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CILIA TRANSPORT OF BIO-FLUID WITH VARIABLE VISCOSITY

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A circular tube, with inner wall ciliated in the form of an axisymmetric metachronal wave, has been taken to be the flow model. The fluid is assumed to be Newtonian, incompressible with viscosity varying across the section of the tube. The analysis is made under the long wavelength approximation. The effects of parameters, involved in the analysis, on the flow flux and friction force at the tube wall have been discussed graphically.

The analysis is applied to understand theoretically the problem of spermatic fluid (semen) transport through vas deferens. The calculated value of flow rate has been compared with the observed value of flow rate in vas deferens of rhesus monkeys.

1. INTRODUCTION

Cilia, the hair like motile appendages, are present in the respiratory, digestive, male and female reproductive tracts and nervous systems of almost all groups of the animal kingdom. The mobility of cilia plays an important role in physiological processes like locomotion, alimentation, circulation, respiration and reproduction. An approximate size, structure and the patterns of motion of cilium are shown in Fig. 1 (Lardner and Shack 1972).

The cilia beatings, being well coordinated, generate a metachronal wave. The envelope of cilia tips, forming the metachronal wave, acts as an extensible wall with a tendency of forward motion always in the same direction.

A recent work on the problem of mucus transport in the trachea, using a mechanical simulation of the cilia in the trachea, has been carried out by Miller (1966, 1967, 1969). Barton and Raynor (1967) have analysed the mucus transport in the trachea where the metachronal wave motion has not been taken into account. Blake (1971) has described a spherical envelope model for studying the swimming of protozoan opalina. In an earlier attempt, Lardner and Shack (1972), studied the effects of cilia motion on spermatic fluid transport in ductus efferentes of male reproductive tract where a two-dimensional channel of infinite length was taken as

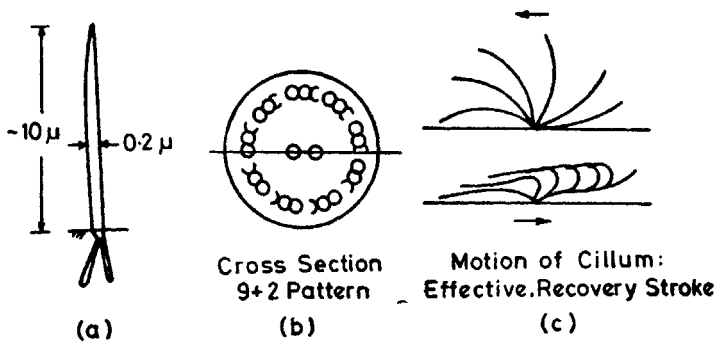

FIG. 1. Approximate size and structure of cilia. (a) Typical dimension, (b) Cross section, (c) Stroking of cilia, (Lardner and shack 1972).

the flow model. In all the cases mentioned above, the viscosity of the concerned fluid has been taken to be constant.

Experiments carried out by Hynes (1960), Bugliarello and Sevilla (1970) and Goldsmith *et al.* (1975), have shown that the viscosity of fluid varies across the thickness of the physiological organs like intestine, the ductus efferentes of the male reproductive tract, and the cervical canal.

Shukla *et al.* (1980) have studied the effects of peripheral layer viscosity on the peristaltic transport of a bio-fluid through a pipe and a channel under the long wavelength approximation used by Barton and Raynor (1968) and applied the analysis to the problems of chyme transport through small intestine and spermatic fluid transport through ductus efferentes of male reproductive tract. They have shown that the flow flux increases and friction force decreases as the thickness of the peripheral layer increases or its fluid viscosity decreases.

Particularly in the physiological process of spermatic fluid transport through vas deferens, the viscosity of the fluid varies across the section of the duct and the walls of the duct (vas deferens) are ciliated. These facts have motivated us to study the effects of cilia motion and viscosity variation on the behaviour of semen transport through vas deferens.

