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P AND *S* WAVES IN A MEDIUM UNDER INITIAL STRESSES AND UNDER GRAVITY

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In this paper the velocities of *P* and *S* waves in an unbounded medium under normal initial stresses and under gravity field are derived and it is shown that these velocities not only depend on the initial stresses and gravity but also on the direction of propagation. The results are shown to tally with the classical case in the absence of initial stresses and gravity. The velocities of *P* and *S* waves along different directions in presence of different combinations of normal initial stresses and in presence of gravity field are calculated numerically and the results presented graphically.

1. INTRODUCTION

The study of stress waves is of central interest to the engineers, physicists and seismologists due to its application in varieties of useful purposes. The extensive literature on the subject is available in the books of Ewing *et al.* (1967), Bullen (1963), Bath (1968), Miklowitz (1966), Achenbach (1973), Kalski (1963) and by many other authors. In their literature the effect of initial stresses has not been considered. Initial stresses are developed in the medium due to many reasons, resulting from difference of temperature,process of quenching, shot pinning and cold working, slow process of creep, differential external forces, gravity variations, etc. The Earth is supposed to be under high initial stresses. It is therefore of great interest to study the effect of these stresses on the propagation of stress waves.

In the last three decades considerable attention has been directed towards the analysis of effects of pre-stressing on the propagation of waves in solids. It was probably first Biot (1940), became interested in the subject. Hughes and Kelly (1953) had shown the effect of uniaxial stress on certain elastic constants. Later Daniels and Smith (1958) considered a problem showing the effect of hydrostatic pressure in the speed of sound.

In 1961, three fundamental studies appeared. Two of them are owed to Hayes and Rivlin (1961a, b), the third to Toupin and Bernstein (1961). In these works the theory of small elastic deformations superimposed on a finite deformation is applied and a rigorous analysis of plane waves travelling in media subject to pure homogeneous deformation is made. Some new phenomena of acousto-elasticity were discovered and the condition for instability of initial state of finite stress was established.

It was the achievement of Biot (1965) to show that acoustic propagation under initial stresses would be fundamentally different from that under stress free state and could not be represented merely by introducing stress dependent elastic coefficients in the classical theory. Biot (1965) has obtained the velocities of longitudinal and transverse waves propagating along the coordinate axes only. Although Dey (1971), Das and Dey (1970), Nowinsky (1978), Dey and Mukherjee (1982), Chakraborty and Dey (1982) have shown the effect of initial stresses on the propagation, reflection and refraction of waves in elastic and an elastic layers or half spaces, yet no attempt has been made so far, to obtain the velocities of longitudinal and transverse waves under initial stresses when the waves propagate not only along the coordinate axes but also along any other direction. Curiously enough, there is no literature available, as yet, to show the effect of gravity on the propagation of body waves.

In this paper, the velocities of longitudinal and transverse waves in an initially stressed medium has been derived following Biot's incremental deformation theory. It is observed that velocities of these waves are very much influenced by the initial stresses present in the medium and also depend on the direction of propagation. The effect of gravity on the propagation of body waves has also been shown. Velocities for different initial stress parameters and gravity parameter have been calculated for different directions of propagation. These results have been presented graphically and compared with the initial stress-free and gravity-free cases. The information obtained in this paper may be used as a non-distructive method for the estimation of pre-stressing present in a medium.

2. *Case I* : *P* AND *S* WAVES IN AN UNBOUNDED MEDIUM UNDER NORMAL INITIAL STRESSES

Consider the medium with normal initial stresses S_{11} and S_{22} in two orthogonal directions x_1 and x_2 respectively (Fig. 1). The equations of motion in two directions under these stresses are (Biot 1965)

$$\left.\begin{aligned} \frac{\partial s_{11}}{\partial x_1} + \frac{\partial s_{12}}{\partial x_2} - P\frac{\partial \omega}{\partial x_2} &= \frac{\partial^2 u_1}{\partial t^2} \\ \frac{\partial s_{21}}{\partial x_1} + \frac{\partial s_{22}}{\partial x_2} - P\frac{\partial \omega}{\partial x_2} &= \frac{\partial^2 u_2}{\partial t^2} \end{aligned}\right\} \quad \ldots(1)$$

