Indian J. pure appl. Math., 20 (2) : 165-180, February 1989

STRESSES IN PRE-STRESSED DRY SANDY SOIL DUE TO NORMAL MOVING LOAD LEADING TO INSTABILITY AND FRACTURE

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(*Received* 25 *August* 1987; *after revision* 4 *July* 1988)

The paper gives a complete study of stresses and displacements induced in a pre-stressed half-space made of dry sandy soil due to concentrated line load moving at a constant speed along the surface. The subsonic, transonic and supersonic cases have been considered. It is seen that sandy parameters and the pre-stress parameters play important roles in the development of the stresses and displacements in the medium. The stress developed at a certain depth have been calculated numerically for the subsonic case due to increasing velocity of the moving load for different values of pre-stress parameters. The velocities of the moving load creating instability in the medium leading to fracture have been calculated for different values of pre-stressing in sandy soil and elastic half-space both and it is observed that sandy soil is less stable to moving loads than the elastic one. Further it is inferred that the pre-stressed medium gets fractured at a less velocity of the moving load in comparison to the pre-stress free case.

INTRODUCTION

To analyse the stresses developed in a body due to a moving source causing fracture is an interesting problem of mechanics having its application towards the stability of a medium. Sneddon[1,2] has developed an analysis which gives us the displacement on the surface of a semi-infinite medium for different kinds of source of disturbance applied on the surface. Of these, the particular kind of source which is acting parallel to the surface and moving with a certain uniform velocity is of special interest to seismologists. In the above paper it has been shown that when the velocity of moving load exceeds the velocity of shear waves in the medium, displacement becomes infinitely large along two lines. The steady state solution of the problem of moving normal load over an elastic half space were given by Cole and Huth[3] and Craggs[4], who derived a relatively simple closed form solution, exhibiting a resonance effect at a critical load velocity, which in this case equals to the velocity of Rayleigh waves. The problem considered by Cole and Huth[3] has been discussed previously by Sneddon[1] by a somewhat different method. However, Sneddon[1] treated only the subsonic case. Ghosh[5] has explained the principle behind the phenomenon of propagation of cracking across the length on the simplified assumption that the crust is lying over a medium (rigid foundation) with shearless contact and a normal point source is moving with a certain velocity. Stresses developed in a transversely isotropic elastic media (under different conditions) due to normal moving load over a rough surface have been discussed by Mukhopadhyay[6], Mukherjee[7] and Dey, *et al.*[8]. Freund[9] discussed wave motion in an elastic half-space subjected to non-uniformly moving surface load.

In this paper attempt has been made to study the stresses and the displacements devel oped leading to fracture of more earthy material, say, dry sandy material under initial stresses. The crust of the earth is not exactly elastic but may be estimated as sandy material whose definition was given by Weiskopf[10]. Further, the normal initial stresses developed in the earth due to many physical causes such as variation of gravity, temperature, slow process of creep etc. deserve its consideration in development of stresses and displacements due to a moving load on the surface.

The relation $E/\mu = 2(1 + \sigma)$ for isotropic elastic solids does not hold good for real earthy materials viz. sand, soil etc. Weiskopf[10] investigated that due to slipping of granules on each other the resistance of shear is much less than that in a solid and the resultant shearing deflection is much greater. For these materials

$$E/\mu > 2(1 + \sigma).$$

So the relation $E/\mu = 2\eta(1 + \sigma)$, where $\eta > 1$ may be considered, where $\eta = 1$ corresponds to elastic case. This relation shows that $\eta\mu$ is the rigidity of the corresponding elastic material when μ is the rigidity of sandy material and hence if μ is considered to be the rigidity of the elastic material then the rigidity of the corresponding sandy material will be μ/η. Also the generalised Lame's constant may be defined as

$$\lambda = E\sigma/\eta(1 + \sigma)(1 - 2\sigma).$$

With the above assumptions and taking into account of the principle of incremental deformation given by Biot[11]. The stresses and displacements produced in a dry sandy half-space under normal initial stresses due to a normal moving load on the rough surface of the half-space have been derived in this paper. The subsonic, transonic and supersonic all the three cases have been discussed. The conditions for instability, due to high stress concentration and the lines of generation of cracks have been obtained. The results in traasonic and supersonic cases have been obtained in terms of Heisenberg delta function and are shown to coincide with the classical result ($\eta = 1$, $S_{11} = S_{33} = 0$) obtained in Cole and Huth[3] in subsonic case and with Ghosh[5] (when $h \to \infty$) in transonic and supersonic cases. In subsonic case the numerical results for stresses developed have been calculated and represented by graphs for different values of sandy parameters η and the initial stress parameters I_1 and I_2.

GOVERNING EQUATIONS AND RELEVANT SOLUTIONS

We consider a homogenecus, isotropic and linearly elastic sandy half space under initial stresses S_{11} and S_{33} along x and z directions respectively. The half space is subjected to a normal load F, independent of y and moving with a constant velocity v in the positive x-direction. The moving load induces a state of plane strain in the half space whereby the y component of displacement vanishes and the remaining displacements and stresses are functions of x, z and t only. The surface of the half space is assumed to be rough.

In the absence of any body force the dynamical equations of motion under initial stress $P = S_{33} - S_{11}$ for two dimensional problems may be written as[11]

$$\frac{\partial s_{11}}{\partial x} + \frac{\partial s_{31}}{\partial z} + P\frac{\partial \omega_y}{\partial z} = \rho\frac{\partial^2 u}{\partial t^2} \qquad \text{(a)}$$

$$\frac{\partial s_{31}}{\partial x} + \frac{\partial s_{33}}{\partial z} + P\frac{\partial \omega_y}{\partial x} = \rho\frac{\partial^2 w}{\partial t^2}. \qquad \text{(b)}$$

The stress-strain relations for sandy medium under initial stresses may be taken as

$$\left.\begin{aligned} s_{11} &= \left(\frac{\lambda}{\eta} + \frac{2\mu}{\eta} + P\right)\frac{\partial u}{\partial x} + \left(\frac{\lambda}{\eta} + P\right)\frac{\partial w}{\partial z} \\ s_{33} &= \left(\frac{\lambda}{\eta} + \frac{2\mu}{\eta}\right)\frac{\partial w}{\partial z} + \frac{\lambda}{\eta}\frac{\partial u}{\partial x} \\ s_{31} &= s_{13} = \frac{\mu}{\eta}\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right) \end{aligned}\right\} \qquad \ldots(1)$$

where s_{ij} are incremental stress components, ω_{ij} the rotational components, e_{ij} the strain components and η is the sandy parameter λ, μ are Lame's constants for the elastic material.