Thus, the aim of the present paper is to discuss the behaviour of fluid transport in a tube, with the consideration of metachronal wave motion due to cilia and variation of viscosity of the fluid under the long wave-length approximation and to apply the analysis to the problem of spermatic fluid (semen) transport through vas deferens. We find here that the flow flux and friction force are linear with pressure drop and eccentricity of the elliptic paths of cilia motion.

2. Formulation of the Problem

We consider the flow of an incompressible, Newtonian fluid with variable viscosity in a uniform tube whose inner wall is ciliated in the form of an axisymmetric metachronal wave moving to the right with speed c (Fig. 2). The envelope of the cilia tips forming the metachronal wave acts as an extensible wall. The geometry of

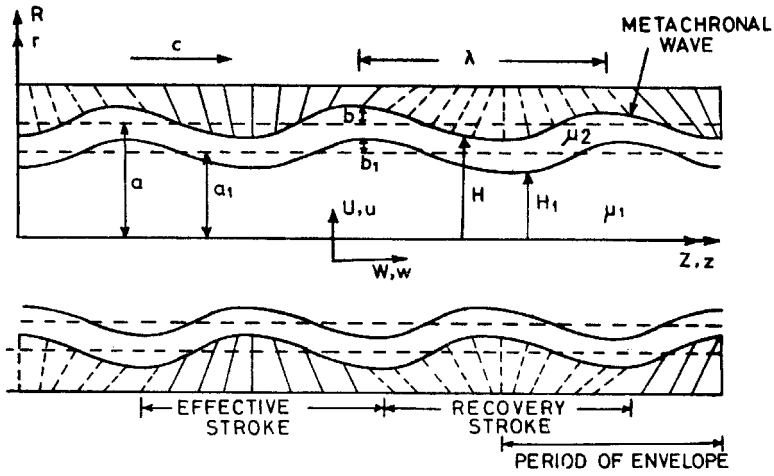

FIG. 2. Cilia induced Peristaltic flow in a tube with peripheral layer adjacent to the metachronal wave.

the envelope of cilia tips and the flow of fluid are described with the help of two co-ordinate systems; one, fixed in the space, is (R, Z) and another is (r, z) moving to the right with a speed c relative to the fixed one. The two co-ordinate systems are connected with the relation

$$R = r, Z = z + ct. \qquad \ldots(1)$$

The velocity components of the fluid in the fixed and moving systems are (U, W) and (u, w) respectively where U, u are in radial direction and W, w in axial direction. The components of velocity of fluid are connected with the relation

$$U = u, W = w + c. \qquad \ldots(2)$$

The envelope of the cilia tips is assumed to be written as (Fig. 2)

$$H = a + a\epsilon \cos\left(\frac{2\pi}{\lambda}\right)(Z - ct) \qquad \ldots(3)$$

or,

$$h = a + a\epsilon \cos\left(\frac{2\pi}{\lambda}\right)(z) \qquad \ldots(4)$$

where equations (3) and (4) are in the fixed and the moving systems respectively, a is the average radius of the ciliated tube, ϵ is the non-dimensional measure with respect to a of the cilia length, λ is wavelength and c is wave-speed of the metachronal wave. Thus eqn. (3) or (4) gives the geometry of the extensible tube wall. Following Sleigh (1968), the cilia tips are assumed to move in elliptic paths so that the horizontal position of a cilia tip is given as :

$$Z_h = Z_0 + \epsilon\, e\, a \sin\left(\frac{2\pi}{\lambda}\right)(Z - ct) \qquad \ldots(5)$$

where Z_0 is a reference position of the cilia and e is a measure of the eccentricity of the elliptic path.