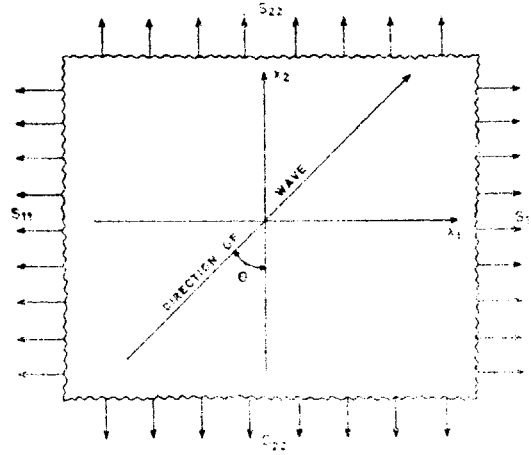

FIG. 1. P and S waves in a medium under initial stresses.

s_{ij} being incremental stress components, $P = S_{22} - S_{11}$, u_i, the displacement components and $\omega = \frac{1}{2}\left(\frac{\partial u_1}{\partial x_2} - \frac{\partial u_2}{\partial x_1}\right)$.

The stress-strain relations are

$$\left.\begin{aligned} s_{11} &= B_{11}\, e_{x_1x_1} + B_{12}\, e_{x_2x_2} \\ s_{22} &= B_{21}\, e_{x_1x_1} + B_{22}\, e_{x_2x_2} \\ s_{12} &= 2Q\, e_{x_1x_2} \end{aligned}\right\} \qquad \ldots(2)$$

where

$$e_{x_ix_j} = \frac{1}{2}\left(\frac{\partial u_j}{\partial x_i} + \frac{\partial u_i}{\partial x_j}\right) \text{ and } i, j = 1, 2.$$

B_{ij} and Q are the elastic coefficients in the pre-stressed medium given by (Biot 1965)

$$\left.\begin{aligned} B_{11} &= \lambda_1 \frac{\partial S_{11}}{\partial \lambda_1},\ B_{12} = \lambda_2 \frac{\partial S_{11}}{\partial \lambda_2} \\ B_{21} &= \lambda_1 \frac{\partial S_{22}}{\partial \lambda_1},\ B_{22} = \lambda_2 \frac{\partial S_{22}}{\partial \lambda_2} \\ Q &= \frac{1}{2}(S_{11} - S_{22})\ \frac{\lambda_1^2 + \lambda_2^2}{\lambda_1^2 - \lambda_2^2} \end{aligned}\right\} \qquad \ldots(3)$$

λ_1 and λ_2 being the extension ratios in the x_1 and x_2 axes respectively in the pre-stressed state.

The existence of strain energy requires the condition

$$B_{12} = B_{21} + P. \qquad \ldots(4)$$

These coefficients alternately may be written in the form Biot (1965)

$$\left.\begin{aligned} B_{11} &= K + N + P, \quad B_{12} = K - N + P \\ B_{21} &= K - N, \quad B_{22} = K + N, \quad Q = N \end{aligned}\right\} \qquad \ldots(5)$$

where $K = \lambda + \mu$ and $N = \mu$; λ, μ being Lame's constants in the initially stressed medium.

Using (5) the stress-strain relations (2) in more practical form may be written as

$$\left.\begin{aligned} s_{11} &= (\lambda + 2\mu + P)\, e_{x_1 x_1} + (\lambda + P)\, e_{x_2 x_2} \\ s_{22} &= \lambda\, e_{x_1 x_1} + (\lambda + 2\mu)\, e_{x_2 x_2} \\ s_{12} &= 2\mu\, e_{x_1 x_2}. \end{aligned}\right\} \qquad \ldots(6)$$