Now from (a) and (b) and (1) the equations of motion for sandy medium under the considered initial stresses can be written in terms of displacement components as

$$(\lambda + 2\mu + \eta P)\frac{\partial^2 u}{\partial x^2} + \left(\mu + \frac{\eta P}{2}\right)\frac{\partial^2 u}{\partial z^2} + \left(\lambda + \mu + \frac{\eta P}{2}\right) \times \frac{\partial^2 w}{\partial x\,\partial z} = \eta\rho\frac{\partial^2 u}{\partial t^2} \qquad \ldots(2)$$

$$\left(\mu - \frac{\eta P}{2}\right)\frac{\partial^2 w}{\partial x^2} + (\lambda + 2\mu)\frac{\partial^2 w}{\partial z^2} + \left(\lambda + \mu + \frac{\eta p}{2}\right) \frac{\partial^2 u}{\partial x\,\partial z} = \eta\rho\frac{\partial^2 w}{\partial t^2}. \qquad \ldots(3)$$

The boundary conditions at the free surface ($z = 0$) prescribe the normal stress to be delta function and the tangential stress to be balanced by the frictional force

i. e $\quad \Delta f_z = -F\delta(x - vt)$

$$\Delta f_x = -RF\delta(x - vt) \qquad \ldots(4)$$

where, Δf_z, Δf_x being the incremental normal and shear boundary forces for unit initial area given by[11]

$$\left.\begin{aligned} \Delta f_z &= \frac{(\lambda + 2\mu)}{\eta} \frac{\partial w}{\partial z} + \left(\frac{\lambda}{\eta} + S_{33} \right) \frac{\partial u}{\partial x} \\ \Delta f_x &= \left(\frac{\mu}{\eta} + \frac{P}{2} \right) \frac{\partial u}{\partial z} + \left(\frac{\mu}{\eta} - \frac{S_{11}}{2} - \frac{S_{33}}{2} \right) \frac{\partial w}{\partial x} . \end{aligned}\right\} \qquad ...(5)$$

Further, R being the coefficient of static friction at the surface and $\delta(x)$ is the Dirac delta function of argument x and is defined by

$$\delta(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} e^{ikx}\, dk.$$

We take the steady state solutions of eqns. (2) and (3) in the form

$$\left.\begin{aligned} u &= \int_0^\infty [A \cos k(x - vt) + B \sin k(x - vt)]\, e^{-kqz}\, dk \\ w &= \int_0^\infty [C \cos k(x - vt) + D \sin k(x - vt)]\, e^{-kqz}\, dk. \end{aligned}\right\} \qquad ...(6)$$

Putting the expressions (6) for u and w in eqns (2) and (3) we get four equations connecting A, D and B, C which are consistent only if

$$q^4 + \left[\frac{\eta\rho v^2 - (\lambda + 2\mu + \eta P)}{(\mu + \frac{1}{2}\eta P)} + \frac{\eta\rho v^2 - (\mu - \frac{1}{2}\eta P)}{(\lambda + 2\mu)} + \frac{(\lambda + \mu + \frac{1}{2}P)^2}{(\mu + \frac{1}{2}P)(\lambda + 2\mu)} \right] q^2 + \frac{[\rho v^2 \eta - \eta(\lambda + 2\mu + \eta P)][\eta\rho v^2 - \eta(\mu - \frac{1}{2}\eta P)]}{(\mu + \frac{1}{2}\eta P)(\lambda + 2\mu)} = 0. \qquad ...(7)$$

If q_1^2 and q_2^2 be the roots of equation (7), then u and w can be written as

$$u = \int_0^\infty [\{A_1 e^{-kq_1z} + A_2 e^{-kq_2z}\} \cos k(x - vt) + \{B_1 e^{-kq_1z} + B_2 e^{-kq_2z}\} \sin k(x - vt)]\, dk \qquad ...(8)$$

$$w = \int_0^\infty [\{m_1 B_1 e^{-kq_1z} + m_2 B_2 e^{-kq_2z}\} \cos k(x - vt) - \{m_1 A_1 e^{-kq_1z} + m_2 A_2 e^{-kq_2z}\} \sin k(x - (vt)]\, dk \qquad ...(9)$$

where

$$m_{1,2} = \frac{(\lambda + 2\mu + \eta P) - \rho\eta v^2 - (\mu + \frac{1}{2}\eta P)\, q_{1,2}^2}{(\lambda + \mu + \frac{1}{2}\eta P)\, q_{1,2}} \cdot$$

Equation (7) is of the form $x^2 + bx + c = 0$. It is observed by calculation that $b^2 - 4c > 0$ and in order that q_1^2 and q_2^2 may both be positive, the expression $-b \pm\sqrt{b_2 - 4c} > 0$, which leads to the condition that $\eta\rho\, v^2$ should be less than both $[\lambda + 2\mu + \eta P]$ ($= T_1$, say) and $\left[\mu - \frac{\eta P}{2}\right]$ ($= T_2$, say) and $[\{\rho v^2\eta - (\lambda + 2\mu + P\eta)\} \times \{v^2\eta\rho - (\mu - \frac{1}{2}\eta P)\}]$ ($= c$, say) should be positive. If q_1^2 and q_2^2 are both negative then $\rho\, v^2\, \eta$ should be greater than both T_1 and T_2 and also c is greater than zero. In this case q_1 and q_2 both are imaginary. In the isotropic case ($\eta = 1$, $S_{11} = S_{33} = 0$), in order that q_1^2 and q_2^2 may be real, the velocity of moving load should be less than shear wave β or should be greater than α. When $\rho\, v^2\eta$ lies between T_1 and T_2, q_1^2 q_2^2 becomes negative. So either q_1^2 or q_2^2 becomes negative in that case, i. e. either q_1 or q_2 is imaginary. The above three cases are known as subsonic, supersonic and transonic case respectively. We shall discuss all the three cases in this paper.

