} \\ (d_j^{(r)}, \delta_j^{(r)})_{1,q_r}\end{matrix}\right] \qquad \ldots(3.3)$$

provided m and N are positive integers, $h > 0$.

PROOF : Expressing the H-function on the left-hand side of (3.1) by Mellin-Barnes contour integral (1.1), changing the order of summation and integration, which is justified due to the absolute convergence involved in the process and finally using (1.5), (1.6) and simplifying with the help of Saalschüitz's theorem (MacRobert 1966, p. 360) and (1.4), the right-hand side of of (3.1) is obtained.

Similarly, we can obtain the formulae (3.2) and (3.3) by using the results

$${}_3F_2\left[\begin{matrix}\alpha, \beta, \gamma \\ \rho, \sigma\end{matrix}; 1\right] = \frac{\Gamma(\rho)\,\Gamma(\alpha-\sigma+1)\,\Gamma(\beta-\sigma+1)\,\Gamma(\gamma-\sigma+1)}{\Gamma(1-\sigma)\,\Gamma(\rho-\alpha)\,\Gamma(\rho-\beta)\,\Gamma(\rho-\gamma)}$$

and [MacRobert 1966, p. 171 (34)] respectively.

Setting $N = 1$, (3.1), (3.2) and (3.3) yields

$$\text{(i)}\ \frac{k(k+1)}{m^2} H\left[Z_1, \ldots, Z_r\right] + H^{\cdots : m_1+2m, n_1; \cdots;\cdots}_{\cdots : p_1+2m, q_1+2m; \cdots;\cdots}$$

$$\times \left[\begin{matrix}Z_1 \\ \vdots \\ Z_r\end{matrix}\right| \begin{matrix}\ldots : (c_j', \gamma_j')_{1,p_1} \\ \ldots : (\Delta(m, 1+\alpha+k), h), \\ (\Delta(m, 1+\alpha+k), h), (\Delta m, \alpha-k), h);\ldots;\ldots \\ (\Delta(m, \alpha+k), h), \quad (d_j', \delta_j')_{1,q_1};\ldots;\ldots\end{matrix}\Bigg]$$

$$= H^{\cdots : m_1+m, n_1; \cdots;\cdots}_{\cdots : p_1+m, q_1+m;\ldots;\ldots} \left[\begin{matrix}Z_1 \\ \vdots \\ Z_r\end{matrix}\right| \begin{matrix}\ldots : (c_j', \gamma_j')_{1,p_1}, (\Delta(m, \alpha), h) \quad ;\ldots;\ldots \\ \ldots : (\Delta(m, 1+\alpha), h), (d_j', \delta_j')_{1,q_1};\ldots;\ldots\end{matrix}\Bigg] \qquad \ldots(3.4)$$

$$\text{(ii)}\ H\left[Z_1, \ldots, Z_r\right] + \frac{k(1-k)}{m^2} H^{\cdots : m_1+m, n_1 \quad ;\ldots;}_{\cdots : p_1+m, q_1+m;\ldots;}$$

(*equation continued on p.* 415)

$$\times \left[\begin{matrix} Z_1 \\ \vdots \\ Z_r \end{matrix}\middle|\begin{matrix} \ldots : (c_j', \gamma_j')_{1,p_1}, (\Delta(m, \alpha+2), h); \ldots; \ldots \\ \ldots : (\Delta(m, \alpha-1), h), (d_j', \delta_j')_{1,q_1}; \ldots; \ldots \end{matrix}\right]$$

$$= H^{\cdots : m_1, n_1+3m}_{\cdots : p_1+3m, q_1+3m; \cdots; \cdots} \left[\begin{matrix} Z_1 \\ \vdots \\ Z_r \end{matrix}\middle|\begin{matrix} \ldots : (\Delta(m, \alpha-k), h), (\Delta(m, 1+\alpha) h), \\ \ldots : (b_j', \delta_j')_{1,q_1}, \quad (\Delta(m, \alpha+k), h), \end{matrix}\right.$$

$$\left.\begin{matrix} (\Delta(m, \alpha+k-1), h), (c_j', \gamma_j')_{1,p_1}; \ldots; \ldots \\ (\Delta(m, \alpha-1), h), (\Delta(m, \alpha-k+1), h); \ldots; \ldots \end{matrix}\right] \quad \ldots(3.5)$$

$$\text{(iii)}\quad H^{\cdots : m_1, n_1+2m; \cdots; \cdots}_{\cdots : p_1+2m, q_1+2m; \cdots; \cdots} \left[\begin{matrix} Z_1 \\ \vdots \\ Z_r \end{matrix}\middle|\begin{matrix} \ldots : (c_j', \gamma_j')_{1,p_1}, (\Delta(m, 1+\alpha-\gamma), h), \\ \ldots : (\Delta(m, \beta), h), (\Delta(m, \beta-\alpha), h), \end{matrix}\right.$$

$$\left.\begin{matrix} (\Delta(m, 1-\gamma), h); \ldots; \ldots \\ (d_j', \delta_j')_{1,q_1}; \quad \ldots; \ldots \end{matrix}\right] - \frac{2\delta\epsilon\left(\frac{1}{2}\alpha+1\right)}{(\alpha+2)(\alpha-\delta+1)(\alpha-\epsilon+1)}$$

$$H^{\cdots ; m_1+2m, n_1}_{\cdots ; p_1+2m, r_1+2m; \cdots; \cdots} \left[\begin{matrix} Z_1 \\ \vdots \\ Z_r \end{matrix}\middle|\begin{matrix} \ldots : (c_j', \gamma_j')_{1,p_1}, \\ \ldots : (\Delta(m, \beta+1), h), \end{matrix}\right.$$

$$\left.\begin{matrix} (\Delta(m, 2+\alpha-\gamma), h), (\Delta(m, -\gamma), h); \ldots; \ldots \\ (\Delta(m, \beta-\alpha-1), h), (d_j', \delta_j')_{1,q_1} \quad ; \ldots; \ldots \end{matrix}\right]$$

$$= \frac{(\alpha+1)(\alpha-\beta-\gamma+1)}{(\alpha-\delta+1)(\alpha-\beta-\gamma-\delta+1)} H^{\cdots ; m_1+4m, n_1}_{\cdots ; p_1+4m, q_1+4m; \ldots; \ldots} \times \left[\begin{matrix} Z_1 \\ \vdots \\ Z_r \end{matrix}\middle|\begin{matrix} \ldots : (c_j', \gamma_j')_{1,p1}, \\ \ldots : (\Delta(m, \beta), h), \end{matrix}\right.$$

$$(\Delta(m, 2+\alpha-\gamma), h), (\Delta(m, 1-\gamma), h), (\Delta(m, 1+\alpha-\gamma-\delta), h),$$

$$(\Delta(m, \beta-\alpha-1), h), (\Delta(m, \beta+\delta-\alpha), h), (\Delta(m, \alpha-\gamma-\delta+2), h),$$

$$\left.\begin{matrix} (\Delta(m, \beta+\delta-\alpha), h); \ldots; \ldots \\ (d_j', \delta_j') 1, q_1; \ldots; \ldots \end{matrix}\right] \quad \ldots(3.6)$$

where

$$2\alpha = \beta + \gamma + \delta + \epsilon.$$

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