The appropriate equations of motion, in the moving (r, z) system for unsteady and axisymmetric flow of a Newtonian, incompressible fluid with variable viscosity, simplified under the long wavelength approximation (in non-dimensional form with dashes dropped) are :

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\mu\frac{-\partial w}{\partial r}\right) = \frac{\partial p}{\partial z} \qquad \ldots(6)$$

$$\frac{\partial p}{\partial r} = 0 \qquad \ldots(7)$$

where,

$$\left.\begin{aligned} r' &= \frac{r}{a}, R' = \frac{R}{a}, z' = \frac{z}{\lambda}, Z' = \frac{Z}{\lambda} \\ t' &= \frac{ct}{\lambda}, \bar{\mu} = \frac{\mu(r)}{\mu_1}, p' = \frac{pa^2}{\lambda c \mu_1} \\ u' &= \frac{u}{\beta c}, U' = \frac{\lambda}{ac} U = \frac{U}{\beta c}, \beta = \frac{a}{\lambda} \\ w' &= \frac{w}{c}, W' = \frac{W}{c} \end{aligned}\right\} \qquad \ldots(8)$$

μ_1 being the viscosity of the central layer, $\mu(r)$ is a viscosity function (i.e. variable viscosity), μ_2 is the viscosity of the peripheral layer.

The non-dimensional boundary conditions in the moving (r, z) system are :

$$\left.\begin{aligned} u &= 0 \\ \partial w/\partial\, r &= 0 \end{aligned}\right\} \text{at } r = 0 \qquad \ldots(9)$$

$$\left.\begin{aligned} u(h) &= (2\pi\epsilon)\sin(2\pi z) + e\beta(2\pi\epsilon)^2 \sin(2\pi z)\cos(2\pi z) \\ w(h) &= -1 - e\beta\, 2\pi\epsilon \cos(2\pi z) \end{aligned}\right\} \text{at } r = 1+\epsilon\cos(2\pi z) = h \qquad \ldots(10)$$

3. Analysis

Equation (7) shows that p is a function of z only. Since $z = Z-ct$, therefore, $p = p(Z, t) = p(z)$.

Solution of equation (6) under the boundary conditions (9) and (10) is

$$w = w(h) - \frac{1}{2}\frac{dp}{dz}\int_r^h \frac{r}{\mu(r)}\, dr. \qquad ...(11)$$

The pressure and flow appear to be stationary to an observer moving at velocity c in the axial direction and therefore, the flow rate q measured in the moving coordinate system is independent of time and position along the axis of the tube (as $z = Z-ct$).

The dimensionless flux q' ($=q/\pi a^2 c$, q being the flux in moving (r, z) system) is given after dropping dashes as :

$$q = \int_0^h 2rw\, dr$$

$$= w(h)\, h^2 - \frac{1}{2}\frac{dp}{dz}\int_0^h \frac{r^3}{\mu(r)}\, dr. \qquad ...(12)$$

The pressure drop across one wavelength is :

$$\Delta p = -\int_0^1 \left(\frac{dp}{dz}\right) dz. \qquad ...(13)$$

The flux in the fixed system (R, Z) in dimensionless form is :

$$Q = 2\int_0^h WR\, dR$$

$$= q + h^2. \qquad ...(14)$$

The time average flux $\bar{Q}$, for a complete period $T = \lambda/c$, is obtained as :

$$\bar{Q} = \frac{1}{T}\int_0^T Q\, dt = \int_0^1 Q\, dt' = q + 1 + \frac{1}{2}\epsilon^2 \qquad ...(15)$$

where

$$t = Tt'.$$

The frictional force

$$F = -\int_0^1 2h.\bar{\mu}_2 \left(\frac{\partial w}{\partial r}\right)_{r=h} dz \text{ (c.f. Schlichting 1978)}$$

and the value of w from eqn. (11) give the dimensionless friction force F' ($= F/\pi c \lambda \mu_1$, F being the friction force at the wall in the stationary system (R, Z)) across one wavelength after dropping the dashes in the form

$$F = -\int_0^1 h^2 \left(\frac{dp_h}{dz}\right) dz. \qquad \ldots(16)$$

4. Effect of Peripheral Layer Viscosity

We consider the viscosity variation in the dimensionless form as under (Shukla *et al.* 1980)

$$\left.\begin{aligned} \bar{\mu} &= \frac{\mu(r)}{\mu_1} = 1, \text{ for } 0 \leqslant r \leqslant h_1 \\ \bar{\mu} &= \bar{\mu}_2 = \frac{\mu_2}{\mu_1}, \text{ for } h_1 \leqslant r \leqslant h \end{aligned}\right\} \qquad \ldots(17)$$