Equations (1) with the help of (6) take the forms

$$\left.\begin{aligned} (\lambda + 2\mu + P)\frac{\partial^2 u_1}{\partial x_1^2} + \left(\lambda + \mu + \frac{1}{2}P\right)\frac{\partial^2 u_2}{\partial x_1 \partial x_2} + \left(\mu + \frac{1}{2}P\right)\frac{\partial^2 u_1}{\partial x_2^2} \\ = \rho\,\frac{\partial^2 u_1}{\partial t^2} \\ (\lambda + 2\mu)\frac{\partial^2 u_2}{\partial x_2^2} + \left(\lambda + \mu + \frac{1}{2}P\right)\frac{\partial^2 u_1}{\partial x_1 \partial x_2} + \left(\mu - \frac{1}{2}P\right)\frac{\partial^2 u_2}{\partial x_1^2} \\ = \rho\,\frac{\partial^2 u_2}{\partial t^2}. \end{aligned}\right\} \qquad \ldots(7)$$

The solution of (7) may be taken as

$$\mathbf{u}^{(n)} = A_n^{\mathbf{d}^{(n)}} \exp(i\eta_n) \qquad \ldots(8)$$

where the index n assigns the direction of propagation of waves, $\mathbf{d}^{(n)}$ is the unit displacement vector and

$$\eta_n = k_n(\mathbf{x} \cdot \mathbf{p}^{(n)} - C_n t) \qquad \ldots(9)$$

$\mathbf{p}^{(n)}$ being the unit propagation vector, C_n the velocity of propagation and k_n the corresponding wavenumber. In two dimensions we have

$$\mathbf{n}^{(n)} = (u_1^{(n)}, u_2^{(n)}),\ \mathbf{d}^{(n)} = (d_1^{(n)}, d_2^{(n)})$$

and ...(10)

$$\mathbf{p}^{(n)} = (p_1^{(n)}, p_2^{(n)}) \text{ such that } p_1^{(n)^2} + p_2^{(n)^2} = 1.$$

The displacement components (8) may be expressed as

$$\begin{bmatrix} u_1^{(n)} \\ u_2^{(n)} \end{bmatrix} = \begin{bmatrix} A_n d_1^{(n)} \\ A_n d_2^{(n)} \end{bmatrix} \exp\left[ik_n (x_1 p_1^{(n)} + x_2 p_2^{(n)} - C_n t)\right]$$

$$= \begin{bmatrix} A \\ B \end{bmatrix} \exp\left[ik_n (x_1 p_1^{(n)} + x_2 p_2^{(n)} - C_n t)\right]. \quad ...(11)$$

Substituting (11) in (7) we get

$$\left.\begin{aligned} A\left[(\lambda + 2\mu + P) p_1^{(n)^2} + \left(\mu + \frac{1}{2} P\right) p_2^{(n)^2} - \rho C_n^2\right] + B\left[\left(\lambda + \mu + \frac{1}{2} P\right) p_1^{(n)} p_2^{(n)}\right] = 0 \\ A\left[\left(\lambda + \mu + \frac{1}{2} P\right) p_1^{(n)} p_2^{(n)}\right] + B\left[(\lambda + 2\mu) p_2^{(n)^2} + \left(\mu - \frac{1}{2} P\right) p_1^{(n)^2} - \rho C_n^2\right] = 0. \end{aligned}\right\} \quad ...(12)$$

For non-zero solution of A and B we have

$$\begin{vmatrix} (\lambda + 2\mu + P) p_1^{(n)^2} + \left(\mu + \frac{1}{2} P\right) p_2^{(n)^2} - \rho C_n^2, & \left(\lambda + \mu + \frac{1}{2} P\right) p_1^{(n)} p_2^{(n)} \\ \left(\lambda + \mu + \frac{1}{2} P\right) p_1^{(n)} p_2^{(n)}, & (\lambda + 2\mu) p_2^{(n)^2} + \left(\mu - \frac{1}{2} P\right) p_1^{(n)^2} - \rho C_n^2 \end{vmatrix} = 0. \quad ...(13)$$

From the above equations two values of C_n^2 may be obtained.