Inserting (8) and (9) in boundary conditions (4, 5) we get the four equations for the constants A_1 and A_2, B_1 and B_2 from which the constants are obtained as

$$A_1 = \frac{2FR\eta}{\pi k}\left[\frac{(\lambda + 2\mu)\, m_2 q_2 - (\lambda + S_{33})}{\Delta^*}\right]$$

$$A_2 = -\frac{2FR\eta}{\pi k}\left[\frac{(\lambda + 2\mu)\, m_1 q_1 - (\lambda + S_{33})}{\Delta^*}\right]$$

$$B_1 = -\frac{F\eta}{\pi k}\left[\frac{(2\mu - S_{33} - S_{11})\, m_2 + (2\mu + P)\, q_2}{\Delta^*}\right]$$

$$B_2 = \frac{F\eta}{\pi k}\left[\frac{(2\mu - S_{33} - S_{11})\, m_1 + (2\mu + P)\, q_1}{\Delta^*}\right] \quad \ldots(10)$$

where

$$\Delta^* = [(2\mu - S_{33} - S_{11})\, m_1 + (2\mu + P)\, q_1][(\lambda + 2\mu)\, m_2\, q_2 - (\lambda + S_{33})] - [(2\mu - S_{33} - S_{11})\, m_2 + (2\mu + P)\, q_2][(\lambda + 2\mu)\, m_1\, q_1 - (\lambda + S_{33})]. \quad \ldots(11)$$

ANALYSIS

Case I Subsonic case $\left(\rho\, v^2 < \frac{1}{\eta}\left(\mu - \frac{\eta P}{2}\right)\right)$

In this case $\rho\, v^2\eta$ is less than both T_1 and T_2 and both the roots q_1 and q_2 are real. In the isotropic case ($\eta = 1$, $S_{11} = S_{33} = 0$) this corresponds to the case that the source moves with a velocity less than velocity of shear waves.

Thus, from (1) using (8), (9), (10) and (11), the expressions for the stresses S_{13} and S_{33} may be written as

$$S_{13} = \frac{F\mu}{\pi\Delta^*}\left[\left\{\frac{M_1 G_2}{Q_1} - \frac{M_2 G_1}{Q_2}\right\}(x - vt) + 2R\left\{\frac{P_1 M_2 q_2 z}{Q_2} - \frac{P_2 M_1 q_1 z}{R_1}\right\}\right] \quad \ldots(12)$$

and

$$S_{33} = \frac{F}{\pi\Delta^*}\left[\frac{P_5 G_1 q_1 z}{Q_1} - \frac{P_6 G_1 q_2 z}{Q_2} + 2R\left\{\frac{P_5 P_2}{Q_1} - \frac{P_6 P_1}{Q_2}\right\}(x - v)\right] \quad \ldots(13)$$

where

$$M_1 = m_1 + q_1,\ M_2 = m_2 + q_2$$

$$Q_1 = (x - vt)^2 + q_1^2 z^2,\ Q_2 = (x - vt)^2 + q_2^2 z^2$$

$$G_1 = P_3 m_1 + P_4 q_1,\ G_2 = P_3 m_2 + P_4 q_2$$

$$P_1 = (\lambda + 2\mu) m_1 q_1 - (\lambda + S_{33}),\ P_2 = (\lambda + (2\mu) m_2 q_2 - (\lambda + S_{33})$$

$$P_3 = (2\mu - S_{33} - S_{11}),\ P_4 = (2\mu + P),\ P_5 = (\lambda + 2\mu) m_1 q_1 - \lambda,$$

$$P_6 = (\lambda + 2\mu) m_2 q_2 - \lambda.$$

$\Delta^* = 0$ is the frequency equation of the Rayleigh waves in the medium under considered initial stresses. Hence, when the velocity of the moving load coincides with the velocity of Rayleigh waves obtained from $\Delta^* = 0$, the stresses will be infinitely large and fracture will take place in the medium

In the absence of initial stresses (i. e. $S_{33} = S_{11} = 0$) when $\eta = 1$, the developed incremental stresses given by (12) and (13) reduce for smooth surface (i. e. $R \to 0$) and the results are seen to coincide with the results obtained by Cole and Huth[3]. The stresses at $x = vt$ i. e., at the point directly below the load may be obtained as

$$s_{13} = \frac{FR}{\pi z \frac{\Delta}{4\mu^2}}\left[\frac{(m_2 + q_2)\{(\theta + 1\} m_1 q_1 - (\theta + I_2)\}}{q_2} - \frac{(m_1 + q_1)\{(\theta + 1) m_2 q_2 - (\theta + I_2)\}}{q_1}\right] \quad \ldots(14)$$

and

$$s_{33} = \frac{F}{\pi z \frac{\Delta^*}{4\mu^2}}\left[\frac{\{(\theta + 1) m_1 q_1 - \theta\}\{(1 - I_2 - I_1) m_2 + (1 + I_2 - I_1) q_2\}}{q_1} - \frac{\{(\theta + 1) m_2 q_2 - \theta\}\{(1 - I_2 - I_1) m_1 (1 + I_2 - I_1) q_1\}}{q_2}\right] \quad \ldots(15)$$

where

$$\theta = \lambda/2\mu,\ I_1 = S_{11}/2\mu,\ I_2 = S_{33}/2\mu.$$

Case II—Transonic case : [i. e. when $\frac{I}{\eta}\left(\mu - \frac{\eta P}{2}\right)\frac{(= T_1)}{\eta} < \rho v^2 < \frac{1}{\eta}(\lambda + 2\mu + \eta P)\frac{(= T_2)}{\eta}$]

Next, we consider the case when $\rho v^2 \gamma$ lies between T_1 and T_2. In the isotropic case this corresponds to the case that the source moves with a velocity greater than the velocity of shear waves but less than the velocity of longitudinal waves.