since we assume the variation of viscosity function $\mu(r)$ as :

$$\left.\begin{aligned} \mu(r) &= \mu_1, \quad \text{for } 0 \leqslant r \leqslant h_1 \\ \mu(r) &= \mu_2, \quad \text{for } h_1 \leqslant r \leqslant h \end{aligned}\right\} \qquad \ldots(18)$$

with $\mu_2 < \mu_1$ i.e. $\bar{\mu}_2 < 1$.

The geometry of the interface in dimensionless form is given by

$$\begin{aligned} h_1 &= \frac{a_1}{a} + \frac{b_1}{a}\cos(2\pi z) \\ &= \alpha + \epsilon_1 \cos(2\pi z) \end{aligned} \qquad \ldots(19)$$

where

$$\alpha = (a_1/a),\ \epsilon_1 = (b_1/a).$$

Obviously the geometry of the ciliated extensible wall in moving (r, z) system given by equation (4) transform in dimensionless from as (dashes dropped) :

$$h = 1 + \epsilon \cos(2\pi z). \qquad \ldots(20)$$

Following Shukla *et al.* (1980), the relations for h_1 and ϵ_1 are as under :

$$h_1 = \alpha h \text{ and } \epsilon_1 = \alpha\epsilon,\ \alpha < 1. \qquad \ldots(21)$$

Introducing the viscosity variation defined by eqn. (17) and the relation for h_1 from eqn. (21), eqn. (12) yields

$$-\frac{dp}{dz} = \frac{1}{g}\left[\frac{g}{h^4} + \frac{1}{h^2} + \frac{2\pi\epsilon\, e\beta \cos(2\pi z)}{h^2}\right] \qquad ...(22)$$

where

$$g = \frac{1-(1-\bar{\mu}_2)\,\alpha^4}{8\,\bar{\mu}_2}. \qquad ...(23)$$

Substituting (dp/dz) from eqn. (22) into eqn. (13), integrating it, and using the relation $q = \bar{Q} - (1 + \epsilon^2/2)$, we get an expression for the time average dimensionless flux as under (dashes dropped) :

$$\bar{Q} = \frac{(2+\epsilon^2)}{2} - \frac{2(1-\epsilon^2)^2}{(2+3\epsilon^2)} + \frac{4\pi\epsilon^2 e\beta(1-\epsilon^2)^2}{(2+3\epsilon^2)} + \frac{\Delta p\, g\, 2(1-\epsilon^2)^{7/2}}{(2+3\epsilon^2)}. \qquad ...(24)$$

Similarly, the dimensionless friction force at the wall is (dash dropped).

$$F = \frac{1}{g}\left[1 - \frac{2(1-\epsilon^2)^{1/2}}{(2+3\epsilon^2)} + \frac{4\pi\epsilon^2 e\beta(1-\epsilon^2)^{1/2}}{(2+3\epsilon^2)}\right] + \frac{\Delta p\, 2(1-\epsilon^2)^2}{(2+3\epsilon^2)}. \qquad ...(25)$$

For $e = 0$, our problem reduces to the problem of Shukla *et al.* (1980) and and the values of $\bar{Q}$ and F as shown by eqns. (24) and (25) respectively are exactly eqns. (36) and (37) respectively of Shukla *et al.*

5. Discussion

To study the effects of peripheral layer viscosity ($\bar{\mu}_2$), eccentricity of cilia paths (e), pressure drop (Δp), non-dimensional measure of the cilia length (ϵ), on the flow flux ($\bar{Q}$) and friction force (F), graphs have been plotted (Figs. 3–5).

Figure 3 (a) reveals the fact that the flow flux $\bar{Q}$ is linear with e, the eccentricity of cilia paths, and Δp, the pressure drop, separately for fixed values of $\bar{\mu}_2$ and ϵ. Figure 3 (b) shows that the wall friction force varies with e and Δp in a fashion similar to that of flow flux.