These two values of C_n^2 are the squares of velocities of longitudinal and transverse waves $C_L^{(n)^2}$ and $C_T^{(n)^2}$ respectively in the direction of $\mathbf{P}^{(n)}$ and are given by

$$C_L^{(n)^2} = \frac{1}{2\rho}\left[\left(\lambda + 3\mu + \frac{1}{2}P\right) + \sqrt{\left(\lambda + \mu - \frac{1}{2}P\right)^2 + 4P\left(\lambda + \mu + \frac{1}{2}P\right)p_1^{(n)^2}}\right]$$

$$C_T^{(n)^2} = \frac{1}{2\rho}\left[\left(\lambda + 3\mu + \frac{1}{2}P\right) - \sqrt{\left(\lambda + \mu - \frac{1}{2}P\right)^2 + 4P\left(\lambda + \mu + \frac{1}{2}P\right)p_1^{(n)^2}}\right] \quad \ldots(14)$$

while calculating these results, relation (10) has been used.

From (14) it is observed that the velocities of longitudinal and transverse waves depend on the initial stresses S_{22} and S_{11} and also on the direction of propagation. In case the direction of propagation of wave makes an angle θ with x_2-direction, then $p_1^{(n)} = \sin\theta$ and hence the velocities of longitudinal and transverse waves in non-dimensionalised form may be written as

$$\frac{C_L^2}{\mu/\rho} = \frac{1}{2}\left[\left\{\delta + 3 + (\zeta_2 - \zeta_1)\right\} + \sqrt{\left\{\delta + 1 - (\zeta_2 - \zeta_1)\right\}^2 + 8(\zeta_2 - \zeta_1)\{\delta + 1 + (\zeta_2 - \zeta_1)\}\sin^2\theta}\right]$$

$$\frac{C_T^2}{\mu/\rho} = \frac{1}{2}\left[\left\{\delta + 3 + (\zeta_2 - \zeta_1)\right\} - \sqrt{\left\{\delta + 1 - (\zeta_2 - \zeta_1)\right\}^2 + 8(\zeta_2 - \zeta_1)\{\delta + 1 + (\zeta_2 - \zeta_1)\}\sin^2\theta}\right] \quad \ldots(15)$$

where

$$\zeta_1 = \frac{S_{11}}{2\mu}, \zeta_2 = \frac{S_{22}}{2\mu}, \delta = \lambda/\mu;$$

ζ_1 and ζ_2 may be termed as pre-stressed parameters.

If the ratio of the velocities C_L and C_T is considered and denoted by $\bar{K}$, i.e., if $C_L/C_T = \bar{K}$, then it is clear from (15) that $\bar{K}$ is not a constant but depends on the initial stresses and the direction of propagation. $\bar{K}$ may be termed as the velocity parameter of an initially stressed medium, $\bar{K}$ is a constant in the absence of initial stresses and coincides with the material constant as defined by Achenbaeh (1973).

Particular Cases

(i) If the wave propagates in the x_1-direction, then $p_1^{(n)} = 1$, and the wave velocities from (14) take the form

$$C_L = \sqrt{\frac{\lambda + 2\mu + P}{\rho}}, \quad C_T = \sqrt{\frac{\mu + 1/2\, P}{\rho}} \qquad \ldots(16)$$

(ii) If the wave propagates in the x_2-direction, then $p_1^{(n)} = 0$, and the wave velocities from (14) take the form

$$C_L = \sqrt{\frac{\lambda + 2\mu}{\rho}}, \quad C_T = \sqrt{\frac{\mu + 1/2\, P}{\rho}}. \qquad \ldots(17)$$

Results (16) and (17) were also obtained by Biot (1965). It is interesting to note that longitudinal wave along x_2-direction is not affected by the initial stress difference $P = S_{22} - S_{11}$. Further if the initial stresses are constant and hydrostatic in nature, then $P = 0$ and so the velocities of longitudinal and shear waves are not affected by such stresses and are independent of direction of propagation also.

It may be noticed that in the absence of initial stresses or when the initial stresses are hydrostatic in nature, the results in (14) coincide with the classical results.

