Indian J. pure appl. Math., **15**(10) : 1124–1127, October 1984

APPLICATIONS OF SOME THEOREMS OF CARLITZ AND SRIVASTAVA

A. K. AGARWAL

Department of Mathematics, R. B. S. College, Agra 282002

(*Received* 2 *November* 1982; *after revision* 6 *September* 1983)

Recently Khanna and Pandey (1982) introduced the polynomials $P_n^{(\alpha,\beta)}(x, y; a, k)$ defined by (1.1) below and obtained various results for these polynomials. In the present paper we prove a theorem on generating functions and give a multivariable extension of this theorem by applying a result of Srivastava (1980). In the last section, following Carlitz (1977), we obtain a class of generating functions for these polynomials.

1. INTRODUCTION

Khanna and Pandey (1982) defined a sequence of polynomials by means of the generating relation

$$\sum_{n=0}^{\infty} P_n^{(\alpha,\beta)}(x, y; a, k)\frac{t^n}{n!} = \frac{2(t/2)^k (1-xt)^{-\alpha}}{(1-yt)^{-\beta}-a} \qquad \text{...(1.1)}$$

where a, α and β are real numbers, k is a non-negative integer, and (for convergence) $|t| < \min\{|x|^{-1}, |y|^{-1}\}$. They also give the following series representation of the polynomials $P_n^{(\alpha,\beta)}(x, y; a, k)$:

$$P_n^{(\alpha,\beta)}(x, y; a, k) = -\frac{(\alpha)_{n-k}\, x^{n-k}\, n!\, 2^{1-k}}{(n-k)!} \times \sum_{m=0}^{\infty} a^{-m-1}\, {}_2F_1\left[\begin{matrix}-n+k, \beta m; \\ 1-\alpha-n+k;\end{matrix}\ \frac{y}{x}\right], \quad k \leqslant n. \qquad \text{...(1.2)}$$

Replacing n by $n+k$ in (1.2), we get

$$\frac{P_{n+k}^{(\alpha,\beta)}(x, y; a, k)}{(n+k)!} = \frac{-(\alpha)_n\, x^n\, 2^{1-k}}{n!}$$

(*equation continued on p.* 1125)

$$\sum_{m=0}^{\infty} a^{-m-1} {}_2F_1 \left[\begin{matrix} -n, \beta m; \\ 1-\alpha-n; \end{matrix} \frac{y}{x} \right], \quad n \geqslant 0. \qquad \ldots(1.3)$$

Equation (1.3) leads us to the following generating function

$$\sum_{n=0}^{\infty} \frac{P_{n+k}^{(\alpha, \beta)} (x, y; a, k) t^n}{(n+k)!} = \frac{2^{1-k} (1-xt)^{-\alpha}}{(1-yt)^{-\beta} - a}. \qquad \ldots(1.4)$$

Making use of (1.4), we in this paper prove a theorem on generating functions, and then apply a result of Srivastava (1980) to give a multivariable extension of the main theorem. In the last section we obtain a class of generating functions making use of Carlitz's theorem (1977).

2. A Theorem on Generating Functions

Applying a well-known technique to (1.4) (i.e., replacing t by $t + u$ in (1.4) and then equating the coefficients of u^m), we readily obtain :

Theorem 2.1—

$$\sum_{n=0}^{\infty} \frac{\binom{n+m}{m} P_{n+k+m}^{(\alpha, \beta)} (x, y; a, k) t^n}{(n+m+k)!}$$

$$= \frac{(1-xt)^{-\alpha}}{(1-yt)^{-\beta}} \frac{P_{m+k}^{(\alpha, \beta)} \left(\frac{x}{1-xt}, \frac{y}{1-yt}; a(1-yt)^{\beta}, k \right)}{(m+k)!},$$

$$m, k = 0, 1, 2, \ldots \qquad \ldots(2.1)$$

Making use of (2.1) and Theorem 2 of Srivastava (1980), we give a multivariable generating function in the following form :

Theorem 2.2—If

$$S_{p,q}^{\mu} [x, y; a, k; u_1, \ldots, u_s, t]$$

$$= \sum_{n=0}^{\infty} a_{n, \mu} P_{nq+k}^{(\alpha, \beta)} (x, y; a, k) G_{\mu+np} (u_1, \ldots, u_s) \frac{t^n}{(nq+k)!},$$

$$a_{n,\mu} \neq 0 \qquad \ldots(2.2)$$

then

$$\sum_{n=0}^{\infty} P_{n+k}^{(\alpha,\beta)}(x, y; a, k)\ M_{n,q}^{p,\mu}(u_1, u_2, \ldots, u_s; v)\frac{t^n}{(n+k)!}$$

$$= \frac{(1-xt)^{-\alpha}}{(1-yt)^{-\beta}}\ S_{p,q}^{\mu}\left[\frac{x}{1-xt}, \frac{y}{1-yt}; a(1-yt)^{\beta}, k; u_1, \ldots, u_s; vt^q\right] \qquad \ldots(2.3)$$

where

$$M_{n,q}^{p,\mu}(u_1, \ldots, u_s; v) = \sum_{m=0}^{[n/q]} a_{m,\mu}\binom{n}{mq} g_{\mu+pm}(u_1, \ldots, u_s)\, v^m. \qquad \ldots(2.4)$$

Remark : An obvious two-variable case of (2.3) would follow if we apply (2.1) and Theorem 2 of Agarwal and Manocha (1980). The details are naturally omitted.

3. A Class of Generating Functions

By applying Carlitz's theorem (1977), below we deduce a new result for the polynomials $P_n^{(\alpha,\beta)}(x, y; a, k)$:

$$\sum_{n=0}^{\infty} P_n^{(\alpha+bm,\beta)}(x, y; a, k)\ \frac{t^n}{n!}$$

$$= \frac{2\left(\frac{v}{2}\right)^k (1-xv)^{-\alpha}}{\{(1-yv)^{-\beta}-a\}\{1-xbv(1-xv)^{-1}\}} \qquad \ldots(3.1)$$

where b is an arbitrary constant, and v is given by

$$v = t(1-xb)^{-b}. \qquad \ldots(3.2)$$

Note: It is worthwhile to remark here that (3.1) can be regarded as a class of generating functions for the polynomials $P_n^{(\alpha,\beta)}(x, y; a, k)$ in view of the fact that by assigning special values to the arbitrary constant b, it is easy to obtain from it a large variety of generating functions. For instance, for $b = 0$, it reduces to (1.1).

Acknowledgement

The author is thankful to the referee for his valuable comments leading to an improvement of the paper.

References

Agarwal, A, K., and Manocha, H. L. (1980). On further extension of bilateral generating functions for certain special functions. *Simon Stevin*, **54**, 179–91.

Carlitz, L. (1977). A class of generating functions. *SIAM J. Math. Anal.*, **8**, 518–32.

Khanna, I. K., and Pandey, Priti (1982). Basic properties of a new type of polynomial set. *Indian J. pure appl. Math.*, **13** (9), 1075–84.

Srivastava, H. M. (1980). Some bilateral generating functions for a certain class of specia. functions-I and II. *Nederl. Akad. Wetensch. Proc. Ser. A* **83** = *Indag. Math.*, **42**, 221–46.