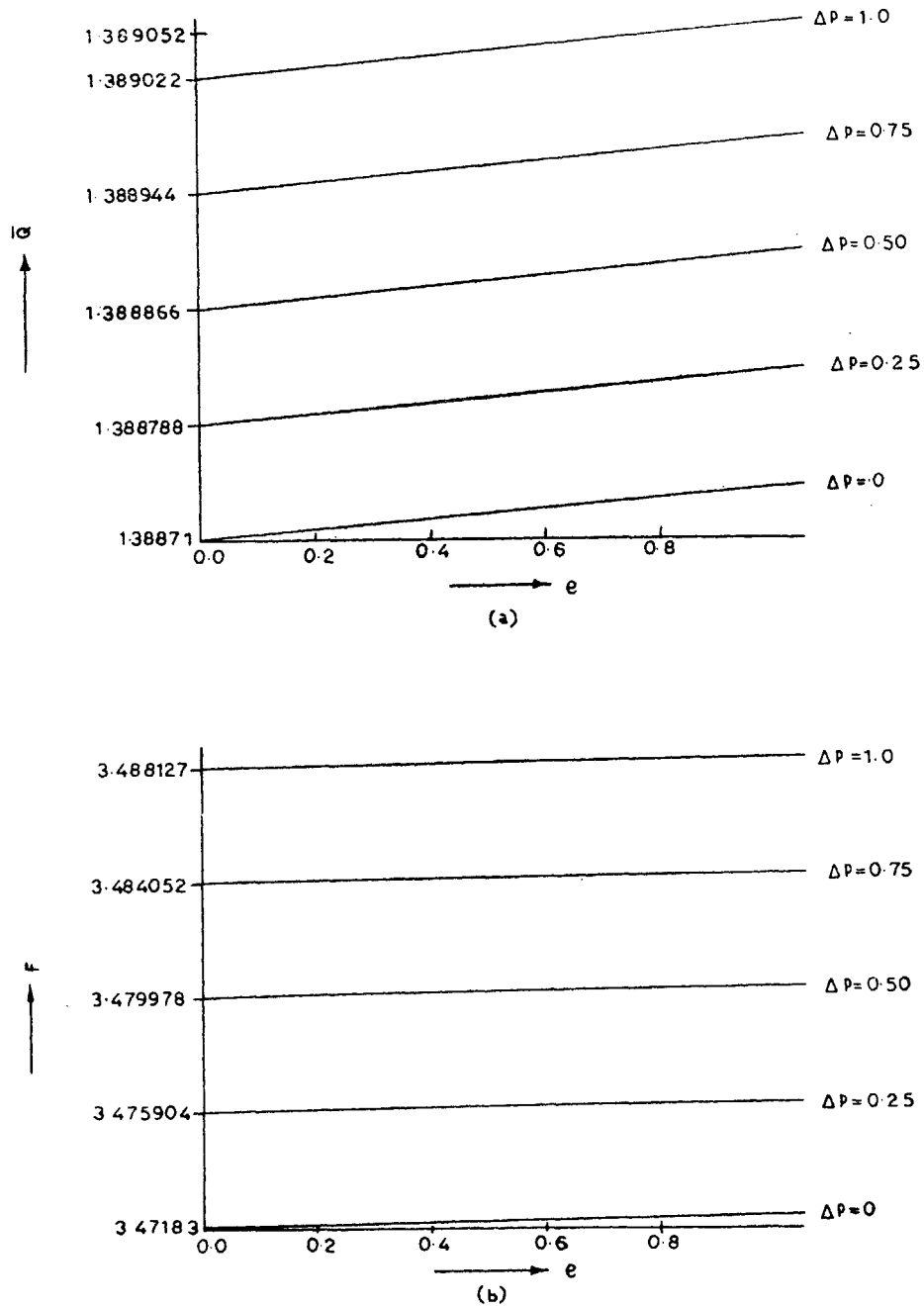

FIG. 3. Variations of $\bar{Q}$ and F with e for different Δp taking $\epsilon=0.9$, $\bar{\mu}_2=0.25$, $\beta=0.0006$, $\alpha=0.92$.