Case II : *P* AND *S* WAVES IN AN UNBOUNDED MEDIUM UNDER GRAVITY FIELD

Biot (1965) has shown that the effect of gravity is to produce hydrostatic initial stresses $S_{11} = S_{22} = -\rho g x_2$. The dynamical equations of motion for compressible medium are (Biot 1965)

$$\left.\begin{aligned} &\frac{\partial}{\partial x_1}(s_{11} + \rho g u_2) + \frac{\partial s_{12}}{\partial x_2} = \rho \frac{\partial^2 u_1}{\partial t^2} \\ &\frac{\partial s_{12}}{\partial x_1} - \rho e g + \frac{\partial}{\partial x_2}(s_{22} + \rho g u_2) = \rho \frac{\partial^2 u_2}{\partial t^2} \end{aligned}\right\}. \qquad \ldots(18)$$

e being the dilatation. Since the gravity produces hydrostatic initial stresses, so in this case the stress-strain relations are obtained from (6) putting $P = 0$. Using these stress-strain relations and solution (11) in eqns. (18) and then eliminating A and B as done in Case I, the square of longitudinal (C_L) and transverse (C_T) wave-velocities are obtained as :

$$\left.\begin{aligned} C_L^2 &= \frac{1}{2\rho}\left[(\lambda + 3\mu) + \sqrt{(\lambda + \mu)^2 + 4\left(\frac{\rho g}{k_n}\right)^2 p_1^{(n)^2}}\right] \\ C_T^2 &= \frac{1}{2\rho}\left[(\lambda + 3\mu) - \sqrt{(\lambda + \mu)^2 + 4\left(\frac{\rho g}{k_n}\right)^2 p_1^{(n)^2}}\right] \end{aligned}\right\}. \qquad \ldots(19)$$

Equation (19) in non-dimensional form may be written as

$$\left.\begin{aligned} C_L^2/(\mu/\rho) &= \frac{1}{2}\left[(\delta + 3) + \sqrt{(\delta + 1)^2 + 4G^2 P_1^{(n)^2}}\right] \\ C_T^2/(\mu/\rho) &= \frac{1}{2}\left[(\delta + 3) - \sqrt{(\delta + 1)^2 + 4G^2 P_1^{(n)^2}}\right] \end{aligned}\right\} \qquad \ldots(20)$$

where $G = \rho g/k_n\mu$ is the dimensionless gravity parameter as introduced by Biot (1965).

The presence of G in (20) shows the effect of gravity in the velocities of body waves. The presence of $p_1^{(n)}$ shows that the velocities depend on the direction of propagation.

Particular Cases

(i) If the waves propagate along x_1-direction then $p_1^{(n)} = 1$ and the wave velocities from (19) take the form,

$$C_L^2, C_T^2 = \frac{1}{2\rho}\left[(\lambda + 3\mu) \pm \sqrt{(\lambda + \mu)^2 + 4G^2}\right]. \qquad ...(21)$$

(ii) If the waves propagates along x_2-direction then $P_1^{(n)} = 0$ and the wave velocities from (19) take the form

$$C_L^2 = \sqrt{\frac{\lambda + 2\mu}{\rho}}, C_T^2 = \sqrt{\mu/\rho}\ . \qquad ...(22)$$

These show that gravity has no effect on the boby waves propagating along x_2-direction.

3. Numerical Calculations and Graphs

The numerical values of $C_L^2/(\mu/\rho)$ and $C_T^2/(\mu/\rho)$ have been computed from the equations (15) corresponding to $\delta = 1$, for different values of ζ_1 and ζ_2 at all directions of propagation from 0° to 90° with x_2-axis. These results have been presented by graphs in Figs. 2–4.

Figure 2 shows deviations of $C_L^2/(\mu/\rho)$ and $C_T^2/(\mu/\rho)$ for $\zeta_1 = 0.4$ and -0.4, and $\zeta_2 = 0.0$ from the initial stress free case i.e. $\zeta_1 = 0.0$, $\zeta_2 = 0.0$, at different directions of propagation from 0° to 90° with x_2-axis. It is observed that the deviations of $C_L^2/(\mu/\rho)$ is noticeable for the angle more than 30° where as for $C_T^2/(\mu/\rho)$ the deviations are prominent for all directions other than the range 40°–50°.