In this case q_2 is imaginary and q_1 is real. Here, we replace q_2 by iq_2' where q_2' is real. The developed incremental stresses S_{13} and S_{33} are given by

$$s_{13} = \frac{F\mu}{\pi\Delta^*}\Big[M_1 G_2 \frac{x - vt}{Q_1} - \frac{M_2 G_1}{2i}\Big\{2\pi\,\delta_+\{(x - vt) - q_2' z\} - 2\pi\,\delta_+\{-(x - vt) - q_2' z\}\Big\} + 2R\Big\{- \frac{P_2 M_1 q_1 z}{Q_1} + \frac{P_1 M_2}{2}[2\pi\,\delta_+\{(x-vt) - q_2' z\} + 2\pi\,\delta_+\{-(x - vt) - q_2' z\}]\Big\}\Big] \quad ...(16)$$

$$s_{33} = \frac{F}{\pi\Delta^*}\Big[\frac{P_5 G_2 q_1 z}{Q_1} - \frac{P_6 G_1}{2}\Big\{2\pi\,\delta_+\{(x - vt) - q_2' z\} + 2\pi\,\delta_+\{-(x - vt) - q_2' z\}\Big\} + 2R\Big\{\frac{P_2 P_5 (x - vt)}{Q_1} - \frac{P_1 P_6}{2i}[2\pi\,\delta_+\{(x - vt) - q_2' z\} - 2\pi\,\delta_+\{(x - vt) - q_2' z\}]\Big\}\Big] \quad ...(17)$$

where, $\delta_+(x)$ is the Heisenberg delta function and is given by

$$\delta_+(x) = \delta(x),\ x = 0$$
$$= -\frac{1}{2\pi ix},\ x \neq 0.$$

The Heisenberg delta functions within the brackets may be written in explicit form as

$$[2\pi\,\delta_+\{(x - vt) - q_2' z\} - 2\pi\,\delta_+\{-(x - vt) - q_2' z\}]$$

(*equation continued on p.* 172)

$$= [2\pi\,\delta\,\{(x - vt) - q_2'\,z\} + 2\pi\,\delta\,\{(x - vt) + q_2'\,z\}$$

$$+ \frac{i}{(x-vt) - q_2'\,z}\,[(x - vt) \neq q_2'\,z] - \frac{i}{(x-vt) + q_2'\,z}$$

$$\times\,[(x - vt) \neq -\,q_2'\,z]]$$

and from equation (7) q_2' may be written as

$$q_2' = \left[\frac{b + \sqrt{b^2 - 4c}}{2}\right]^{1/2} \qquad \ldots(18)$$

$$b = \frac{\frac{v^2}{\beta^2} - \left(\frac{\theta}{\eta} + \frac{2}{\eta} + 2I_2 - 2I_1\right)}{\left(\frac{1}{\eta} + I_2 - I_1\right)} + \frac{\eta\,\frac{v^2}{\beta^2} - \eta\left(\frac{1}{\eta} - I_2 + I_1\right)}{(\theta + 2)} + \frac{(\theta + 1 + I_2 - I_1)^2}{(1 + I_2 - I_1)\,(\theta + 2)}$$

$$c = \frac{\eta\left[\frac{v^2}{\beta^2} - \left(\frac{\theta}{\eta} + \frac{2}{\eta} + 2I_2 - 2I_1\right)\right]\left[\frac{v^2}{\beta^2} - \left(\frac{1}{\eta} - I_2 + I_1\right)\right]}{\left(\frac{1}{\eta} + I_2 - I_1\right)(\theta + 2)}. \qquad \ldots(19)$$

From expression (17) with the help of (18), it is observed that when $\frac{1}{\eta}\left(\mu - \frac{\eta P}{2}\right) < \rho v^2 < \frac{1}{\eta}\,(\lambda + 2\mu + P\eta)$, s_{33} becomes infinite along the lines $(x - vt) = \pm\,q_2'\,z$. Hence crakcs are produced along these lines. From (19) it is seen that the slope of the crack lines q_2' depends on the initial stress parameters I_1 and I_2. However, the initial stresses have no effect on the cracklines when $I_1 = I_2$ i. e. the initial stresses are hydrostatic in nature. It is easy to calculate the slope of the cracklines from equation (19) for particular values of the initial stress parameters and at any time t.

Case III—Supersonic case : $\rho\,v^2 > \frac{1}{\eta}\,(\lambda + 2\mu + P\eta)\,\frac{(= T_2)}{\eta}$

Consider the case when $\rho\,v^2\eta$ is greater than T_2. In the classical case ($\eta = 1$, $I_1 = I_2 = 0$) this corresponds to the velocity of source is greater than the shear wave as well as P-wave velocity in the medium.