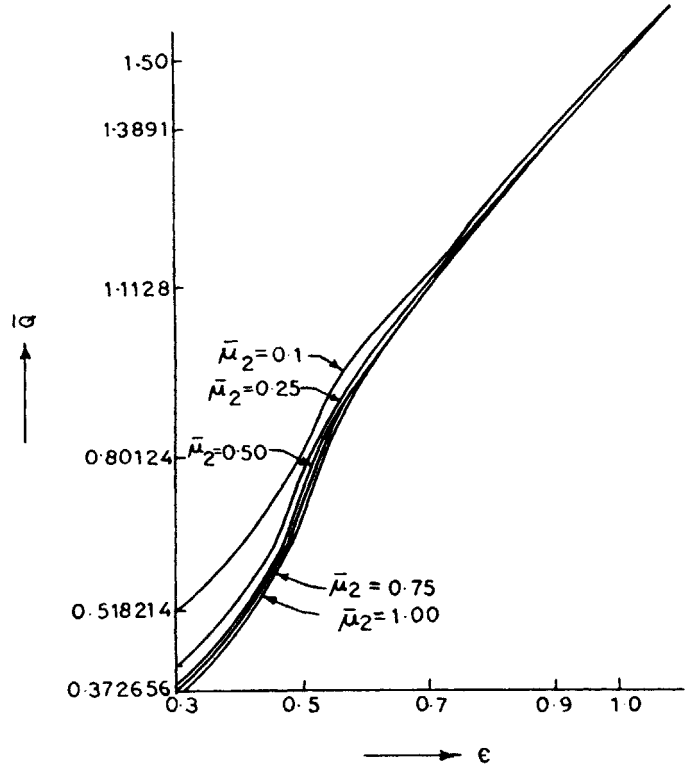

FIG. 4. Variation of $\bar{Q}$ with ϵ for different $\bar{\mu}_2$ with $\beta = 0.0006$, $\alpha = 0.92$, $\Delta p = 0.72044$.

Figure (4) shows that $\bar{Q}$ increases with increase in ϵ, the non-dimensional measure of cilia length. $\bar{Q}$ has higher values for lower values of $\bar{\mu}_2$.

From Figure (5), we learn that the flow flux $\bar{Q}$ decreases with an increase in the peripheral layer viscosity $\bar{\mu}_2$. $\bar{Q}$ is maximum for the least value of $\bar{\mu}_2$ ($\bar{\mu}_2$ is assumed to vary from 0.1 to 1 since the viscosity of semen of rhesus monkey is of the order 4 c. P., as suggested by Gupta and Sheshadri (1976), which is taken as the viscosity of fluid in the core region and the peripheral layer viscosity is assumed to be 1 c. P. (of water) giving the value of $\bar{\mu}_2$ as 0.25) and decreases rapidly as $\bar{\mu}_2$ increases from 0.1 to 0.50. The decrease in the value of $\bar{Q}$ is negligible for increase in $\bar{\mu}_2$ from 0.50 to 1.0. The increase in e lessens the decrease in $\bar{Q}$ with increasing $\bar{\mu}_2$ as the curves flatten.

6. APPLICATION

Since cilia are present in the vas deferens and viscosity of the spermatic fluid in core region is different from that in the peripheral layer, our present analysis, taking