Figure 3 gives a comparison between the velocity curves for cases involving different combinations of biaxial initial stresses and the case when the medium is free of initial stresses. Longitudinal waves have been represented by curves 1–3 and the curves 4–6 represent the transverse waves. Deviation of values of velocities of

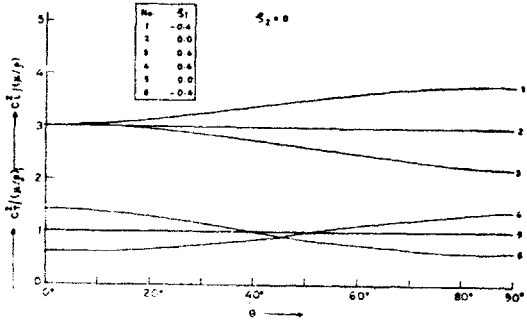

FIG. 2. Variations of $C_L^2/(\mu/\rho)$ and $C_T^2/(\mu/\rho)$ with respect to θ for different values of ζ_1 when $\zeta_2 = 0$.

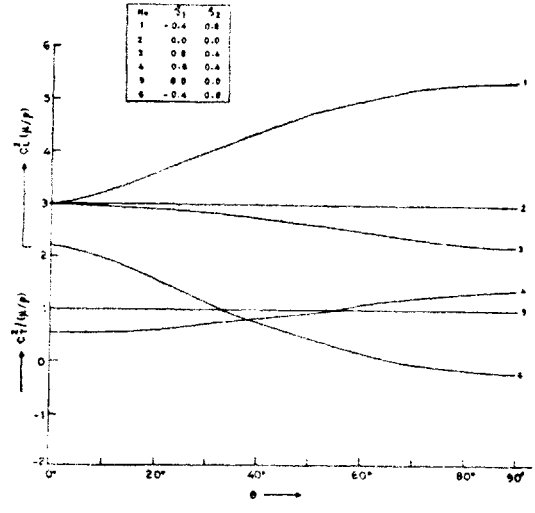

FIG. 3. Variations of $C_L^2/(\mu/\rho)$ and $C_T^2/(\mu/\rho)$ with respect to θ for different combinations of ζ_1 and ζ_2.

longitudinal waves for $\zeta_1 = -0.4$ and $\zeta_2 = 0.8$ (curve 1) from the initial-stress-free case (curve 2) is considerable when $\theta = 10°$ even, and gradually increases as θ increases. The deviation for the case when $\zeta_1 = 0.8$ and $\zeta_2 = 0.4$ (curve 3) is noticiable only for $\theta > 30°$. In case of transverse waves, the velocities for cases $\zeta_1 = -0.4$, $\zeta_2 = 0.8$ (curve 6) and $\zeta_1 = 0.8$, $\zeta_2 = 0.4$ (curves 4) show remarkable departure from initial-stress-free case (curve 5) for all range of values of θ, other than 35°–45°.

In Fig. 4 also comparison of velocity curves has been made varying ζ_1 for two fixed directions of propagations $\theta = 90°$ and $30°$ when $\zeta_2 = 0.8, 0.0$ and -0.8.

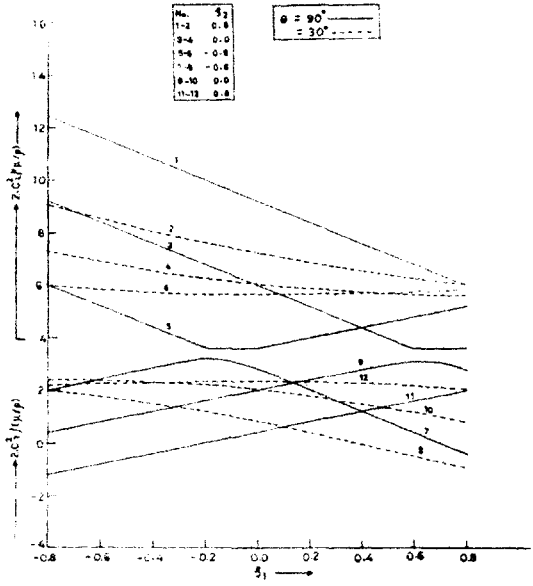

FIG. 4. Variations of $2C_L^2/(\mu/\rho)$ and $2C_T^2/(\mu/\rho)$ with respect to ζ_1 for fixed values of $\theta = 90°$ and $30°$ when $\zeta_2 = 0.8, 0.0, -0.8$.