In this case q_1 and q_2 both are imaginary. Replacing q_1 and q_2 by iq_1' and iq_2'

respectively, where q_1' and q_2' are real, the expressions for s_{13} and s_{33} are obtained as

$$s_{13} = \frac{F\mu}{\pi\Delta^*}\Big[\frac{M_1 G_2}{2i}\{2\pi\,\delta_+(x-vt) - q_1' z\} - 2\pi\,\delta_+\{-(x-vt) - q_1' z\}\} - \frac{M_2 G_1}{2i}\{2\pi\,\delta_+(x-vt) - q_2' z\} - 2\pi\,\delta_+\{-(x-vt) - q_2' z\}\} + 2R\Big\{\frac{P_1 M_2}{2}[2\pi\,\delta_+\{(x-vt) - q_2' z\} + 2\pi\,\delta_+\{(x-vt) - q_2' z\}] - \frac{P_2 M_1}{2}[2\pi\,\delta_+\{(x-vt) - q_1' z\} + 2\pi\,\delta_+\{-(x-vt) - q_1' z\}]\Big\}\Big] \quad \ldots(20)$$

and

$$s_{33} = \frac{F}{\pi\Delta^*}\Big[\frac{P_5 G_2}{2}\{2\pi\,\delta_+\{(x-vt) - q_1' z\} + 2\pi\,\delta_+\{-(x-vt) - q_1' z\}\} - \frac{P_6 G_1}{2}\{2\pi\,\delta_+\{(x-vt) - q_2' z\} + 2\pi\,\delta_+\{-(x-vt) - q_2' z\}\} + 2R\Big\{\frac{P_2 P_5}{2i}[2\pi\,\delta_+\{(x-vt) - q_1' z\} - 2\pi\,\delta_+\{-(x-vt) - q_1' z\}] - \frac{P_1 P_6}{2i}\Big[2\pi\,\delta_+\{(x-vt) - q_2' z\} - 2\pi\,\delta_+\{-(x-vt) - q_2' z\}]\Big\}\Big] \quad \ldots(21)$$

where q_1' as obtained from (7) is given by

$$q_1' = \left[\frac{b - \sqrt{b^2 4c}}{2}\right]^{1/2}. \quad \ldots(22)$$

From eqn. (21) it is observed that when the point source moves with a velocity greater than $\left[\frac{\frac{1}{\eta}(\lambda + 2\mu + \eta P)}{\rho}\right]^{1/2}$, then four cracks are produced along the lines $(x - vt) = \pm q_{1,2}' z$ instead of two cracks as obtained in case II. Beyond the crack-lines the stresses may be calculated from equation (21) using the definition of Heisenberg delta function. These cracklines will be very much affected by the presence of initial stresses because their slopes $\pm q_{1,2}'$ are functions of initial stresses present in medium. The slopes of cracklines may be calculated from (18) and (22) for different values of initial stress parameters and elastic constants.

Numerical Calculation and Discussions

The values of s_{33}/F and s_{13}/FR have been calculated (ICL 1901A) for subsonic case when $z = 100$ from eqns. (14) and (15) (in dimensionless form) in elastic ($\eta = 1.0$) and sandy materials ($\eta = 1.5$) taking $\lambda = \mu$, as a particular case, due to increasing velocity of moving load in presence of pre-comperssive stress along x-direction ($I_1 = 0.0$, -0.2, -0.4, -0.6) and pre-tensile stress along z-direction ($I_2 = 0.0$, 0.2, 0,4, 0.6, 0.8) and also for th e case free from pre-stresses to facilitate the comparison. These results have been presented in Tables I and II. It is observed that the stresses produced in elastic medium is always less than that in sandy soil.

Table I

Values of normal stresses and shear stresses due to increasing velocity of moving load in an elastic as well as sandy half space under compressive stress along x-direction.

$z = 100$, $I_2 = 0.0$

I_1	v/β	s_{33}/F		s_{13}/FR	
		$\eta = 1.0$	$\eta = 1.5$	$\eta = 1.0$	$\eta = 1.5$
(1)	(2)	(3)	(4)	(5)	(6)
0.0	0.1	0.641×10^{-2}	0.643×10^{-2}	0.242×10^{-4}	0.366×10^{-4}
0.0	0.2	0.655×10^{-2}	0.555×10^{-2}	0.101×10^{-3}	0.156×10^{-3}
0.0	0.3	0.681×10^{-2}	0.707×10^{-2}	0.246×10^{-3}	0.396×10^{-3}
0.0	0.4	0.723×10^{-2}	0.781×10^{-2}	0.490×10^{-3}	0.850×10^{-3}
0.0	0.5	0.790×10^{-2}	0.920×10^{-2}	0.903×10^{-3}	0.178×10^{-2}
0.0	0.6	0.902×10^{-2}	0.124×10^{-1}	0.165×10^{-2}	0.421×10^{-2}
0.0	0.7	0.111×10^{-1}	0.275×10^{-1}	0.321×10^{-2}	0.177×10^{-1}
0.0	0.8	0.167×10^{-1}	infinitely large	0.778×10^{-2}	infinitely large
0.0	0.9	0.761×10^{-1}	infinitely large	0.650×10^{-1}	infinitely large
−0.2	0.1	0.833×10^{-2}	0.101×10^{-1}	0.122×10^{-2}	0.220×10^{-2}
−0.2	0.2	0.862×10^{-2}	0.108×10^{-1}	0.139×10^{-2}	0.265×10^{-2}
−0.2	0.3	0.918×10^{-2}	0.124×10^{-1}	0.172×10^{-2}	0.365×10^{-2}
−0.2	0.4	0.101×10^{-1}	0.162×10^{-1}	0.232×10^{-2}	0.616×10^{-2}
−0.2	0.5	0.119×10^{-1}	0.307×10^{-1}	0.346×10^{-2}	0.165×10^{-2}
−0.2	0.6	0.156×10^{-1}	infinitely large	0.608×10^{-2}	infinitely large
−0.2	0.7	0.284×10^{-1}	infinitely large	0.158×10^{-1}	infinitely large
−0.4	0.1	0.127×10^{-1}	0.361×10^{-1}	0.368×10^{-2}	0.171×10^{-1}
−0.4	0.2	0.136×10^{-1}	0.578×10^{-1}	0.420×10^{-2}	0.305×10^{-1}
−0.4	0.3	0.154×10^{-1}	infinitely large	0.534×10^{-2}	infinitely large
−0.4	0.4	0.194×10^{-1}	—do—	0.789×10^{-2}	—do—
−0.4	0.5	0.311×10^{-1}	—do—	0.158×10^{-1}	—do—
−0.4	0.6	0.209	—do—	0.207	—do—
−0.6	0.1	0.347×10^{-1}	—do—	0.163×20^{-1}	—do—
−0.6	0.2	0.454×10^{-1}	—do—	0.228×10^{-1}	—do—
−0.6	0.3	0.192	—do—	0.579×10^{-1}	—do—