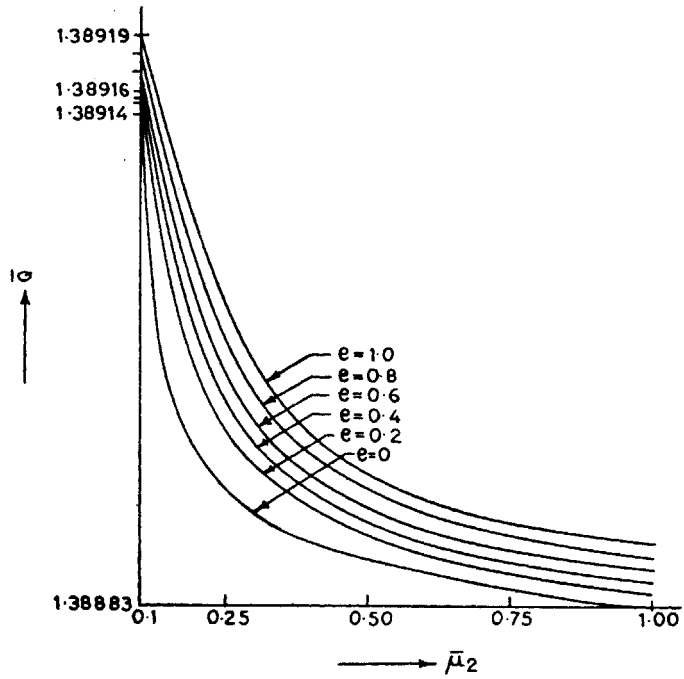

FIG. 5. Variation of Q with μ_2 for different values of e, with $\epsilon=0.9$, $\beta=0.0006$, $\alpha=0.92$, $\Delta p=0.72044$.

uniform cylindrical tube with ciliated inner wall in form of metachronal wave, may be applied to the physiological problem of fluid transport in vas deferens. According to Gupta and Sheshadri (1976), the parameters for flow in the vas deferens of a rhesus monkey are as under :

$$a = 0.012 \text{ cm}, \lambda = 20 \text{ cm}, c = 10 \text{ cms/sec}.$$

$$\mu_1 = 4 \text{ c. P.}, \quad \alpha = 0.92, \quad \beta = \frac{a}{\lambda} = 0.0006.$$

Assuming the value of μ_2 as 1 c. P. (of water)

$$\bar{\mu}_2 = 0.25.$$

Considering the value of b as 0.0108 cm (Srivastava *et al.* 1982), the value of $\epsilon = 0.9$.

Taking the above mentioned values of the various parameters, the pressure drop of 30 mm Hg corresponds to the dimensionless pressure drop

$$\Delta p' = \frac{\Delta p \times a^2}{\lambda c \mu_1}$$

$$= 0.72044.$$

i.e. $\Delta p = 0.72044$ (after dropping the dash).

Assuming e to be 0.9 and using the values of parameters assigned above, the average flow rate is found to be

$$\bar{Q} = .00628 \text{ ml/sec.} \qquad \text{(dimensional form)}$$

where,

$$[\bar{Q}]' = 1.38898 \qquad \text{(non-dimensional form)}$$

The experimental value of average flow rate observed by Guha *et al.* (1975) in the vas deferens of rhesus monkeys for 30 mm Hg pressure drop is 0.02 ml/sec.

The calculated value of flow rate in the present analysis differs from the observed value by 68.60%.

7. Conclusion

Considering a circular tube model, with ciliated inner wall (acting as an extensible wall) in the form of a metachronal wave, the effects of peripheral layer viscosity, eccentricity of paths of cilia motion, pressure drop and amplitude of the metachronal wave (cilia length in non-dimensional form) on the flow flux and wall friction force have been discussed graphically. The flow flux and friction force at the wall increase with an increase in pressure drop or eccentricity. The flow flux increases with an increase in the cilia length or decrease in the peripheral layer viscosity. Further it has been concluded that the decrease in flow flux due to an increase in peripheral layer viscosity is reduced by increasing the eccentricity of the cilia paths.

Our analysis has been applied to the problem of semen transport through vas deferens and the calculated value of flow rate has been compared with the observed value. The calculated value differs from the observed value by 68.60%. This difference is expected to be caused by the use of assumed values of some parameters, uniform shape of the tube (in practical, vas deferens is non-uniform diverging tube) and negligence of the complex wave motion (as suggested by Wilson *et al.* (1977), the wave motion in vas deferens is complex rather than peristaltic) in the vas deferens.

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