The numerical values of $\bar{K}^2$ have been calculated for different values of $P/2\mu$ ($P = S_{22} - S_{11}$) and for different directions of propagation, θ. The results have been shown in Fig. 5. It is observed that values of $\bar{K}^2$ depend on both the direction of propagation and the initial stresses present in the medium. $\bar{K}^2$ is a constant, and is independent of direction of propagation when there is no initial stresses in the medium. When the direction of propagation $\theta = 0°$, values of $\bar{K}^2$ increase as $P/2\mu$ decreases from 1.6 to its negative values and the value tends to infinity when $P/2\mu \to -1.0$. The values of $\bar{K}^2$ for other directions such as $\theta = 30°, 45°, 60°$ and $90°$, however, are finite when $P/2\mu = -1.0$, and do not exist when $P/2\mu$ is less than -1.0. This is because of the fact that the material does not remain stable for the initial stresses $P/2\mu < -1.0$. i.e. $\zeta_1 - \zeta_2 > 1.0$. For all positive values of $P/2\mu$ i.e. for all $\zeta_2 > \zeta_1$, the values of $\bar{K}^2$ exist but become high at different values of $P/2\mu$ depending on the direction of propagation.

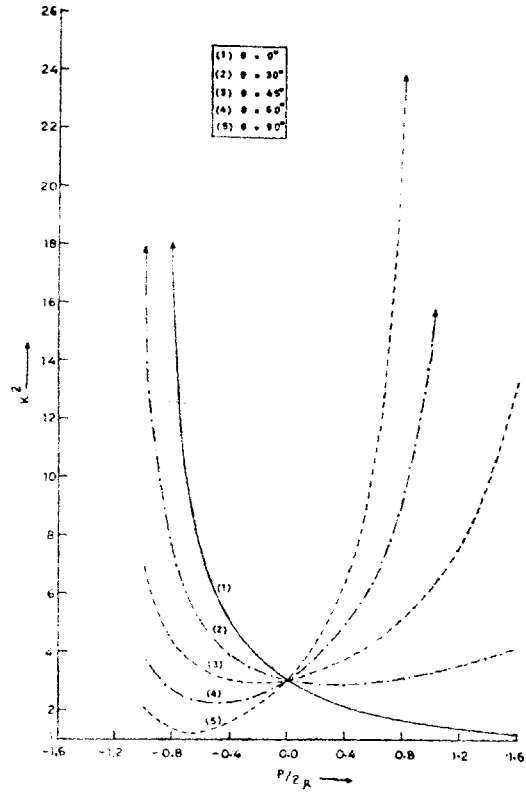

FIG. 5. Variations of $\bar{K}^2$ with respect to $P/2\mu$ for some fixed values of θ.

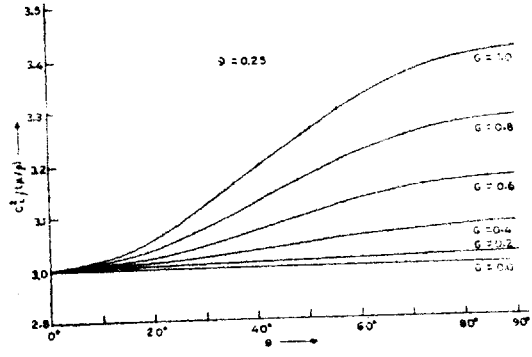

FIG. 6. Variations of $C_L^2/(\mu/\rho)$ with respect to θ for different values of G when $\nu = 0.25$.

Figs. 6-9 show the effect of gravity parameter $G\left(=\frac{\rho g}{k_n \mu}\right)$ on $C_L^2/(\mu/\rho)$ and $C_T^2/(\mu/\rho)$ at different directions of propagation.