TABLE II

Values of normal stresses and shear stresses due to increasing velocity of moving load in an elastic as well as sandy half space under tensile stress along z-direction

$z = 100$, $I_1 = 0.0$

I_2	v/β	s_{33}/F		s_{13}/FR	
		$\eta = 1.0$	$\eta = 1.5$	$\eta = 1.0$	$\eta = 1.5$
(1)	(2)	(3)	(4)	(5)	(6)
0.2	0.1	0.653×10^{-2}	0.686×10^{-2}	0.244×10^{-4}	0.391×10^{-4}
0.2	0.2	0.670×10^{-2}	0.714×10^{-2}	0.103×10^{-3}	0.170×10^{-3}
0.2	0.3	0.698×10^{-2}	0.769×10^{-2}	0.251×10^{-3}	0.445×10^{-3}
0.2	0.4	0.746×10^{-2}	0.874×10^{-2}	0.510×10^{-3}	0.102×10^{-2}
0.2	0.5	0.824×10^{-2}	0.110×10^{-1}	0.968×10^{-3}	0.244×10^{-2}
0.2	0.6	0.962×10^{-2}	0.200×10^{-1}	0.186×10^{-2}	0.900×10^{-2}
0.2	0.7	0.126×10^{-1}	infinitely large	0.407×10^{-2}	infinitely large
0.2	0.8	0.242×10^{-1}	—do—	0.148×10^{-1}	—do—
0.4	0.1	0.732×10^{-2}	0.918×10^{-2}	0.284×10^{-4}	0.601×10^{-4}
0.4	0.2	0.754×10^{-2}	0.983×10^{-2}	0.121×10^{-3}	0.276×10^{-3}
0.4	0.3	0.795×10^{-2}	0.113×10^{-1}	0.303×10^{-3}	0.816×10^{-3}
0.4	0.4	0.866×10^{-2}	0.150×10^{-1}	0.640×10^{-2}	0.253×10^{-2}
0.4	0.5	0.994×10^{-2}	0.512×10^{-1}	0.131×10^{-2}	0.351×10^{-1}
0.4	0.6	0.126×10^{-1}	infinitely large	0.295×10^{-2}	infinitely large
0.4	0.7	0.222×10^{-1}	—do—	0.107×10^{-1}	infinitely large
0.6	0.1	0.912×10^{-2}	0.219×10^{-4}	0.395×10^{-4}	0.269×10^{-3}
0.6	0.2	0.952×10^{-2}	0.323×10^{-1}	0.173×10^{-3}	0.232×10^{-2}
0.6	0.3	0.103×10^{-1}	infinitely large	0.458×10^{-3}	infinitely large
0.6	0.4	0.119×10^{-1}	—do—	0.108×10^{-2}	—do—
0.6	0.5	0.155×10^{-1}	—do—	0.283×10^{-2}	—do—
0.6	0.6	0.359×10^{-1}	—do—	0.186×10^{-1}	—do—
0.8	0.1	0.139×10^{-1}	—do—	0.789×10^{-4}	—do—
0.8	0.2	0.152×10^{-1}	—do—	0.378×10^{-3}	—do—
0.8	0.3	0.185×10^{-1}	—do—	0.126×10^{-2}	—do—
0.8	0.4	0.317×10^{-1}	—do—	0.643×10^{-2}	—do—

TABLE III

Critical values of v/β at which $\Delta^* = 0$

I_1	I_2	v/β	
		$\eta = 1.0$	$\eta = 1.5$
−0.4	0.0	0.6098	0.2920
−0.2	0.0	0.7838	0.5721
0.0	0.0	0.3187	0.7505
0.0	0.2	0.8625	0.6650