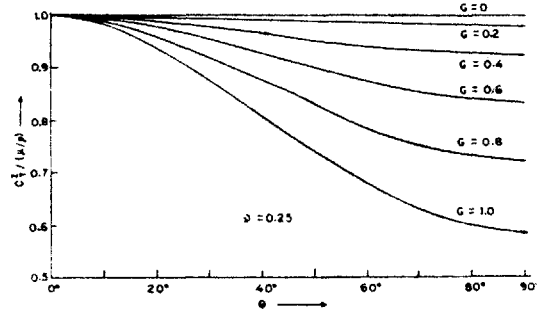

FIG. 7. Variations of $C_T^2/(\mu/\rho)$ with respect to θ for different values of G when $\nu = 0.25$.

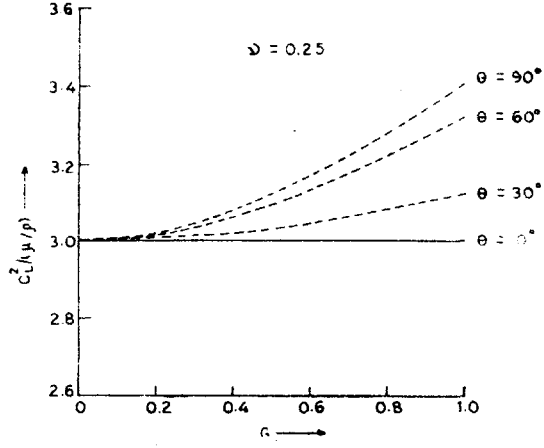

FIG. 8. Variations of $C_L^2/(\mu/\rho)$ with respect to G for different values of θ when $\nu = 0.25$.

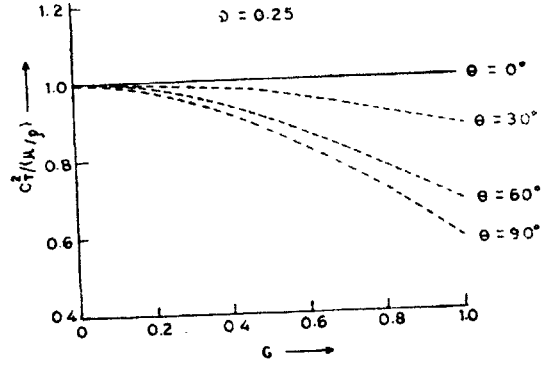

FIG. 9. Variations of $C_L^2/(\mu/\rho)$ with respect to G for different values of θ when $\nu = 0.25$.

In Fig. 6. the values of $C_L^2 / (\mu/\rho)$ for $G = .2(.2)\ 1$ have been compared with initial gravity-free case $Ge = 0.0$, corresponding to poisson ratio $\nu = 0.25$. It is observed that the variations increase as θ increases.

Fig. 7 gives the value of $C_T^2 /(\mu/\rho)$ when $\nu = 0.25$. For different valuse of G. It is observed that $C_T^2 /(\mu/\rho)$ decreases as θ increases.

Lastly, Figs. 8 and 9 show variations of $C_L^2 /(\mu/\rho)$ and $C_T^2 / (\mu/\rho)$ respectively for different values of θ due to variation in G from 0 to 1 when poison ratio $\nu = 0.25$. These show that the variations increase as θ increases.

4. Conclusion

Figures 2–9 show considerable effects of initial-stresses and gravity-field on the velocities of body waves. It is interesting to note that when initial stresses are hydrostatic and constant in magnitude, the velocities of longitudinal and the transverse waves are independent of the initial stresses and the direction of propagation also. But this is not the case under gravity field which produces hydrostatic initial stresses whose magnitude is a function of depth. However, the velocities are independent of gravity field only when the body wave propagates along the direction of g, the acceleration due to gravity. It is worth mentioning that Figs. 2-4. may also be utilised to estimate the magnitude of initial stresses present in a medium without destroying it but measuring velocities of *P* and *S* waves in the medium with the available experimental methods. Similarly Figs. 6–9 may be used to estimate the gravity field at a place inside the earth or on the surface of the earth by measuring the velocities of *P* and *S* waves at the place concerned.

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