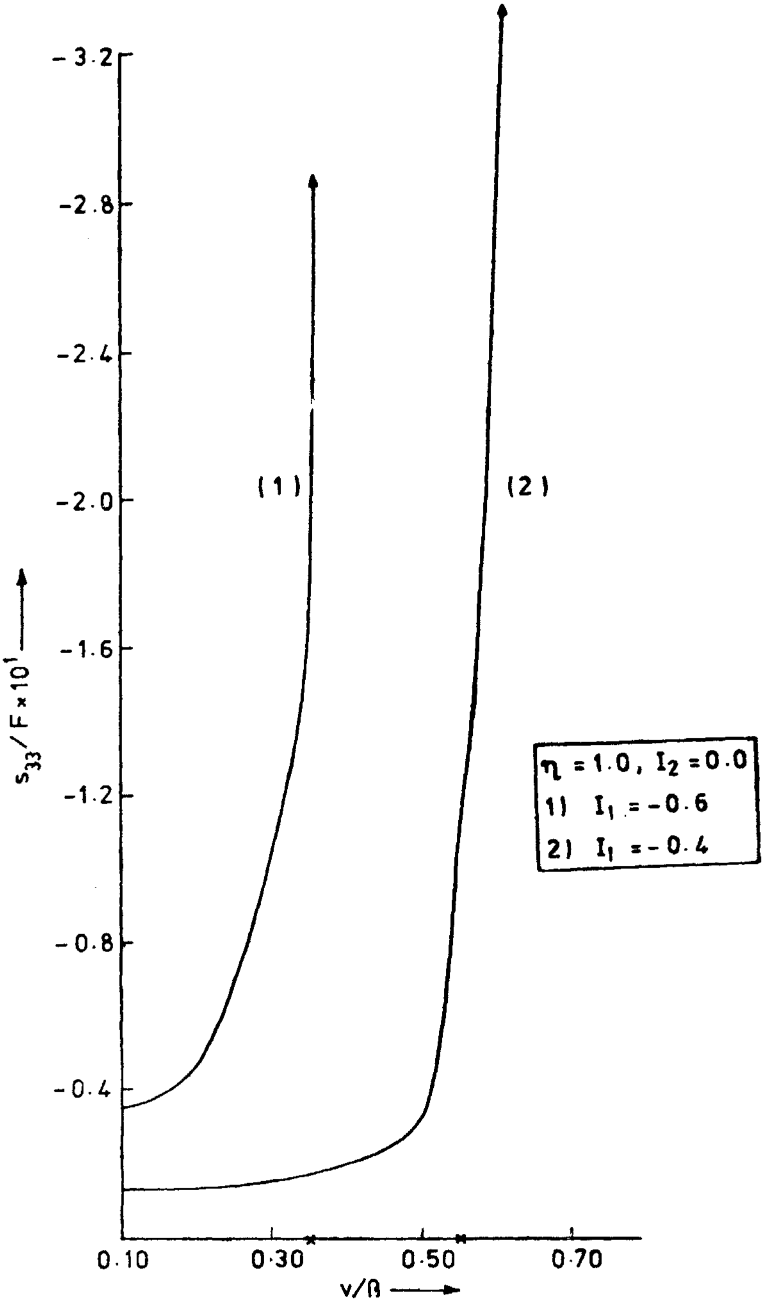

FIG. 1. Development of normal stress s_{33} in elastic half space (η = 1.0) under compressive pre-stresses along the x-direction.

The critical values of v/β have also been calculated from $\Delta^* = 0$ at which the stresses developed will be infinitely large creating fracture in the medium. The critical

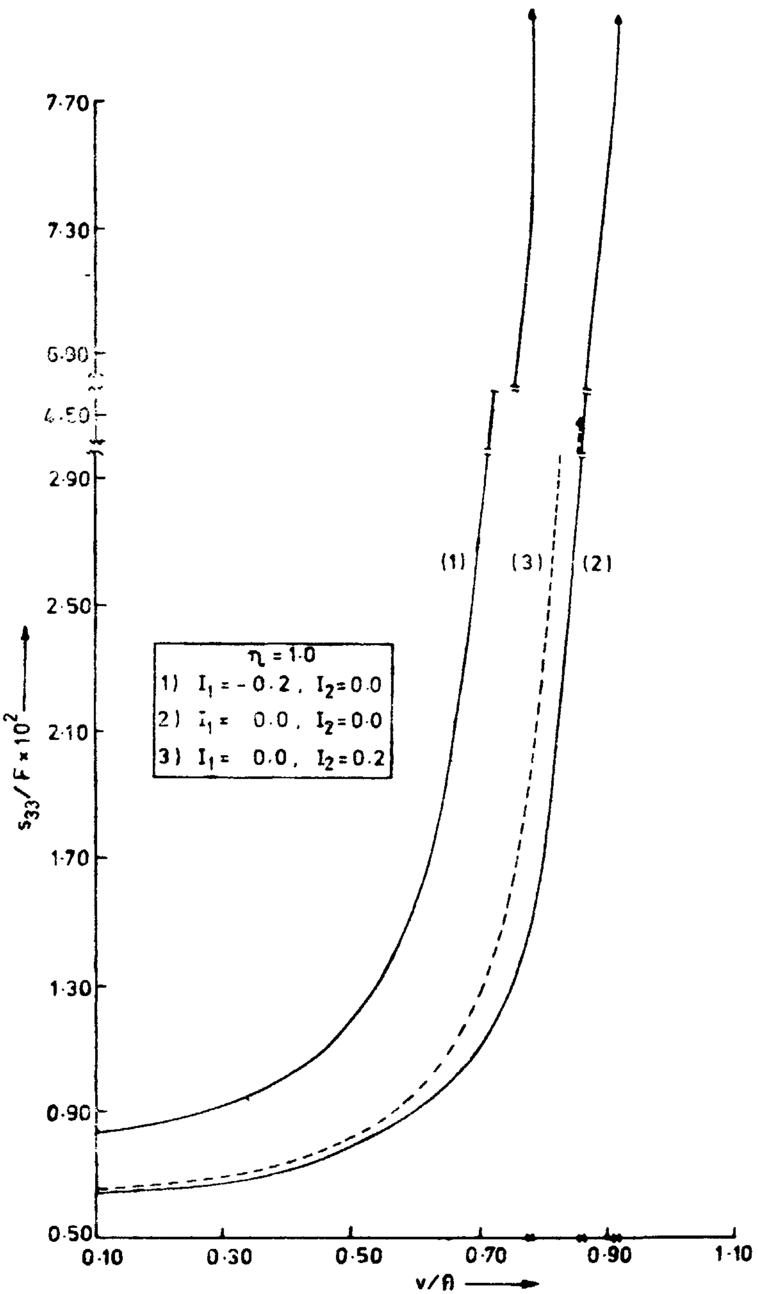

FIG. 2. A comparison in the development of normal stresses in pre-stressed and free from pre-stresses half space due to increasing velocity of moving load.

velocities of moving load giving rise to fracture for a few values of pre-stress parameters in elastic and sandy material have been presented in Table III. It is found that

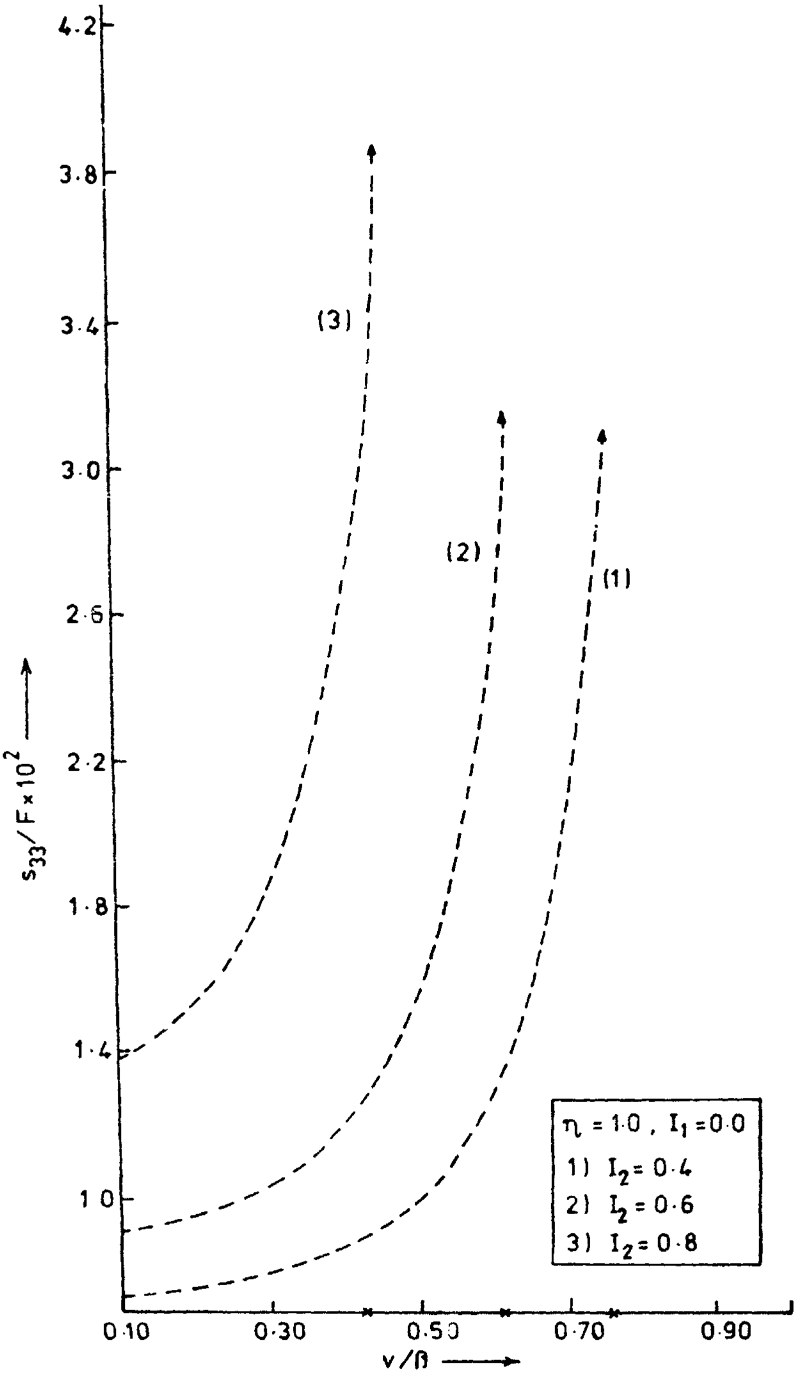

FIG. 3. Development of normal stresses in an elastic half space moderate and high tensile-stress z-direction.

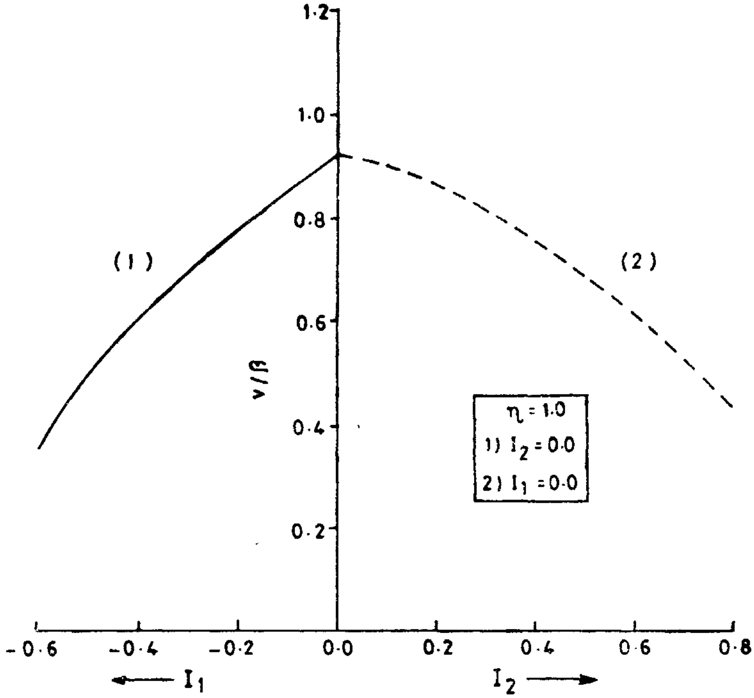

FIG. 4. Critical velocities of the moving load creating fracture in a pre-stressed elastic half space.

the sandy material gets fractured at lesser velocity of moving load in comparison with the elastic material i. e. sandy materials are less stable to moving load than the elastic one.

Figures 1-3 to show the nature of the curves of the normal stresses (s_{33}) developed in a pre-stressed elastic halfspace due to increasing velocities of moving load for compressive stress along x-directions and tensile stress along z-direction. It may be that the curves are asymptotic for certain values of v/β depending upon the pre-stress parameters I_1 and I_2 suggesting that the stresses are very high creating instability in the medium causing fracture at the corresponding critical velocity of the moving load.

Observing Figs. 1 and one infers that high pre-comp ressive stresses along x-direction will cause the instability at lesser velocity of moving load than the classical case. From Figs. 2 and 3 the same remarks can be made for the case when the elastic medium is under pre-tensile stress along z-direction. Also from the above figures it is clear that as the magnitude of compressive stress along x-direction or tensile stress along z-direction increases the fracture takes place at less and velocities of the moving load.

Figure 4 represents the curves for the critical values of v/β at which the stresses developed in a pre-stressed elastic material will be infinitely large creating fracture in the medium.

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